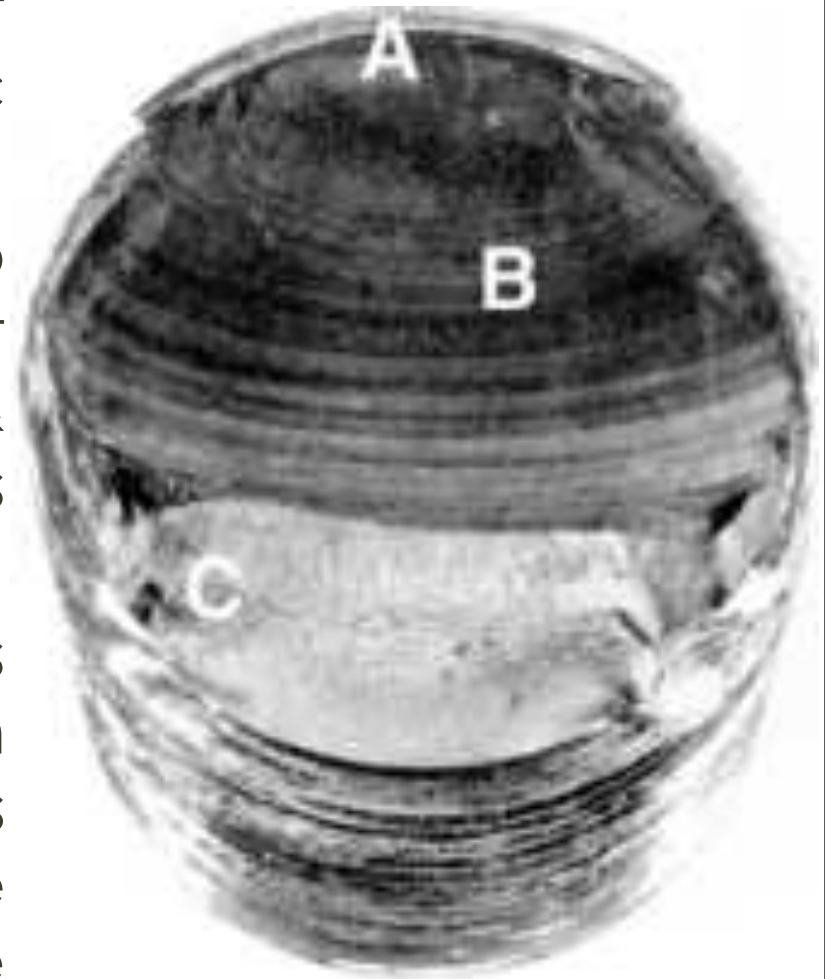


# Introduction to Fatigue in Metals

- ❑ Loading produces stresses that are variable, repeated, alternating, or fluctuating
- ❑ Maximum stresses well below yield strength
- ❑ Failure occurs after many stress cycles
- ❑ Failure is by sudden ultimate fracture
- ❑ No visible warning in advance of failure

# Stages of Fatigue Failure

- ❑ **Stage I:** Initiation of micro-crack due to cyclic plastic deformation
- ❑ **Stage II:** Progresses to macro-crack that repeatedly opens & closes, creating bands called *beach marks*
- ❑ **Stage III:** Crack has propagated far enough that remaining material is insufficient to carry the load, and fails by simple ultimate failure



# Fatigue Fracture Example

- ❑ AISI 4320 drive shaft
- ❑ B: crack initiation at stress concentration in keyway
- ❑ C: Final brittle failure



# Fatigue-Life Methods

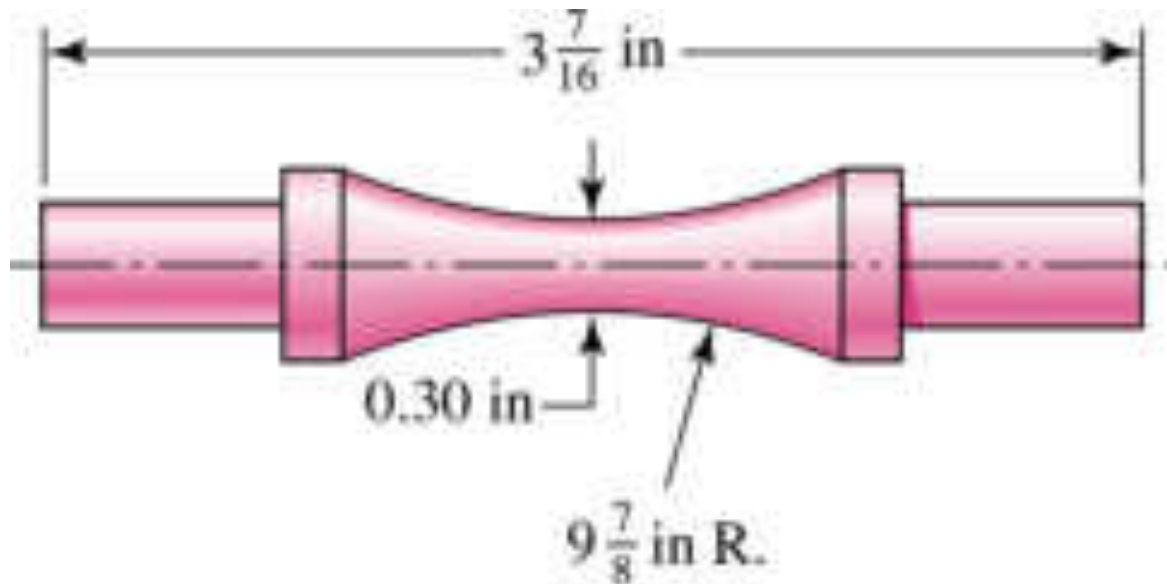
- Three major fatigue life models
- Methods predict life in number of cycles to failure,  $N$ , for a specific level of loading

# Fatigue-Life Methods

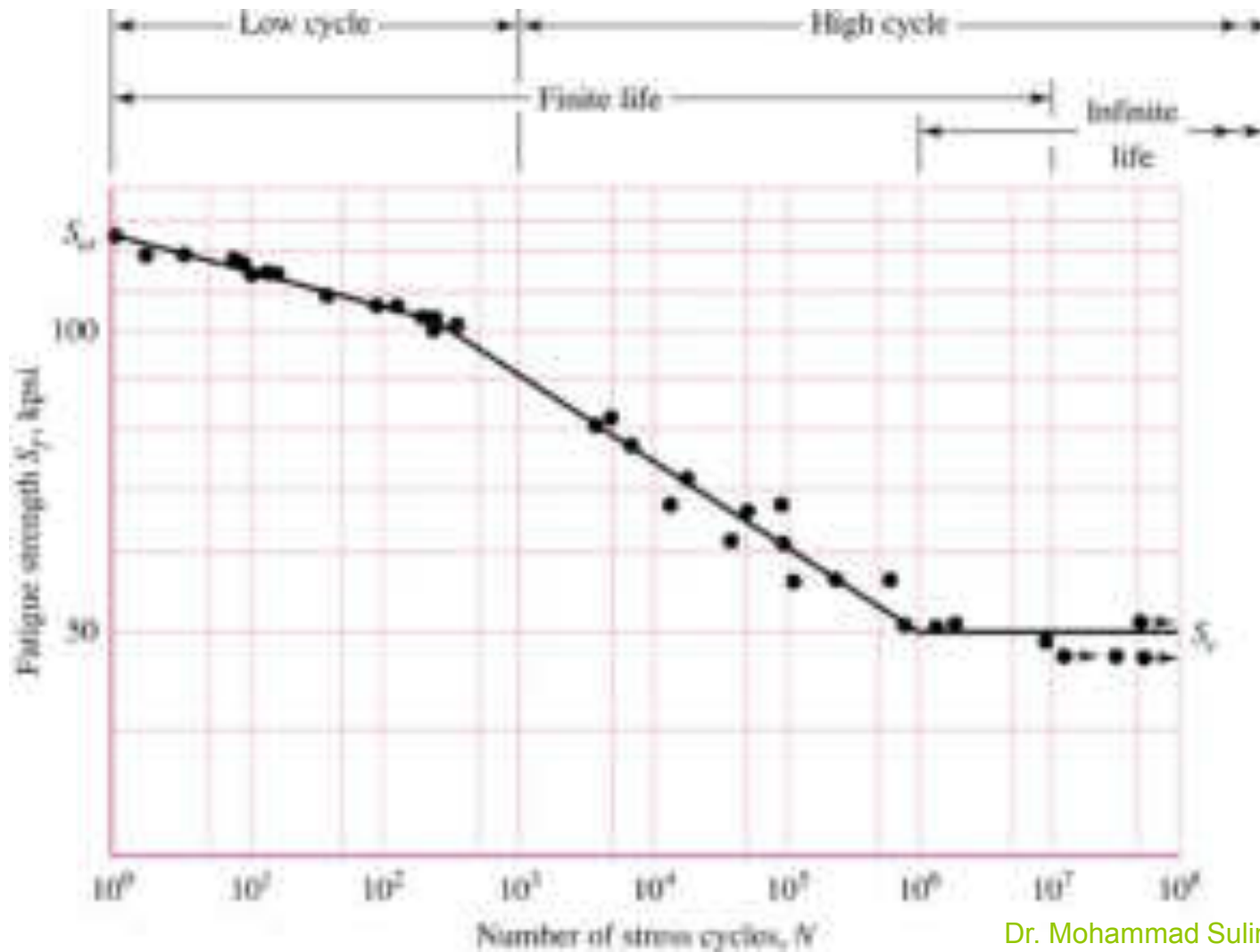
1. **Stress-life method**
2. **Strain-life method**
3. **Linear-elastic fracture mechanics method**

# 1. Stress-Life Method

- ❑ Test specimens subjected to repeated stress while counting cycles to failure
- ❑ Pure bending with no transverse shear
- ❑ *completely reversed* stress cycling



# S-N Diagram



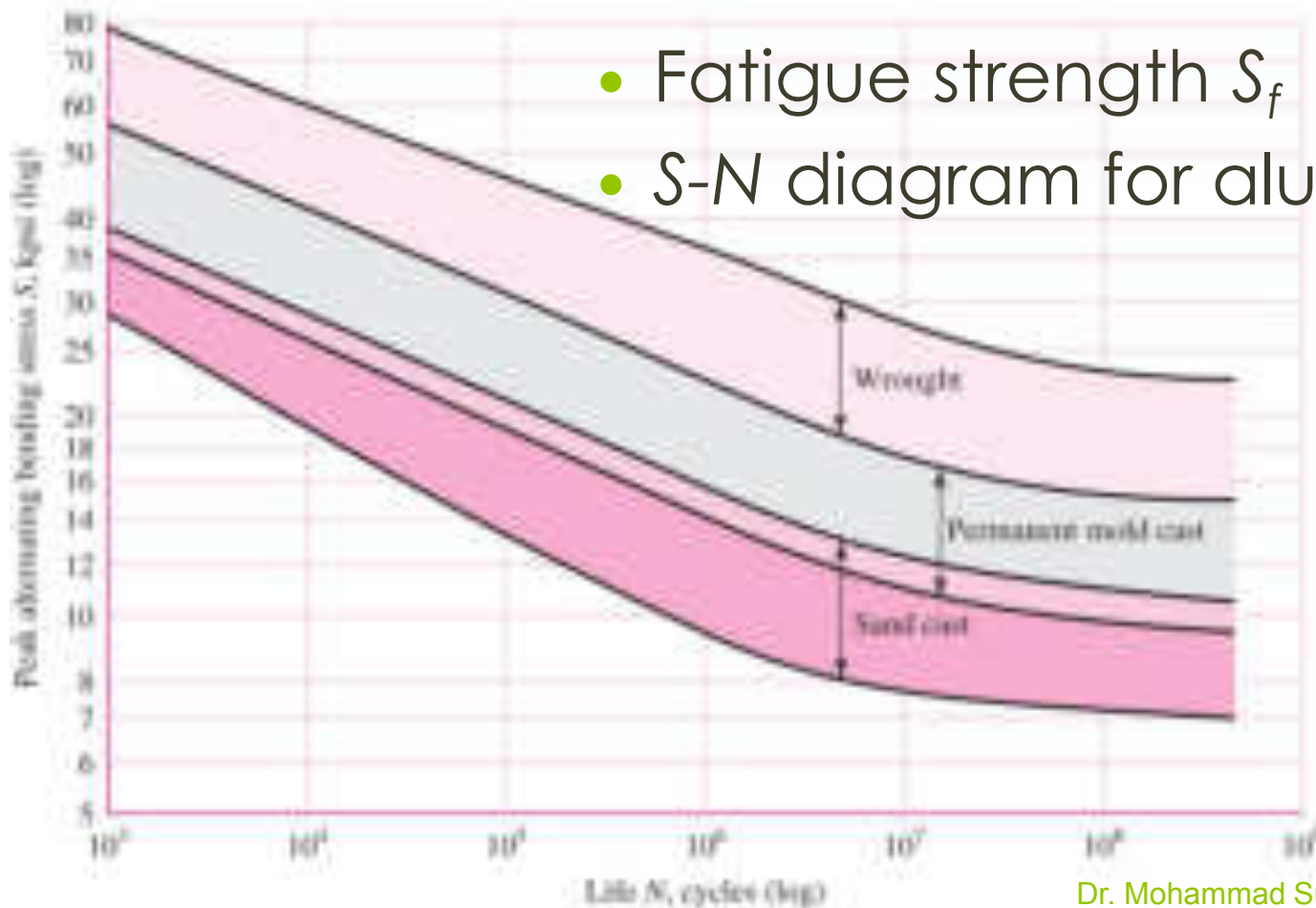
# S-N Diagram for Steel

- ❑ Stress levels below  $S_e$  predict infinite life
- ❑  $10^3$  to  $10^6$  cycles: finite life
- ❑ Below  $10^3$  cycles: *low cycle*
  - ✓ quasi-static
  - ✓ Yielding usually occurs before fatigue



# S-N Diagram for Nonferrous Metals

- no endurance limit
- Fatigue strength  $S_f$
- S-N diagram for aluminums

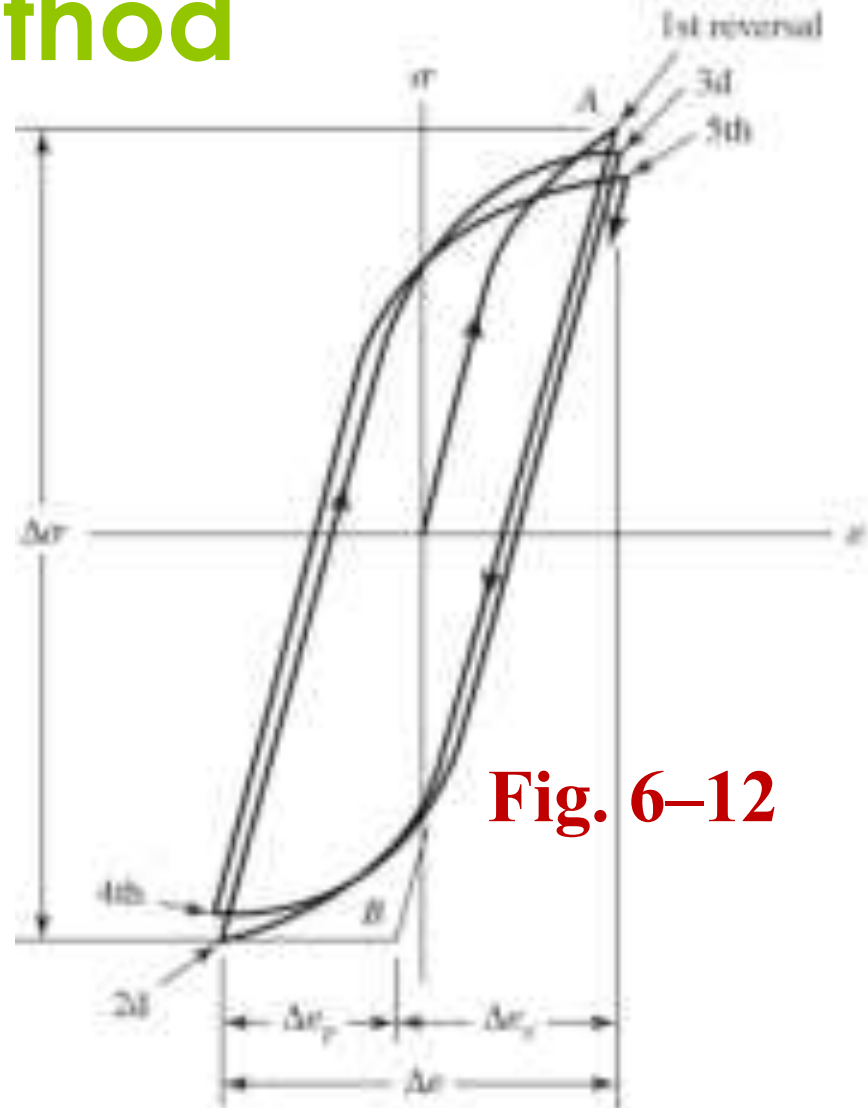


## 2. Strain-Life Method

- ❑ Detailed analysis of plastic deformation at localized regions
- ❑ Compounding of several idealizations leads to significant uncertainties in numerical results
- ❑ Useful for explaining nature of fatigue

## 2. Strain-Life Method

- ❑ Fatigue failure begins at a local discontinuity
- ❑ When stress at discontinuity exceeds elastic limit, plastic strain occurs
- ❑ Cyclic plastic strain can change elastic limit, leading to fatigue

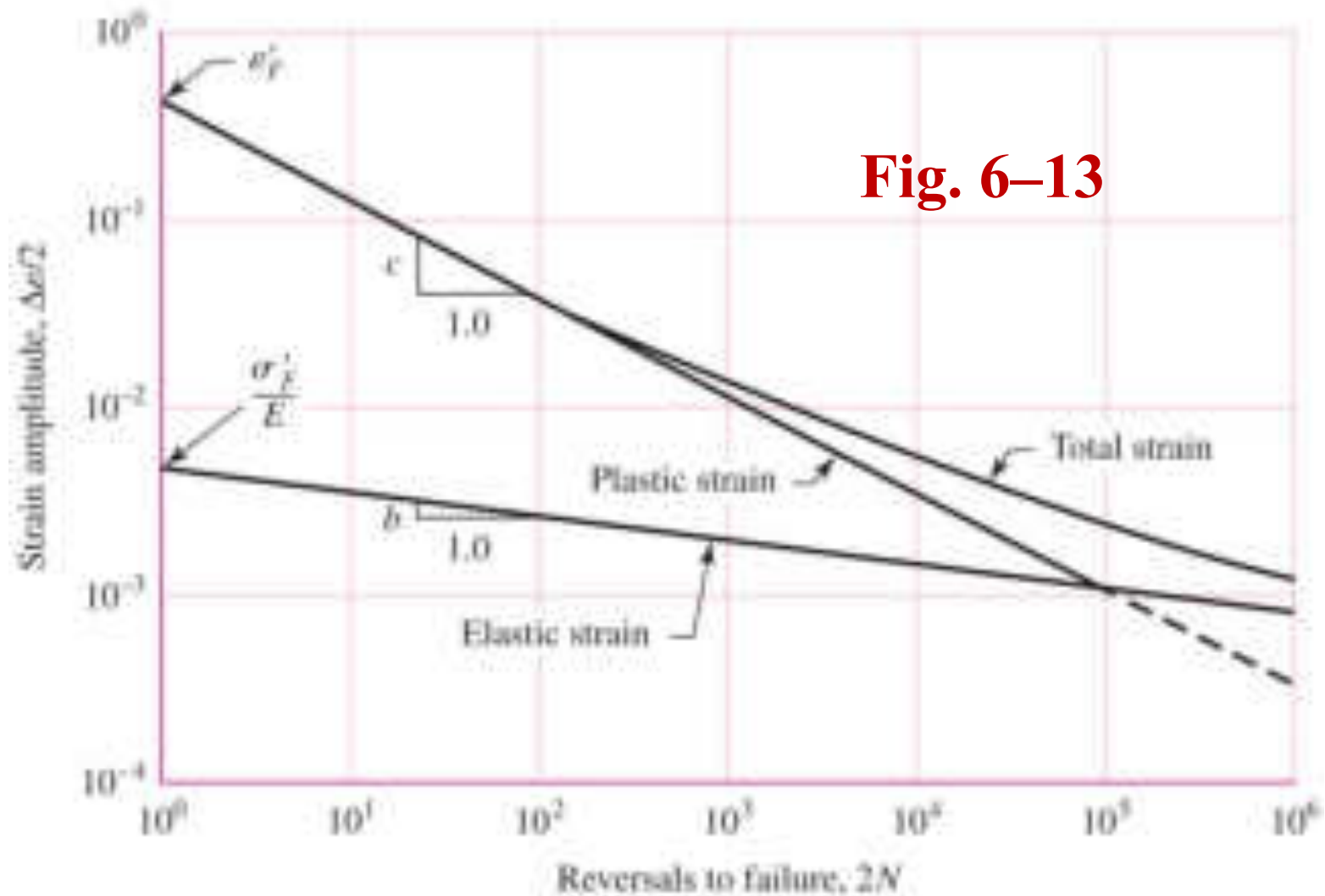


**Fig. 6-12**

# Relation of Fatigue Life to Strain

- ❑ **Figure 6–13:** relationship of fatigue life to true-strain amplitude
- ❑ **Fatigue ductility coefficient  $\epsilon'_F$**  = true strain at which fracture occurs in one reversal (point A in Fig. 6–12)
- ❑ **Fatigue strength coefficient  $\sigma'_F$**  = true stress corresponding to fracture in one reversal (point A in Fig. 6–12)

# Relation of Fatigue Life to Strain



## Relation of Fatigue Life to Strain

- Equation of plastic-strain line in Fig. 6–13

$$\frac{\Delta \varepsilon_p}{2} = \varepsilon'_p (2N)^c \quad (6-1)$$

- Equation of elastic strain line in Fig. 6–13

$$\frac{\Delta \varepsilon_e}{2} = \frac{\sigma'_F}{E} (2N)^b \quad (6-2)$$

$$\frac{\Delta \varepsilon}{2} = \frac{\Delta \varepsilon_e}{2} + \frac{\Delta \varepsilon_p}{2}$$

$$\frac{\Delta \varepsilon}{2} = \frac{\sigma'_F}{E} (2N)^b + \varepsilon'_p (2N)^c \quad (6-3)$$

# Relation of Fatigue Life to Strain

- **Fatigue ductility exponent  $c$**  = slope of plastic-strain line
- $2N$  stress reversals =  $N$  cycles
- **Fatigue strength exponent  $b$**  = slope of elastic-strain line

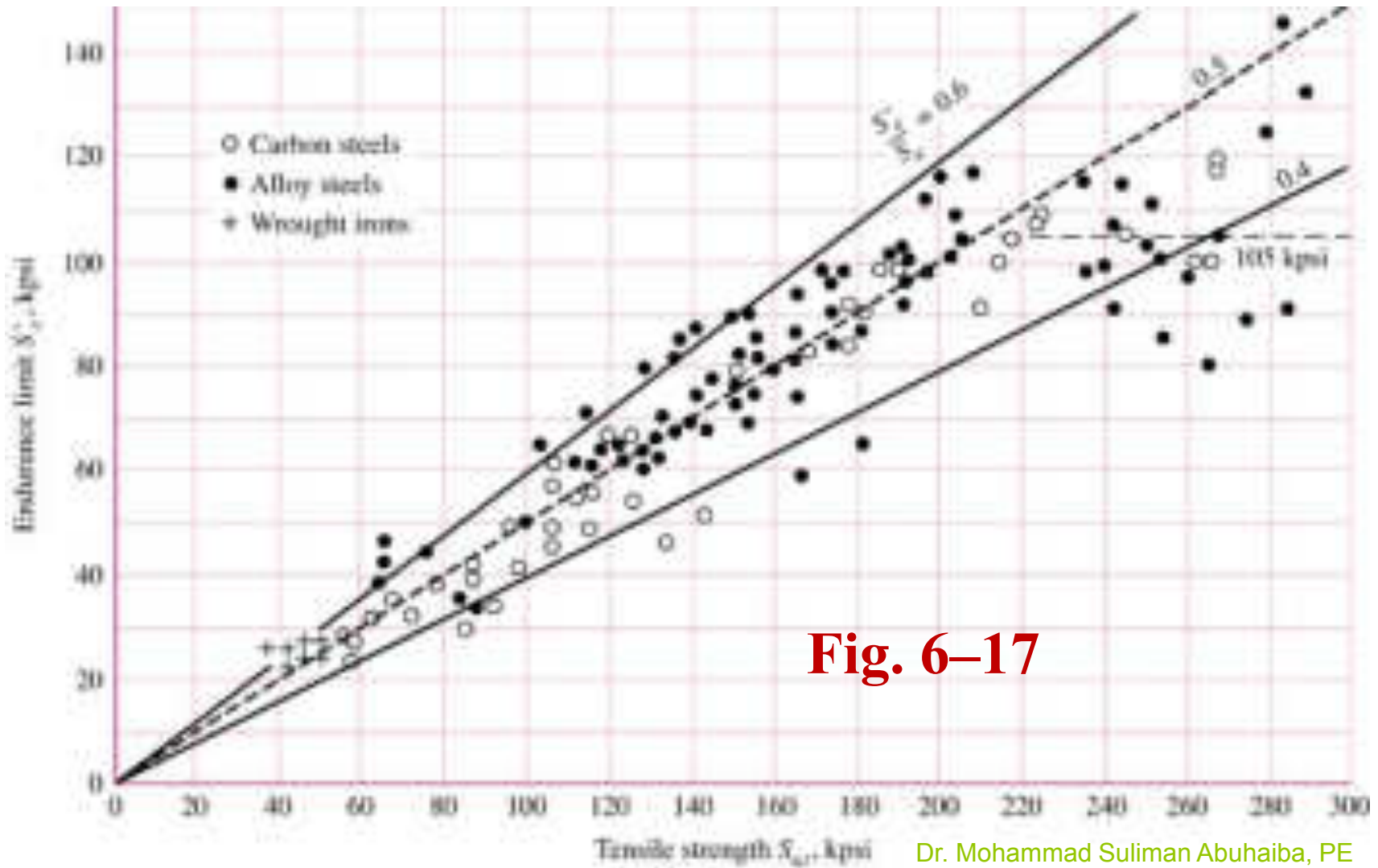
# Relation of Fatigue Life to Strain

$$\frac{\Delta \epsilon}{2} = \frac{\sigma'_F}{E} (2N)^b + \epsilon'_F (2N)^c \quad (6-3)$$

- **Manson-Coffin:** relationship between fatigue life and total strain
- **Table A-23:** values of coefficients & exponents
- Equation has **limited use for design**
  - ✓ Values for total strain at discontinuities are not readily available



# The Endurance Limit



**Fig. 6-17**

# The Endurance Limit

Simplified estimate of endurance limit for steels for the rotating-beam specimen,  $S'_e$

$$S'_e = \begin{cases} 0.5S_{UT} & S_{UT} \leq 200 \text{ kpsi (1400 MPa)} \\ 100 \text{ kpsi} & S_{UT} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{UT} > 1400 \text{ MPa} \end{cases} \quad (6-8)$$

# Fatigue Strength

Dr. Mohammad Suliman Abuhaiba, PE

- For design, an approximation of idealized S-N diagram is desirable.
- To estimate fatigue strength at  $10^3$  cycles, start with Eq. (6-2)

$$\frac{\Delta \epsilon_e}{2} = \frac{\sigma'_F}{E} (2N)^b \quad (6-2)$$

- Define specimen fatigue strength at a specific number of cycles as

$$(S'_f)_N = E \Delta \epsilon_e / 2$$

$$(S'_f)_N = \sigma'_F (2N)^b \quad (6-9)$$

# Fatigue Strength

- At  $10^3$  cycles,  $(S'_f)_{10^3} = \sigma'_F (2 \cdot 10^3)^b = f S_{ut}$
- $f$  = fraction of  $S_{ut}$  represented by  $(S'_f)_{10^3}$

$$f = \frac{\sigma'_F (2 \cdot 10^3)^b}{S_{ut}} \quad (6-10)$$

- SAE approximation for steels with  $H_B \leq 500$  may be used.

$$\sigma'_F = S_{ut} + 50 \text{ kpsi}$$

$$\sigma'_F = S_{ut} + 345 \text{ MPa}$$

# Fatigue Strength

- To find  $b$ , substitute endurance strength and corresponding cycles into Eq. (6-9) and solve for  $b$

$$b = -\frac{\log(\sigma'_F/S'_e)}{\log(2N_e)}$$

(6-12)

# Fatigue Strength

$$(S'_f)_N = \sigma'_F (2N)^b \quad (6-9)$$

$$f = \frac{\sigma'_F}{S_{ut}} (2 \cdot 10^3)^b \quad (6-10)$$

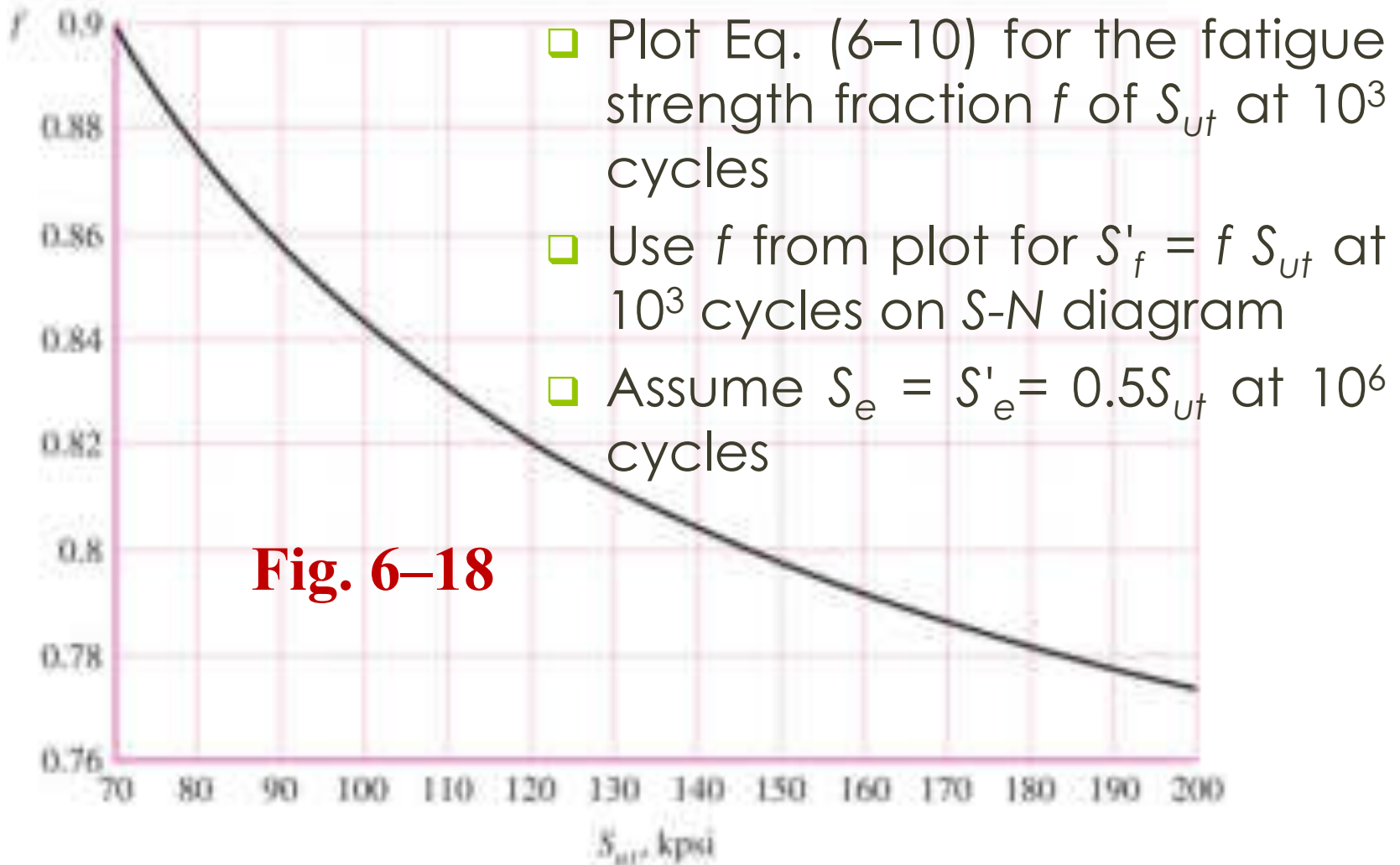
$$\sigma'_F = S_{ut} + 50 \text{ kpsi}$$

$$\sigma'_F = S_{ut} + 345 \text{ MPa}$$

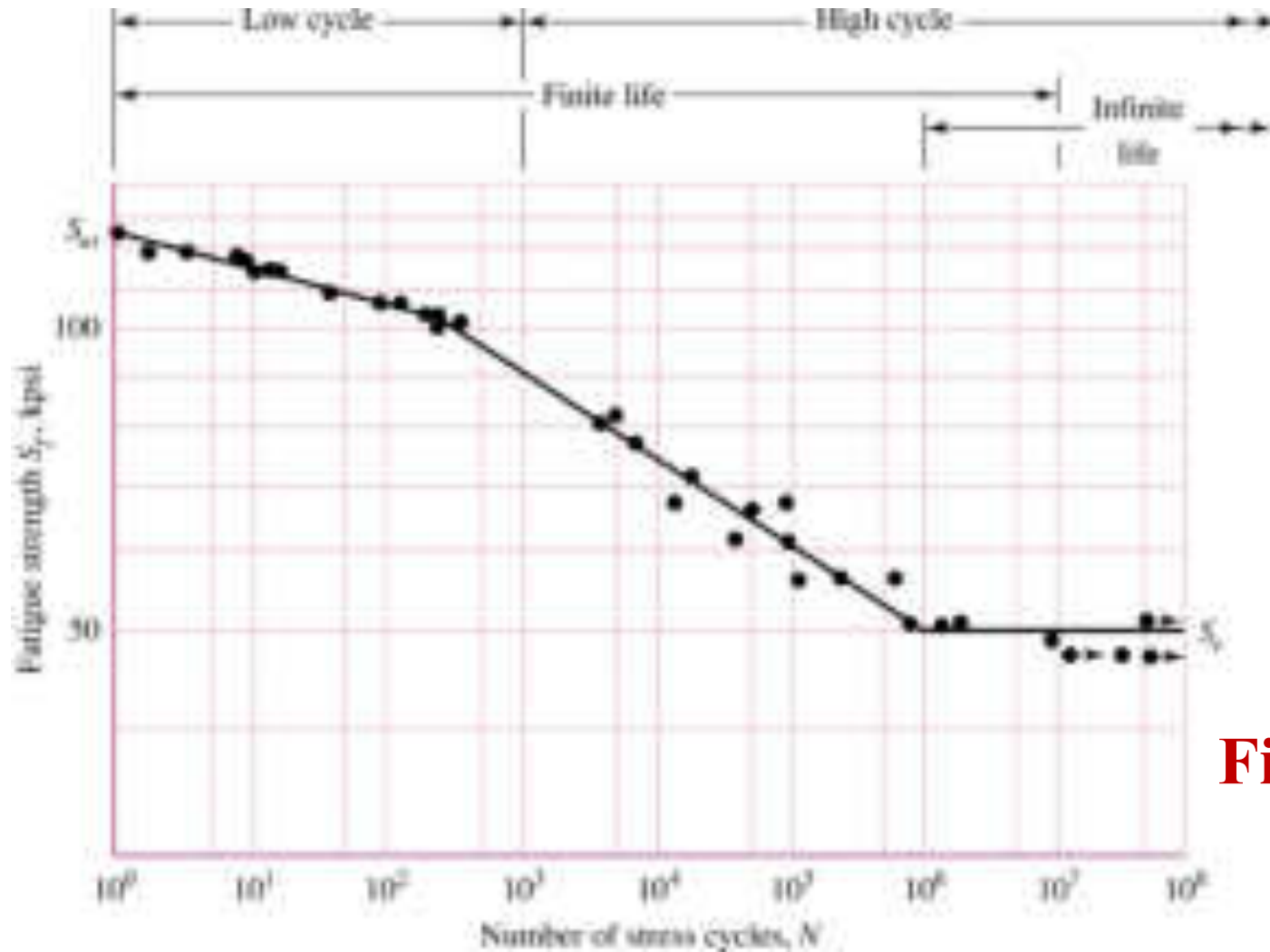
$$b = -\frac{\log(\sigma'_F/S'_c)}{\log(2N_c)} \quad (6-12)$$

Substitute Eqs. 6-11 & 6-12 into Eqs. 6-9 and 6-10 to obtain expressions for  $S'_f$  and  $f$

# Fatigue Strength Fraction $f$



# Equations for S-N Diagram



**Fig. 6-10**



# Equations for S-N Diagram

- Write equation for S-N line from  $10^3$  to  $10^6$  cycles
- Two known points
  - ✓ At  $N=10^3$  cycles,  $S_f = f S_{ut}$
  - ✓ At  $N=10^6$  cycles,  $S_f = S_e$
- Equations for line:

$$S_f = a N^b$$

(6-13)

$$a = \frac{(f S_{ut})^2}{S_e}$$

(6-14)

$$b = -\frac{1}{3} \log \left( \frac{f S_{ut}}{S_e} \right)$$

(6-15)

# Low-cycle Fatigue

- $1 \leq N \leq 10^3$
- On the idealized  $S$ - $N$  diagram on a log-log scale, failure is predicted by a straight line between two points  $(10^3, f S_{ut})$  and  $(1, S_{ut})$

$$S_f \geq S_{ut} N^{(\log f)/3}$$

$$1 \leq N \leq 10^3$$

[6-17]

## Example 6-2

Given a 1050 HR steel, *estimate*

- a. the rotating-beam endurance limit at  $10^6$  cycles.
- b. the endurance strength of a polished rotating-beam specimen corresponding to  $10^4$  cycles to failure
- c. the expected life of a polished rotating-beam specimen under a completely reversed stress of 55 kpsi.

# Endurance Limit Modifying Factors

- ❑ Endurance limit  $S'_e$  is for carefully prepared and tested specimen
- ❑ If warranted,  $S_e$  is obtained from testing of actual parts
- ❑ When testing of actual parts is not practical, a set of *Marin factors* are used to adjust the endurance limit

# Endurance Limit Modifying Factors

$$S_e = k_a k_b k_c k_d k_r k_f S'_e \quad (6-18)$$

$k_a$  = surface condition modification factor

$k_b$  = size modification factor

$k_c$  = load modification factor

$k_d$  = temperature modification factor

$k_r$  = reliability factor<sup>1)</sup>

$k_f$  = miscellaneous-effects modification factor

$S'_e$  = rotary-beam test specimen endurance limit

$S_e$  = endurance limit at the critical location of a machine part in the geometry and condition of use

# Surface Factor $k_a$

- Surface factor is a function of ultimate strength
- Higher strengths are more sensitive to rough surfaces

$$k_a = aS_{ut}^b$$

(6-19)

**Table 6-2**

Parameters for Mean Surface Modification Factor, Eq. (6-19)

Surface Finish	Factor $a$		Exponent $b$
	$S_{ut}$ kpsi	$S_{ut}$ MPa	
Ground	1.34	1.58	-0.085
Machined or cold-drawn	2.70	4.51	-0.265
Hot-rolled	14.4	57.7	-0.718
As-forged	39.9	272	-0.995

## Example 6-3

A steel has a min ultimate strength of 520 MPa and a machined surface. Estimate  $k_a$ .

# Size Factor $k_b$ rotating & Round

- Larger parts have greater surface area at high stress levels
- Likelihood of crack initiation is higher For bending and torsion loads, the size factor is given by

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \leq d \leq 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \leq 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \leq d \leq 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \leq 254 \text{ mm} \end{cases} \quad (6-20)$$



## Size Factor $k_b$ rotating & Round

- Applies only for round, rotating diameter
- For axial load, there is no size effect,
  - ✓  $k_b = 1$

## Size Factor $k_b$ not round & rotating

- ❑ An **equivalent round rotating** diameter is obtained.
- ❑ Volume of material stressed at and above 95% of max stress = same volume in rotating-beam specimen.
- ❑ Lengths cancel, so **equate areas**.

## Size Factor $k_b$ not round & rotating

- For a **rotating round section**, the **95% stress area** is the area of a ring,

$$A_{0.95\sigma} = \frac{\pi}{4} [d^2 - (0.95d)^2] = 0.0766d^2 \quad (6-22)$$

- Equate 95% stress area for other conditions to Eq. (6-22) and solve for  $d$  as the equivalent round rotating diameter

## Size Factor $k_b$ round & not rotating

- For non-rotating round,

$$A_{0.95\sigma} = 0.01046d^2 \quad (6-23)$$

- Equating to Eq. (6-22) and solving for equivalent diameter,

$$d_e = 0.370d \quad (6-24)$$

## Size Factor $k_b$ not round & not rotating

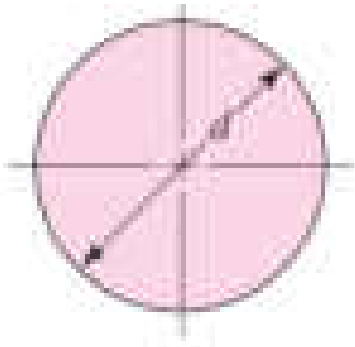
- For rectangular section  $h \times b$ ,  $A_{95\sigma} = 0.05 hb$ . Equating to Eq. (6-22),

$$d_e = 0.808(hb)^{1/2}$$

(6-25)

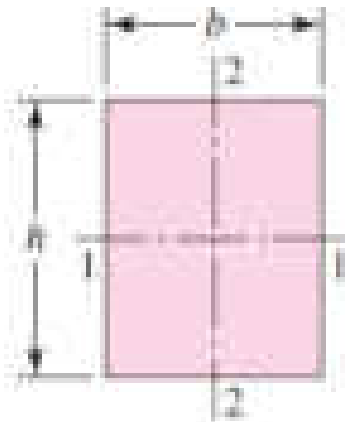
# Size Factor $k_b$

**Table 6–3:**  $A_{95\sigma}$  for common non-rotating structural shapes undergoing bending



$$A_{0.95\sigma} = 0.01046d^2$$

$$d_e = 0.370d$$

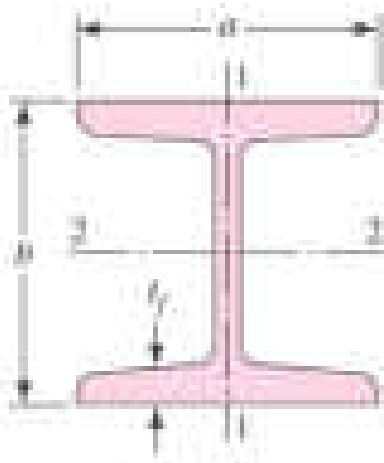


$$A_{0.95\sigma} = 0.05hb$$

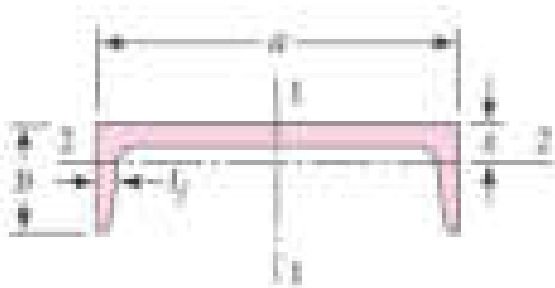
$$d_e = 0.808\sqrt{hb}$$

# Size Factor $k_b$

**Table 6-3:**  $A_{95\sigma}$  for common non-rotating structural shapes undergoing bending



$$A_{95\sigma} = \begin{cases} 0.10ty & \text{axis 1-1} \\ 0.05bt & \text{axis 2-2} \end{cases} \quad t_f > 0.025a$$



$$A_{95\sigma} = \begin{cases} 0.05ab & \text{axis 1-1} \\ 0.052xt + 0.1ty(b-x) & \text{axis 2-2} \end{cases}$$

## Example 6-4

A steel shaft loaded in bending is 32 mm in diameter, abutting a filleted shoulder 38 mm in diameter. The shaft material has a mean ultimate tensile strength of 690 MPa. Estimate the Marin size factor  $k_b$  if the shaft is used in

- a. A rotating mode.
- b. A nonrotating mode.



# Loading Factor $k_c$

- Accounts for changes in endurance limit for different types of fatigue loading.
- Only to be used for **single load types**.
- Use **Combination Loading method** (Sec. 6–14) when **more than one load type** is present.

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion}^{17} \end{cases}$$

(6-26)

# Temperature Factor $k_d$

- Endurance limit appears to maintain same relation to ultimate strength for elevated temperatures as at RT
- **Table 6-4:** Effect of Operating Temperature on Tensile Strength of Steel.\*  
( $S_T$  = tensile strength at operating temperature (OT);  $S_{RT}$  = tensile strength at room temperature;  $0.099 \leq \hat{\sigma} \leq 0.110$ )

# Temperature Factor $k_d$

## Table 6–4

Temperature, °C	$S_T/S_{RT}$	Temperature, °F	$S_T/S_{RT}$
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

# Temperature Factor $k_d$

- If ultimate strength is known for OT, then just use that strength. Let  $k_d = 1$ .
- If ultimate strength is known only at RT, use Table 6-4 to estimate ultimate strength at OT. With that strength, let  $k_d = 1$ .
- Use ultimate strength at RT and apply  $k_d$  from Table 6-4 to the endurance limit.

$$k_d = \frac{S_T}{S_{RT}}$$

(6-28)

# Temperature Factor $k_d$

- A fourth-order polynomial curve fit of the data of Table 6-4 can be used in place of the table,

$$k_d = 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 \\ + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4$$

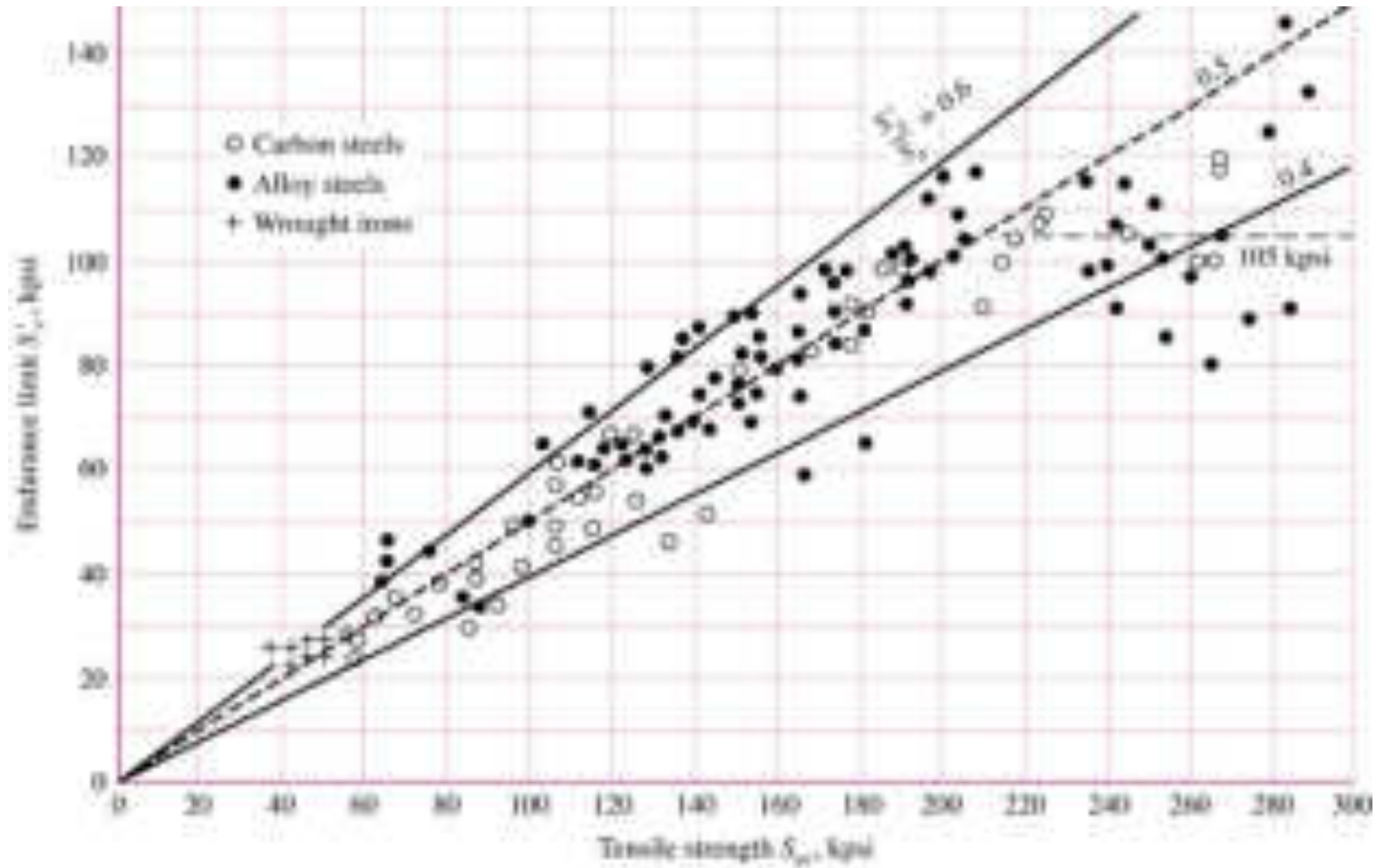
(6-27)

# Reliability Factor $k_e$

- Fig. 6–17,  $S'_e = 0.5 S_{ut}$  is typical of the data and represents 50% reliability.
- Reliability factor adjusts to other reliabilities.

# Reliability Factor $k_e$

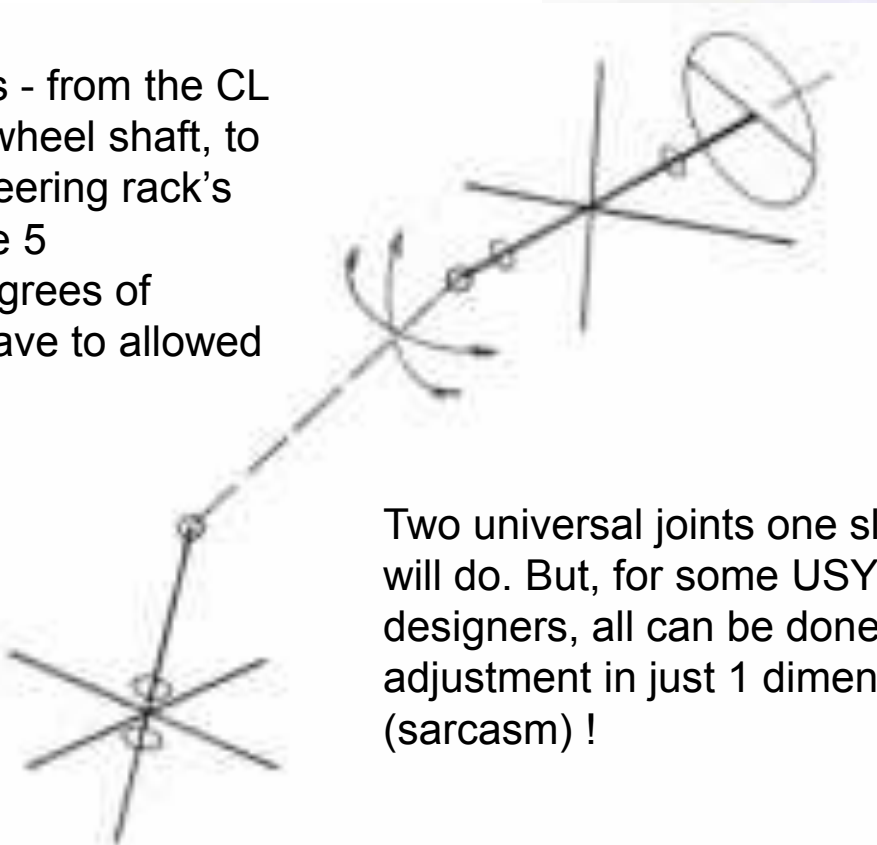
Fig. 6-17



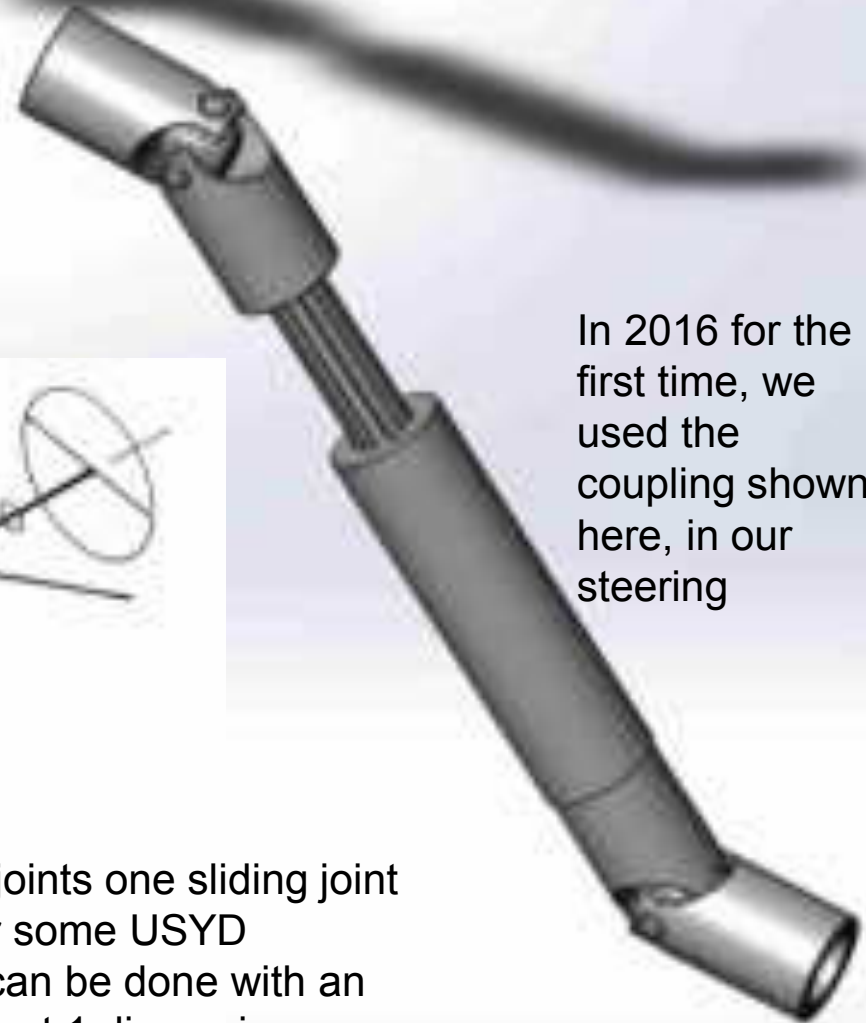


2 Dimensional misalignments

In 3 Dimensions - from the CL of the steering wheel shaft, to the CL of the steering rack's pinion, there are 5 independent degrees of freedom, that have to allowed for, exactly !



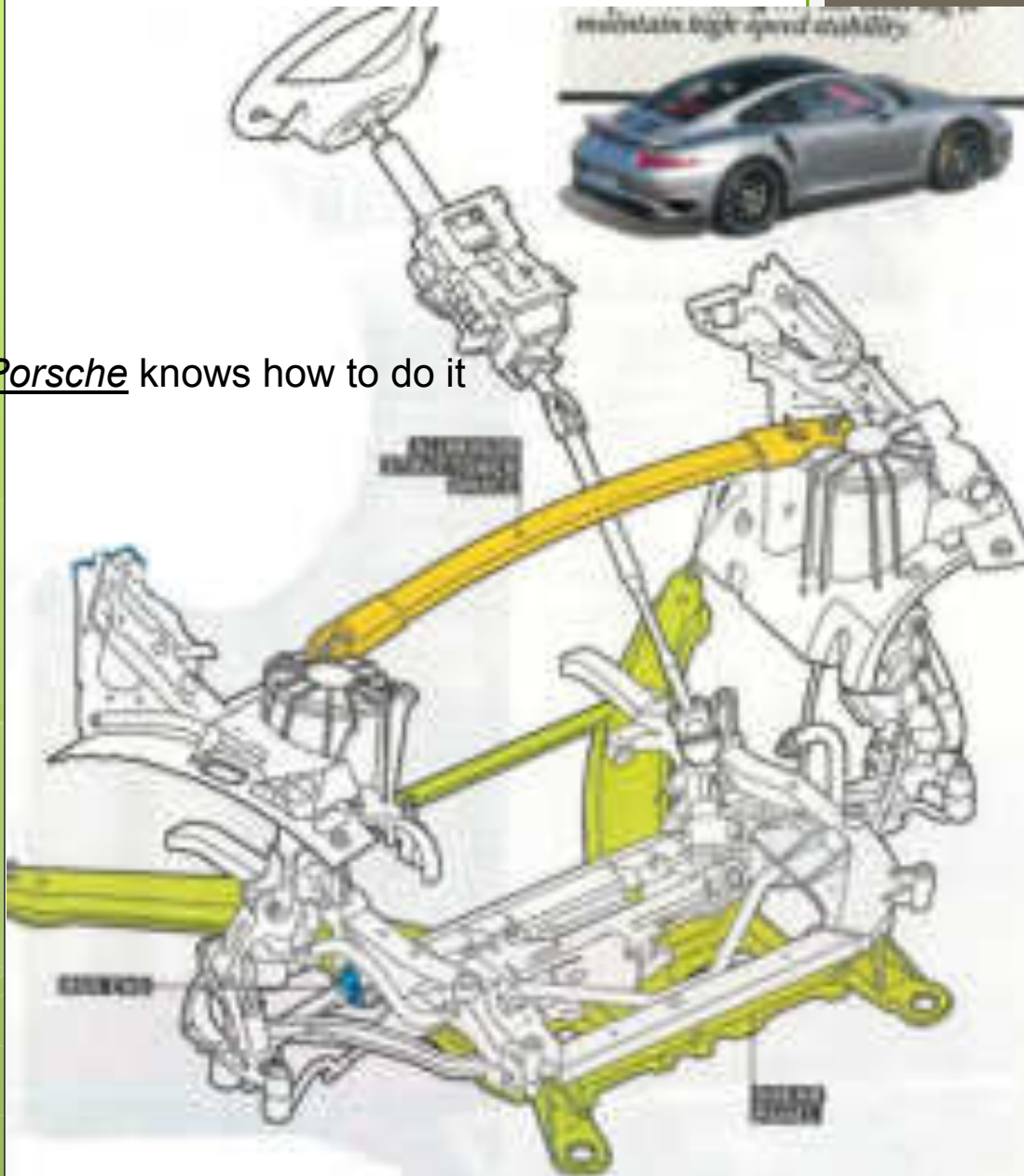
Two universal joints one sliding joint will do. But, for some USYD designers, all can be done with an adjustment in just 1 dimension (sarcasm) !



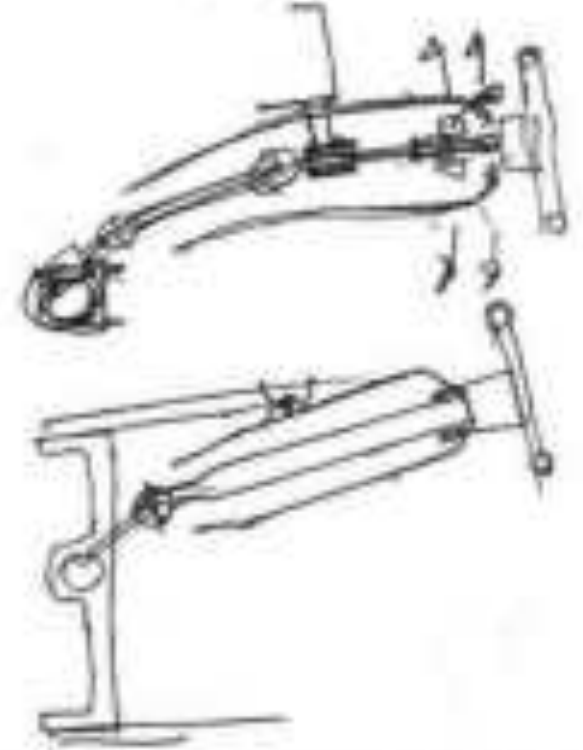
In 2016 for the first time, we used the coupling shown here, in our steering



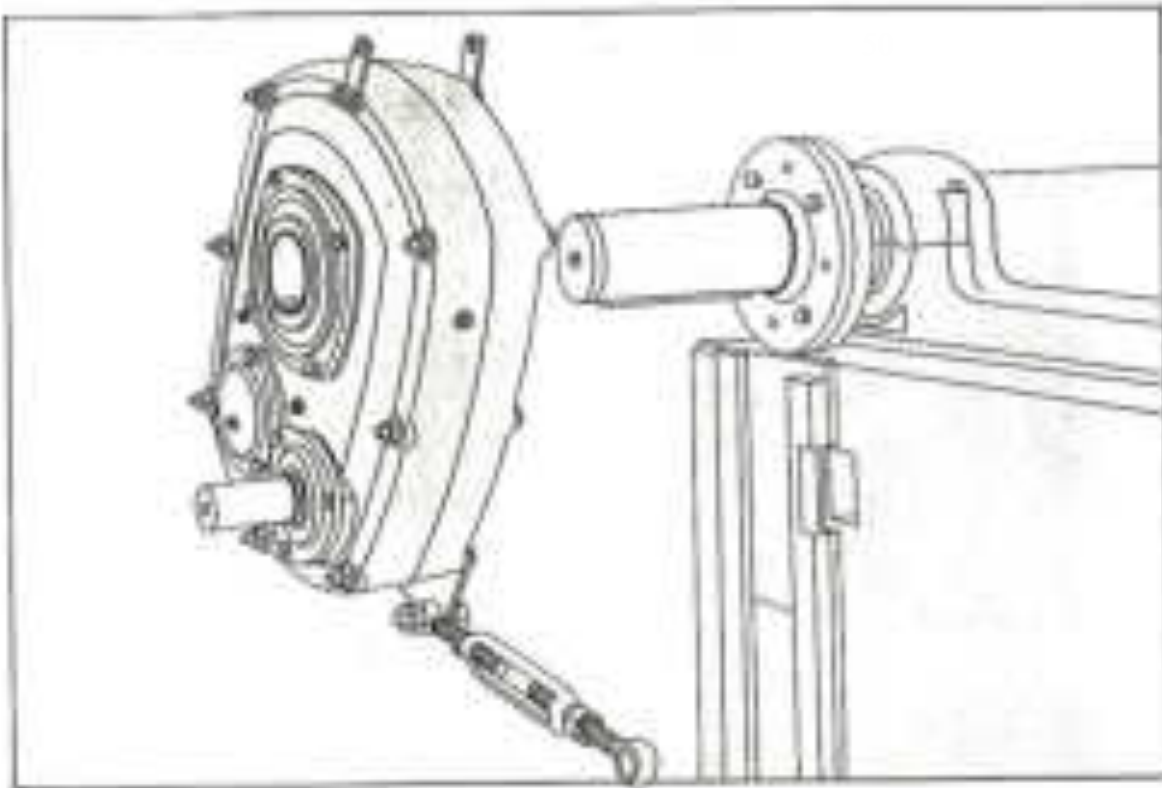
Porsche knows how to do it



Even Gordon Murry an Englishman, for the McLaren F1 team understood the problem and the solution.



In F1 \$12000 is spent to save 1 kg (the Economist magazine).



The use of shaft mounted gearbox as shown here, have become very popular. This is largely because if the box and the shaft are not mounted separately their alignment is difficult to guarantee. The weight due to the box typically causes an insignificant increase in shaft stresses.



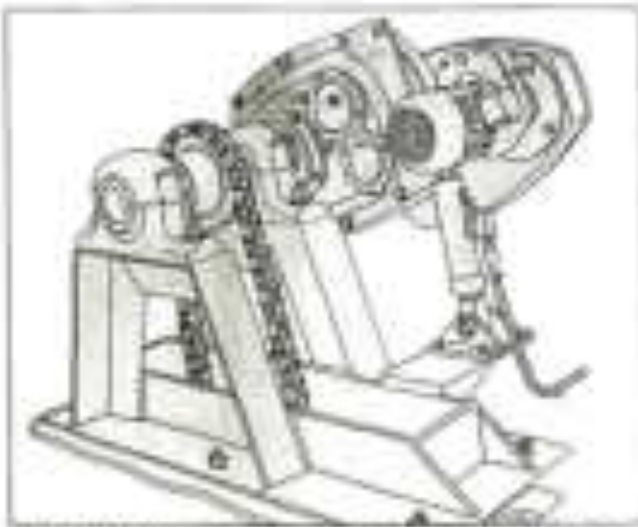
Nominal ratio 5:1  
Efficiency 98%



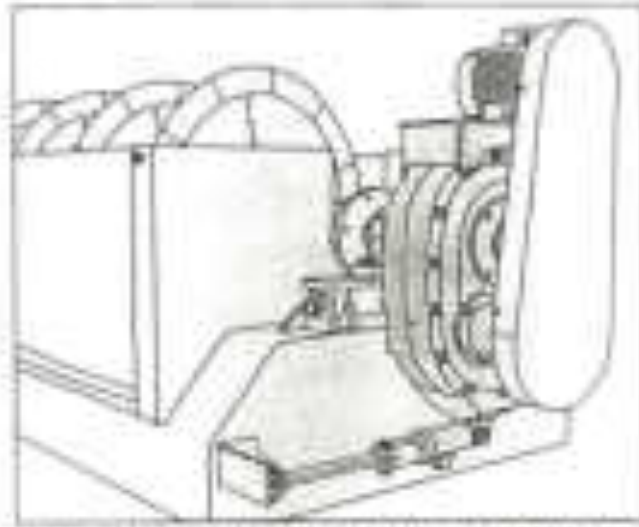
Nominal ratio 15:1, 20:1  
Efficiency 97%



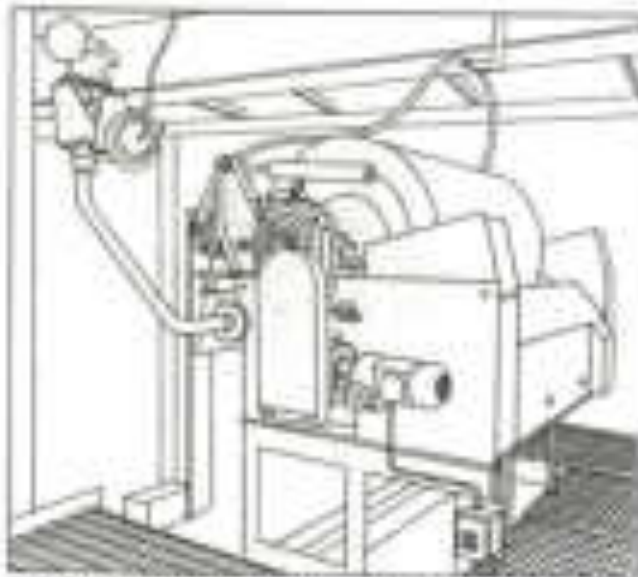
Nominal ratio 70:1, 130:1  
Efficiency 96%



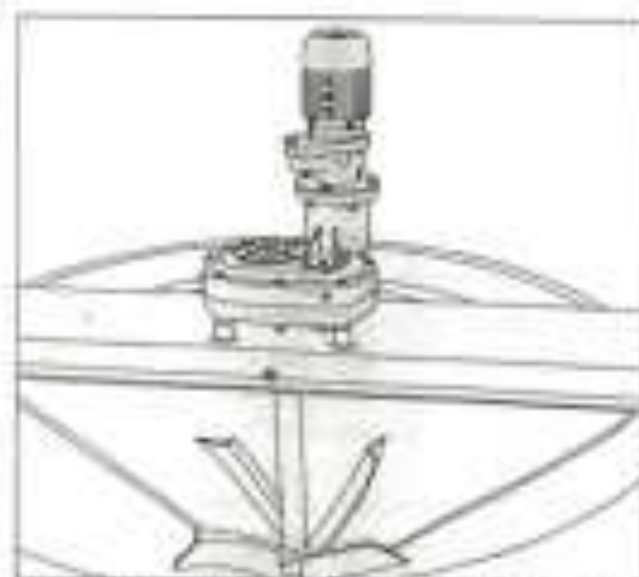
*NT-SALA Speed Reducer TV 72 SC with overload release driving a chain scraper in a water treatment plant.*



*Screw conveyor with NT-SALA Speed Reducer TV 200 and overload release.*



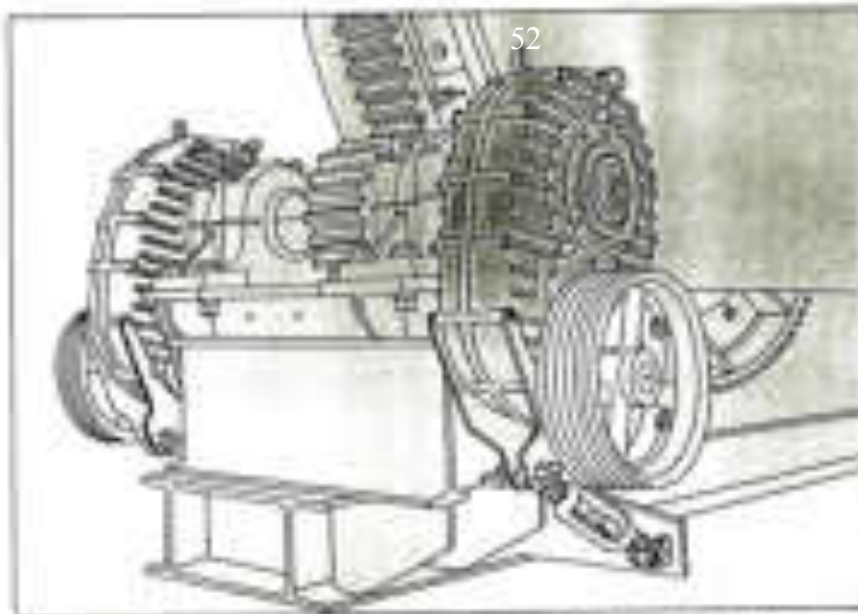
*NT-SALA Speed Reducer with turbine for filter drive in the mining industry.*



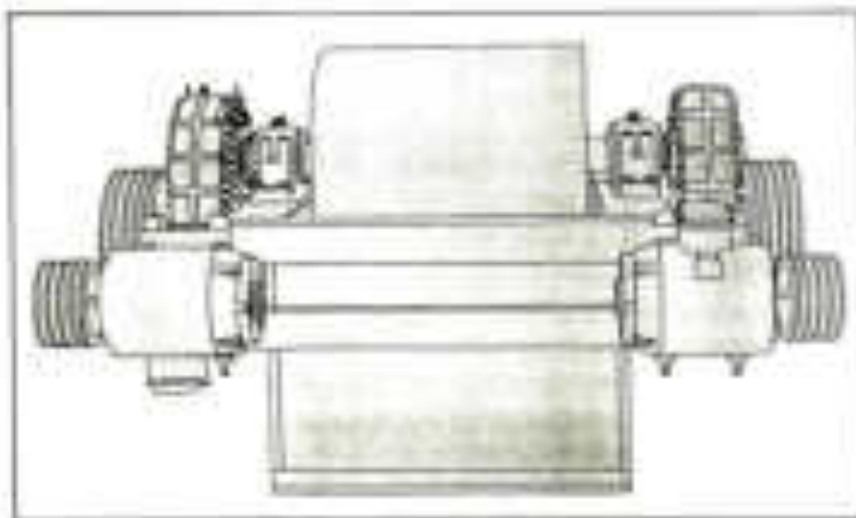
*Vertically mounted NT-SALA Speed Reducer with large connected gear motor used on agitator.*

## Dual drive

52

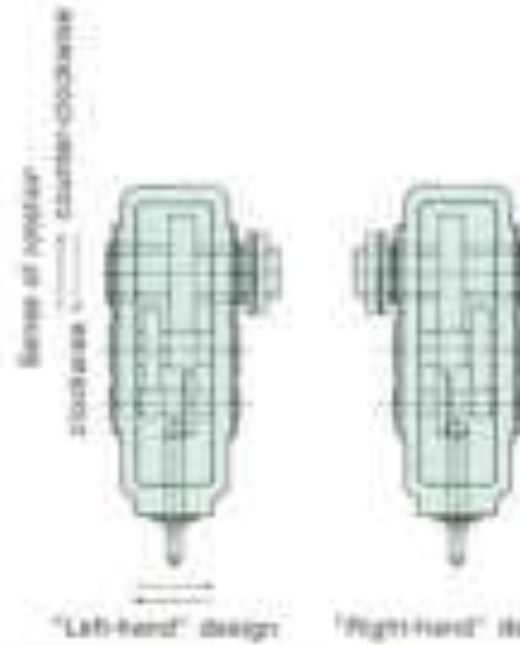
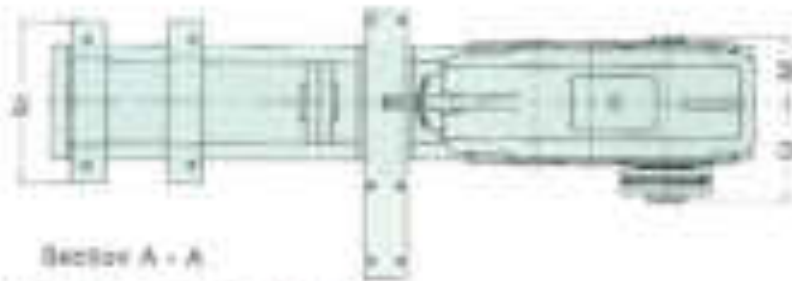
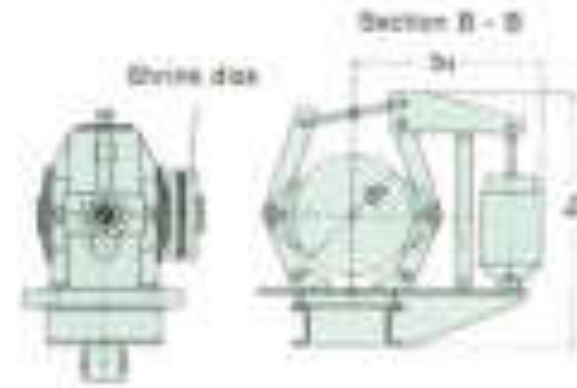
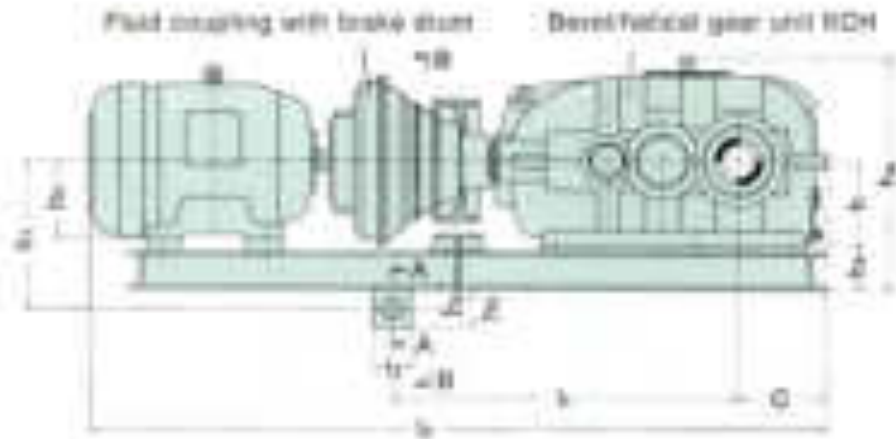


Two NI-3AL4 7F 602 Speed Reducers with dual 27 kW 1470 RPM electric motors driving a 3.1 m long, 2.3 m diameter drying drum.



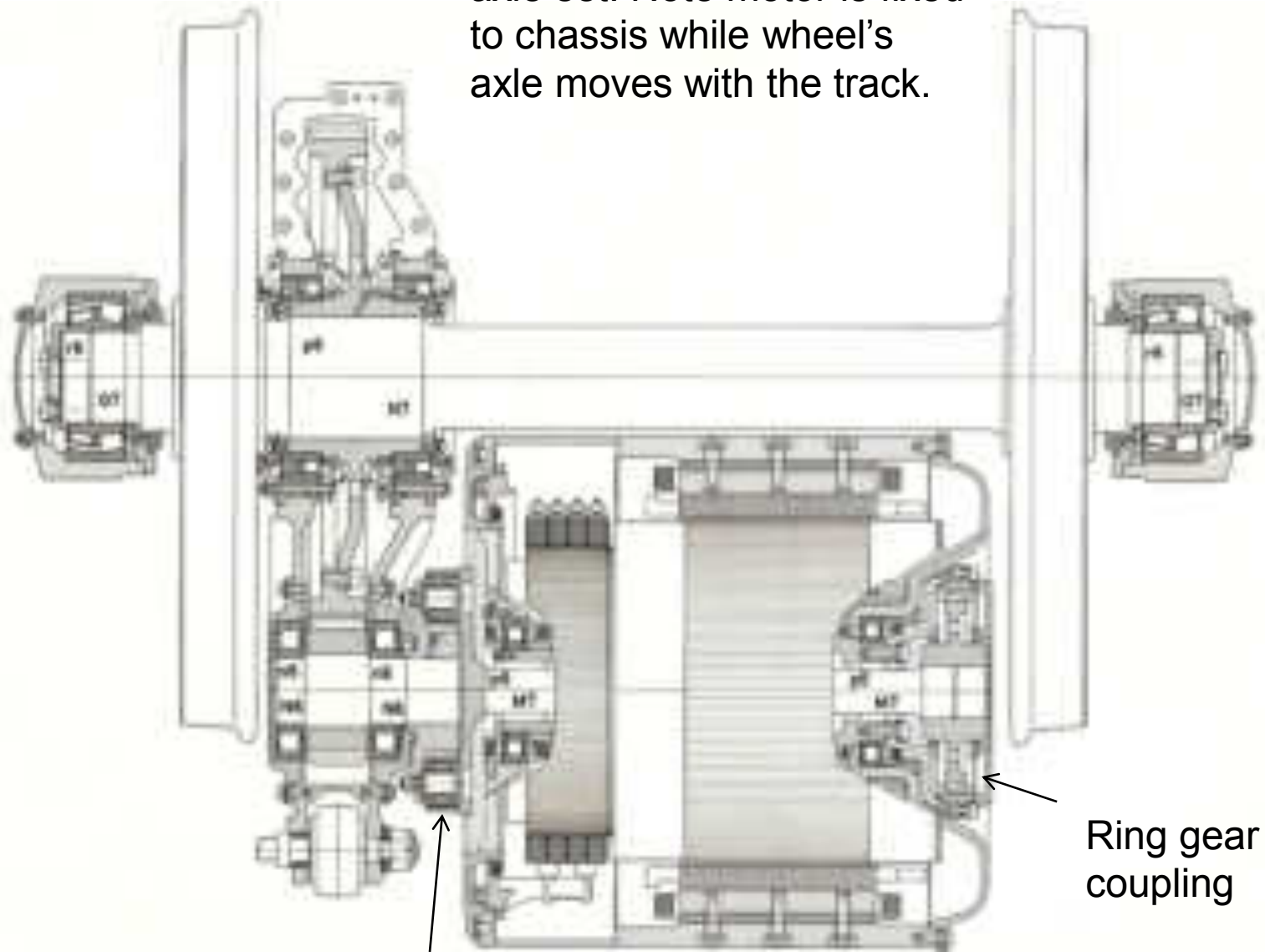
Typical dual drive for conveyor using two NI-3AL4 7F 602 Speed Reducers.





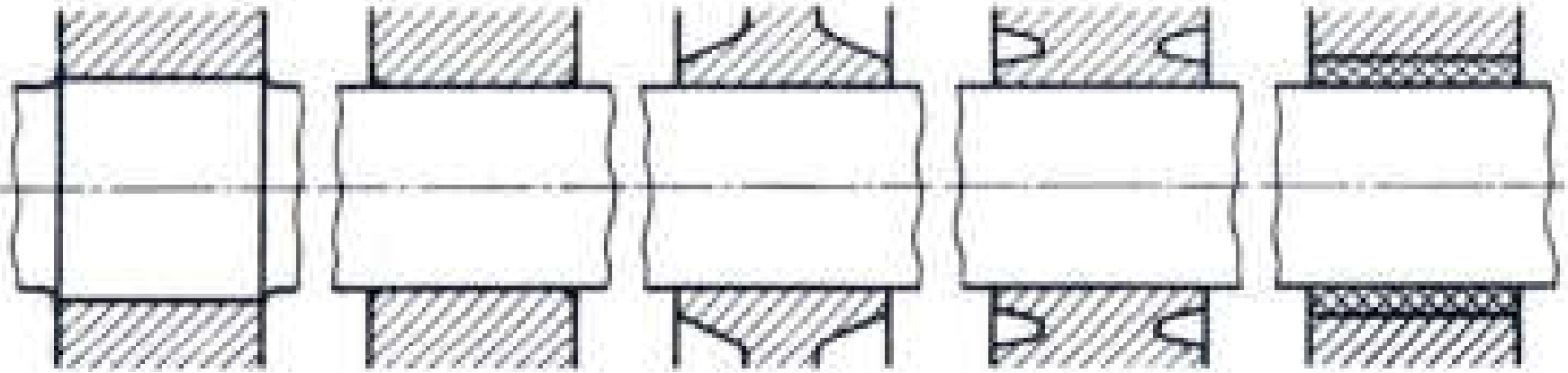
Helix shaft design and dimensions see pages 37 and 38

Locomotive drive motor and axle set. Note motor is fixed to chassis while wheel's axle moves with the track.

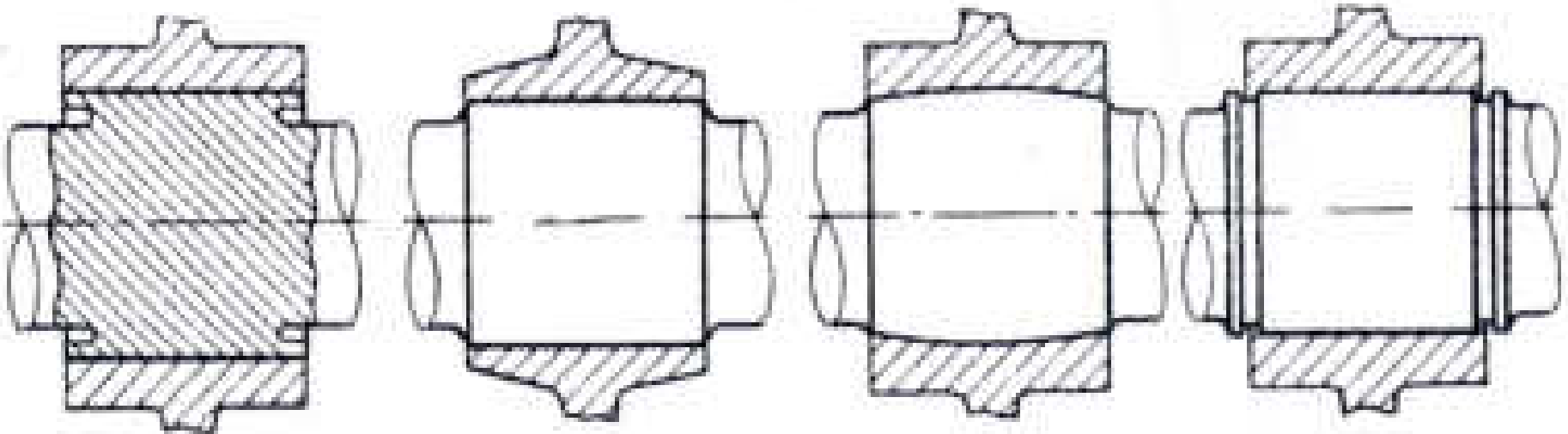


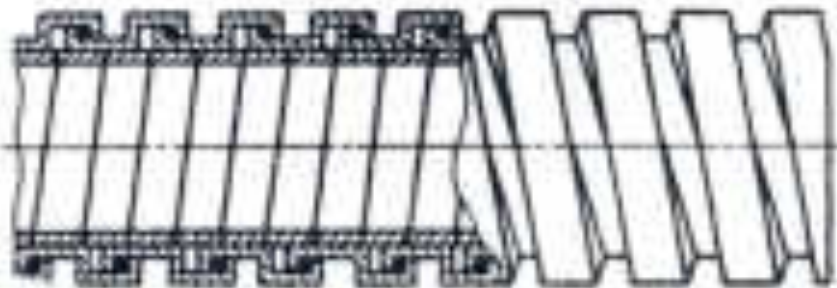
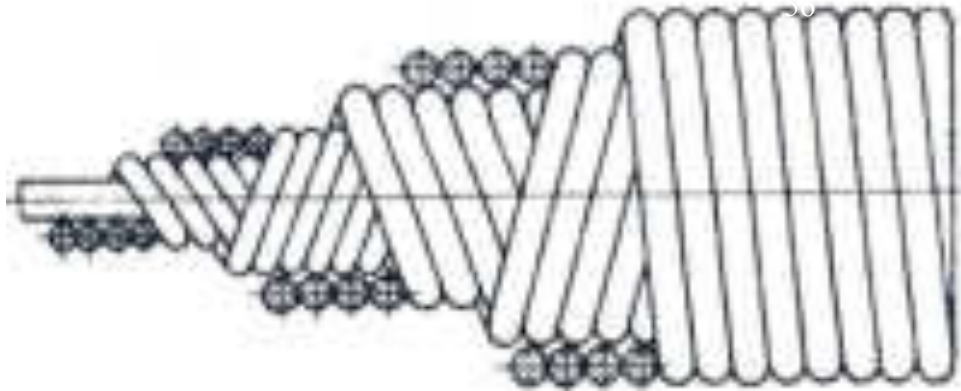
Pin and rubber  
coupling

Ring gear  
coupling



Means of reducing bending moments and stress concentration where a shaft enters a boss or housing





In nearly all shafts deflection is closely constrained, yet there are instances where the shaft is designed to be rigid in torsion but not in bending.

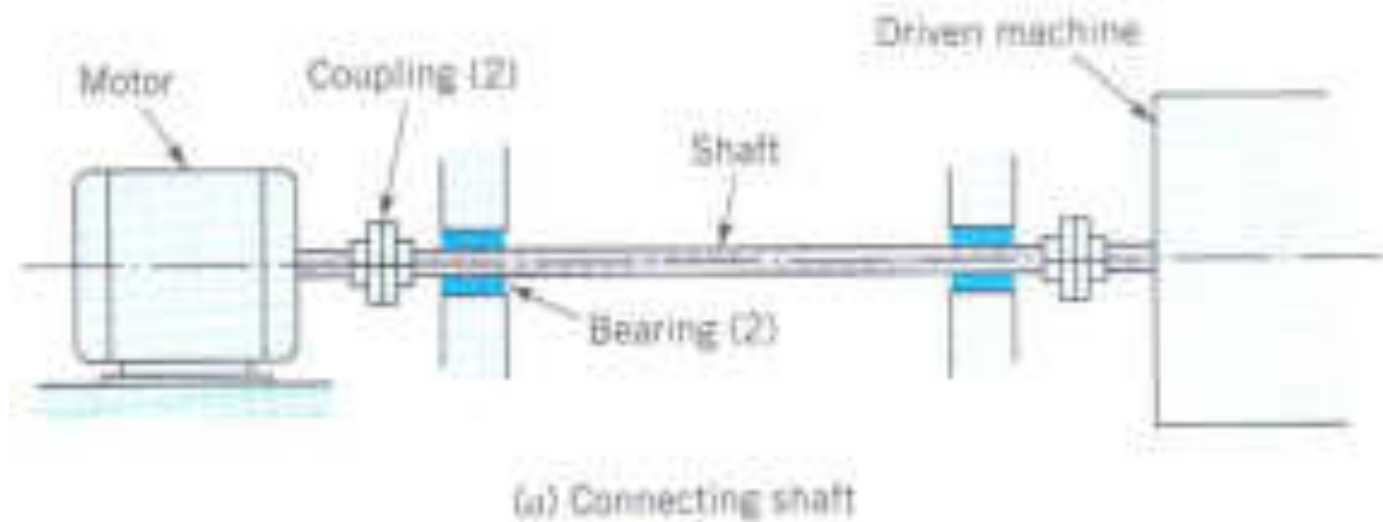
In some instances some specific bending is part of the shaft function, and allowed in its analysis.



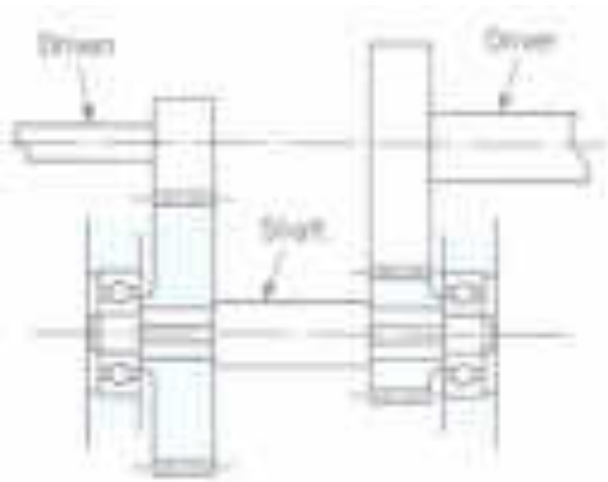
# Design of Shaft

- A shaft is a rotating member usually of circular cross-section (solid or hollow), which transmits power and rotational motion.
- Machine elements such as **gears, pulleys (sheaves), flywheels, clutches, and sprockets** are mounted on the shaft and are used to transmit power from the driving device (motor or engine) through a machine.
- **Press fit, keys, dowel, pins and splines** are used to attach these machine elements on the shaft.
- The shaft rotates on **rolling contact bearings or bush bearings**.
- Various types of **retaining rings, thrust bearings, grooves and steps** in the shaft are used to take up axial loads and locate the rotating elements.
- **Couplings** are used to transmit power from drive shaft (e.g., motor) to the driven shaft (e.g.

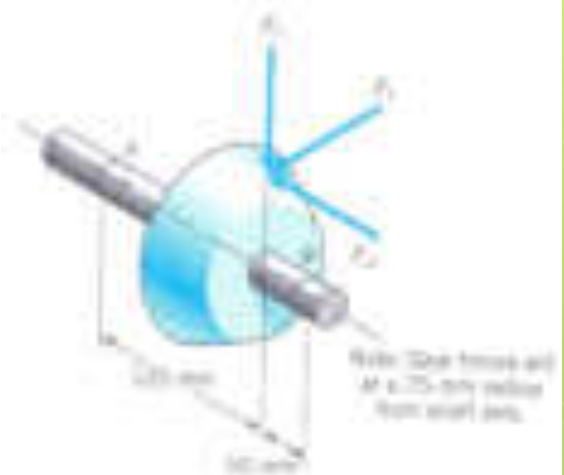
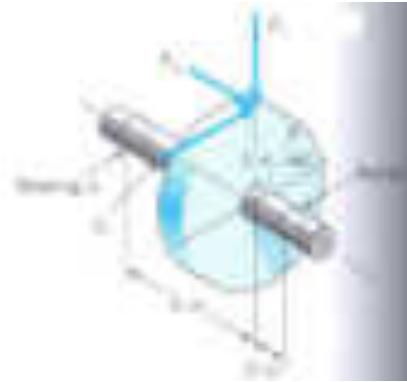
**The connecting shaft is loaded primarily in torsion.**



# Combined bending and torsion loads on shaft: Shaft carrying gears.



Gear shaft



With the same forces and at a 75-degree angle from each other.

From power and rpm find the torque ( $T$ ), which gives rise to shear stress.

From Torque ( $T$ ) and diameter ( $d$ ), find  $F_t = 2T/d$ . From  $F_t$  and pressure angles of gears you can find  $F_r$  and  $F_a$ .

$F_r$  and  $F_t$  are orthogonal to each other and are both transverse forces to the shaft axis, which will give rise to normal bending stress in the shaft. When shaft rotates, bending stress changes from tensile to compressive and then compressive to tensile, ie, completely reversing state of stress.

$F_a$  will give rise to normal axial stress in the shaft.

# Loads on shaft due to pulleys

Pulley torque ( $T$ ) = Difference in belt tensions in the tight ( $t_1$ ) and slack ( $t_2$ ) sides of a pulley times the radius ( $r$ ), ie

$$T = (t_1 - t_2) \times r$$

Left pulley torque

$$T_1 = (7200 - 2700) \times 380 = 1,710,000 \text{ N-mm}$$

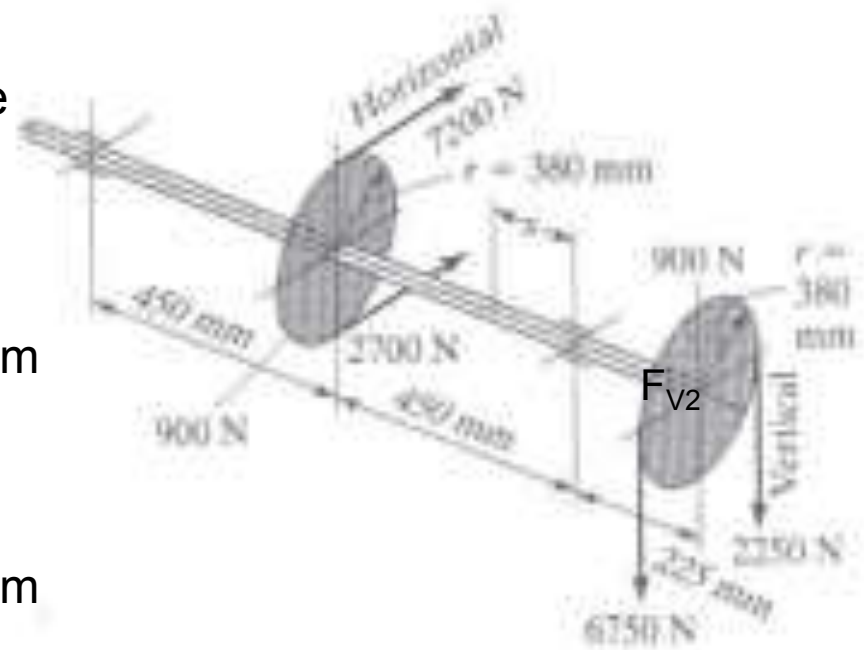
Right pulley has exactly equal and opposite torque:

$$T_2 = (6750 - 2250) \times 380 = 1,710,000 \text{ N-mm}$$

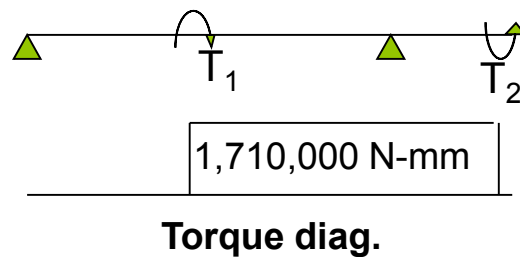
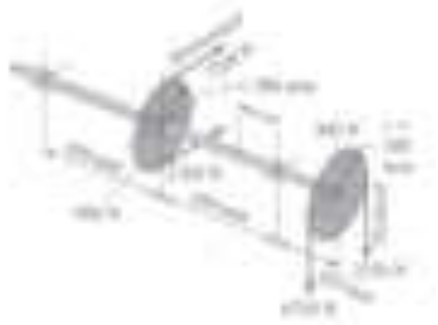
Bending forces in vertical ( $F_v$ ) and horizontal ( $F_H$ ) directions:

At the left pulley:  $F_{v1} = 900 \text{ N}$ ;  $F_{H1} = 7200 + 2700 = 9900 \text{ N}$

At the right pulley:  $F_{v2} = 900 + 6750 + 2250 = 9900 \text{ N}$ ;  $F_{H2} = 0$



# Torque and Bending moment diagrams for the pulley system

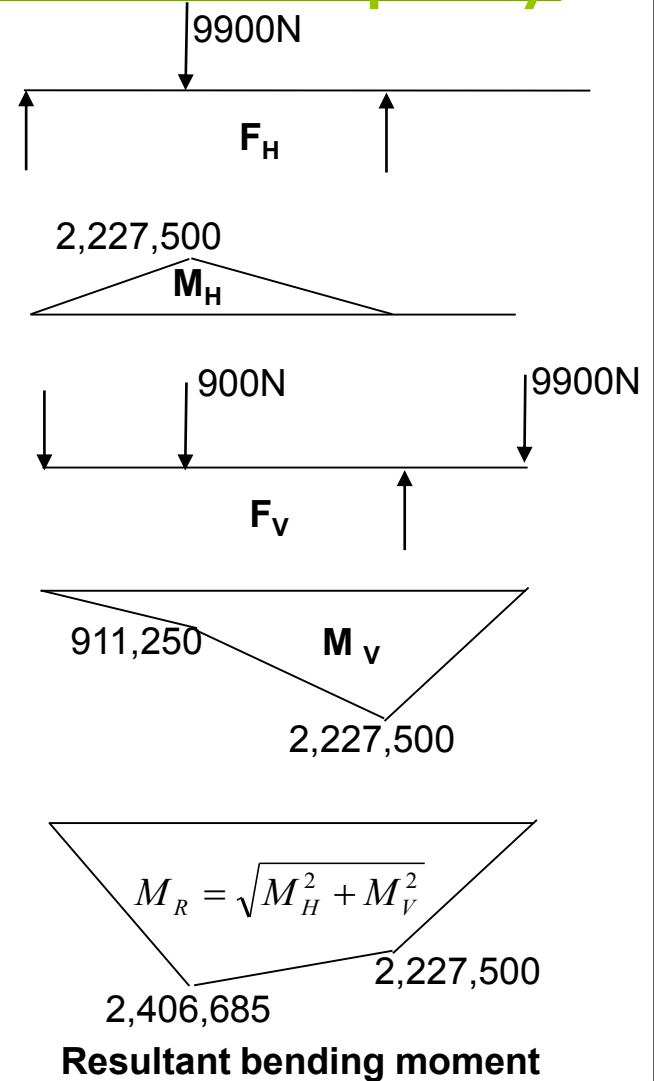


From Horizontal forces ( $F_H$ ) and vertical forces ( $F_V$ ), Bending moments  $M_H$  &  $M_V$  are drawn separately.

Then the resultant moments at various points on the shaft can be found from

$$M_R = \sqrt{M_H^2 + M_V^2}$$

The section of shaft where the left pulley is located has obviously the highest combination of Torque (1,710,000 N-mm) and Bending moment (2,406,685 N-mm)



# Power, torque & speed

For linear motion:

$$\text{Power} = F.v \text{ (force x velocity)}$$

For rotational motion

$$\text{Power } P = \text{Torque x angular velocity}$$

$$= T \text{ (in-lb)} \cdot \omega \text{ (rad/sec) in-lb/sec}$$

$$= T \cdot (2 \pi n/60) \text{ in-lb/sec} \quad [n=\text{rpm}]$$

$$= T \cdot (2 \pi n/(60 \cdot 12 \cdot 550)) \text{ HP} \quad [\text{HP}=550 \text{ ft-lb/sec}]$$

$$= T \cdot n/63,025 \text{ HP}$$

or,  $T = 63,025 \text{ HP}/n \text{ (in-lb)}$ , where  $n = \text{rpm}$

Similarly,  $T = 9,550,000 \text{ kW}/n \text{ (N-mm)}$ , where  $n = \text{rpm}$

Shear ( $\tau$ ) and bending ( $\sigma$ ) stresses on the outer surface of a shaft, for a torque (T) and bending moment (M)

$$\tau = \frac{T}{J} r = \frac{T}{(\pi d^4 / 32)} \frac{d}{2} = \frac{16T}{\pi d^3}$$

**For solid circular section:**

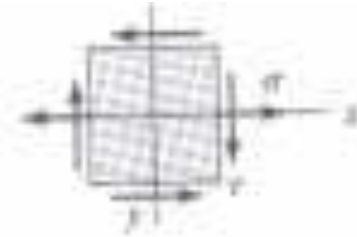
$$\sigma = \frac{M}{I} r = \frac{M}{(\pi d^4 / 64)} \frac{d}{2} = \frac{32M}{\pi d^3}$$

**For hollow circular section:**

$$\tau = \frac{T}{J} r = \frac{T}{(\pi (d_o^4 - d_i^4) / 32)} \frac{d_o}{2} = \frac{16T}{\pi (d_o^4 - d_i^4)} d_o = \frac{16T}{\pi d_o^3 (1 - \lambda^4)} \quad \text{where, } \lambda = \frac{d_i}{d_o}$$

$$\sigma = \frac{M}{I} r = \frac{M}{(\pi (d_o^4 - d_i^4) / 64)} \frac{d_o}{2} = \frac{32M}{\pi (d_o^4 - d_i^4)} d_o = \frac{32M}{\pi d_o^3 (1 - \lambda^4)} \quad \text{where, } \lambda = \frac{d_i}{d_o}$$

The stress at a point on the shaft is normal stress ( $\sigma$ ) in X direction and shear stress ( $\tau$ ) in XY plane.



From Mohr Circle:

$$S_1 = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} \text{ and } S_2 = \frac{\sigma}{2} - \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

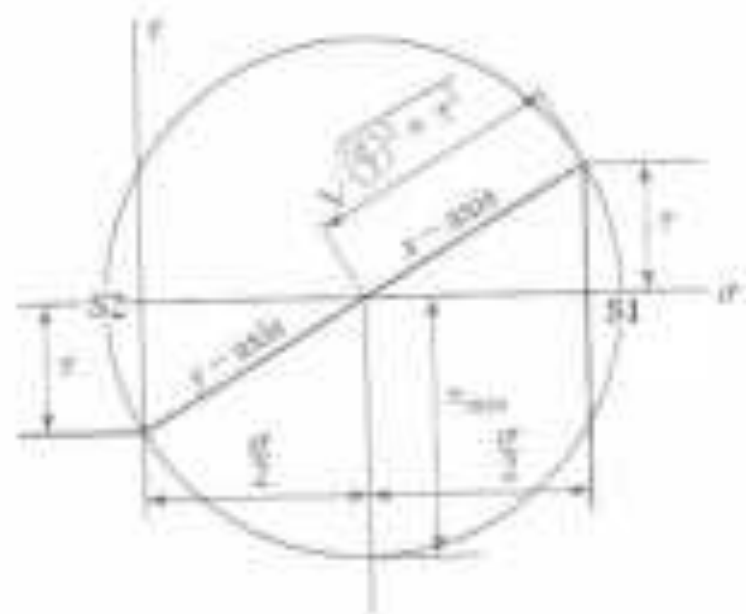
Max Distortion Energy theory:

$$S_1^2 + S_2^2 - S_1 S_2 \leq \left(\frac{S_{yp}}{N_{fs}}\right)^2$$

Putting values of  $S_1$  &  $S_2$  and simplifying:

$$\sigma^2 + 3\tau^2 \leq \left(\frac{S_{yp}}{N_{fs}}\right)^2$$

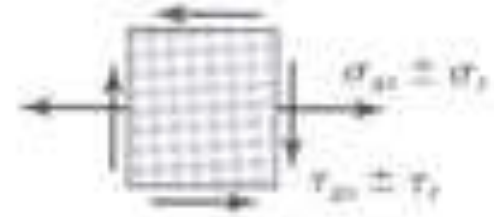
**This is the design equation for non rotating shaft**





## Design of rotating shafts and fatigue consideration

The most frequently encountered stress situation for a rotating shaft is to have completely reversed bending and steady torsional stress. In other situations, a shaft may have a reversed torsional stress along with reversed bending stress.



The most generalized situation the rotating shaft may have both steady and cyclic components of bending stress ( $s_{av}, s_r$ ) and torsional stress ( $t_{av}, t_r$ ).

From Soderberg's fatigue criterion, the equivalent static bending and torsional stresses are:

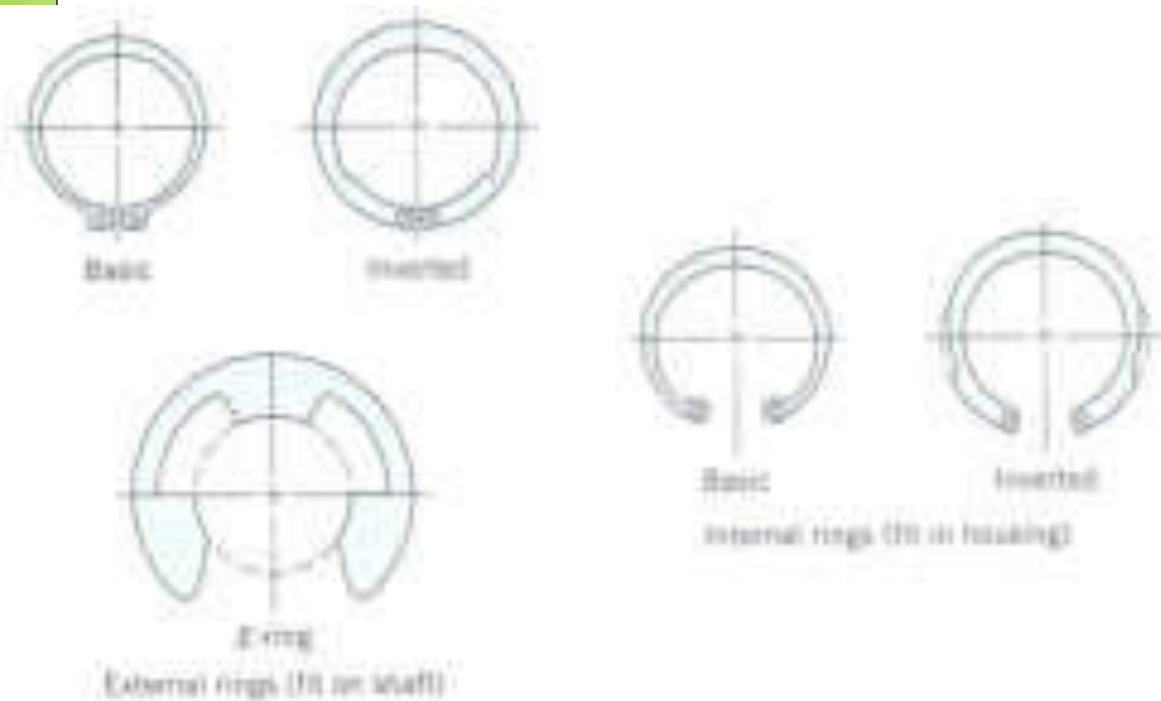
$$\sigma_{equivalent} = \sigma_{av} + \sigma_r K_{fs} \left( \frac{S_{yp}}{S_e} \right)$$

$$\tau_{equivalent} = \tau_{av} + \tau_r K_{ft} \left( \frac{S_{yp}}{S_e} \right)$$

Using these equivalent static stresses in our static design equation, the equation for rotating shaft is:

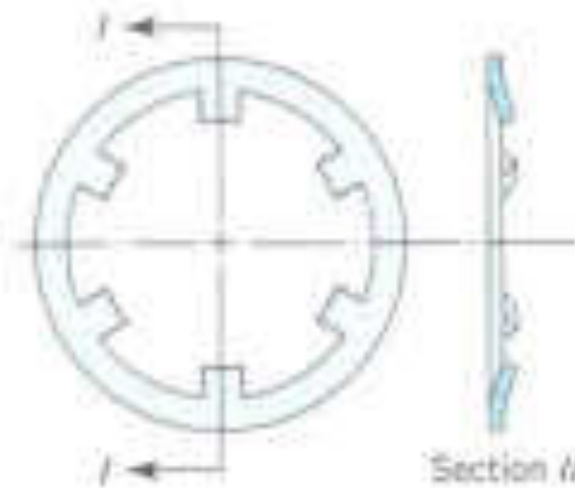
$$\left( \sigma_{av} + \sigma_r K_{fs} \left( \frac{S_{yp}}{S_e} \right) \right)^2 + 3 \left( \tau_{av} + \tau_r K_{ft} \left( \frac{S_{yp}}{S_e} \right) \right)^2 \leq \left( \frac{S_{yp}}{N_{fs}} \right)^2$$

**Conventional retaining (or snap) rings fit in grooves and take axial load, but grooves cause stress concentration in shaft**

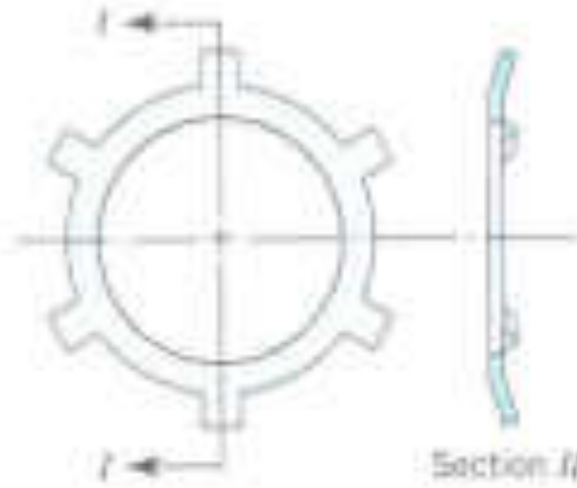


Retaining rings are standardized items, available in various standard sizes with various axial load capacities.

# Push type retaining rings – no grooves required, less stress concentration, but less axial support



External ring (fit on shaft)

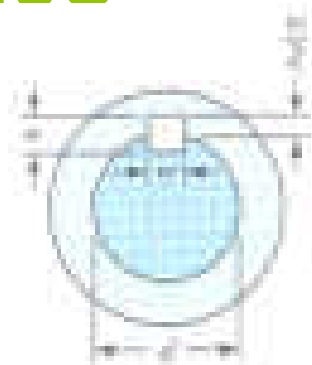


Internal ring (fit in housing)

(A) Fish-on type – no grooves required

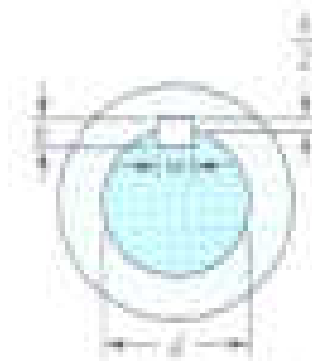
Teeth deflect when installed to "bite in" and resist removal  
(less positive than conventional type)

# Various types of keys for transmitting torque



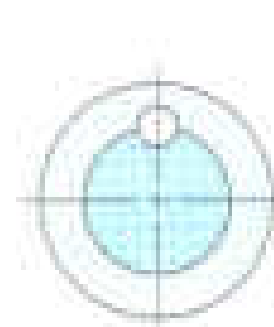
$w = d/4$

(a) Square key



$w = d/4$ ,  $h = 3w/4$

(b) Flat key



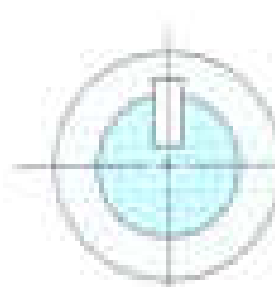
Key usually has drive fit, is often tapered

(c) Round key



Keys are tapered and driven tightly for heavy-duty service

(d) Tapered keys

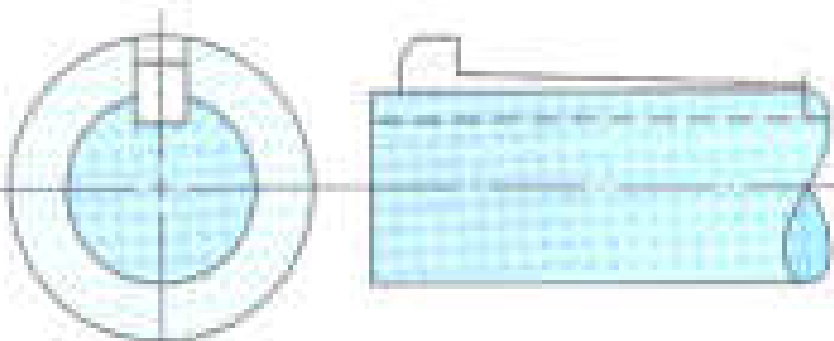


Widely used in automotive and machine tool industries

(e) Woodruff key

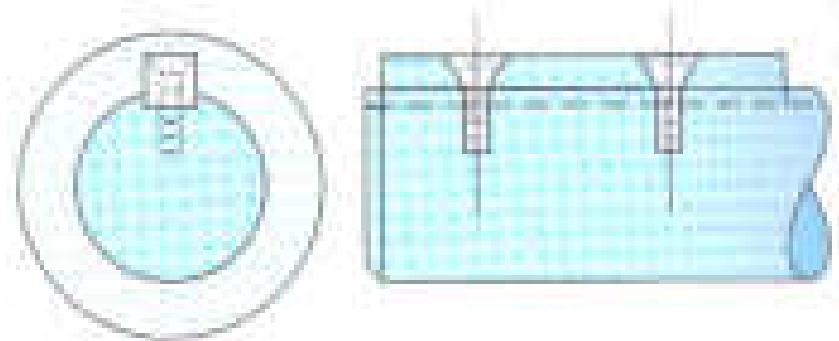


## Other common types of keys



Usually tapered, giving tight fit when driven into place; gb head facilitates removal

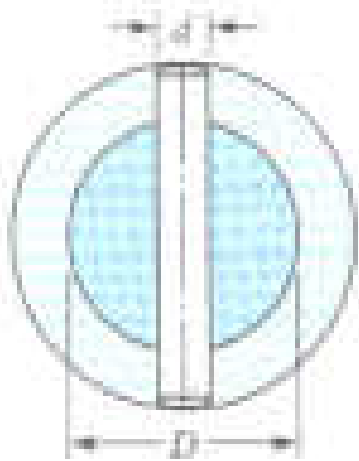
(f) Gib-head key



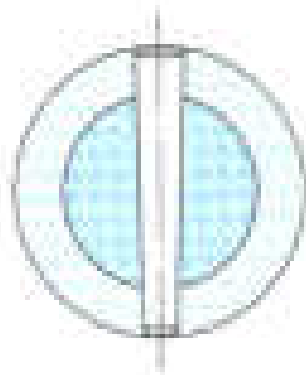
Key is screwed to shaft; hub is free to slide axially – easier sliding is obtained with two keys spaced 180° apart

(g) Feather key

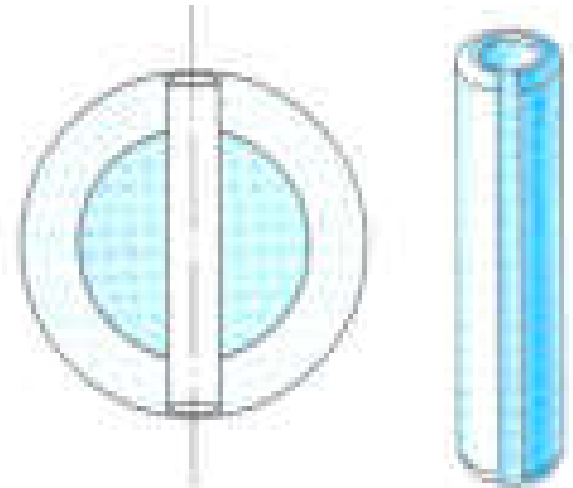
# Various types of collar pins



(a) Straight round pin

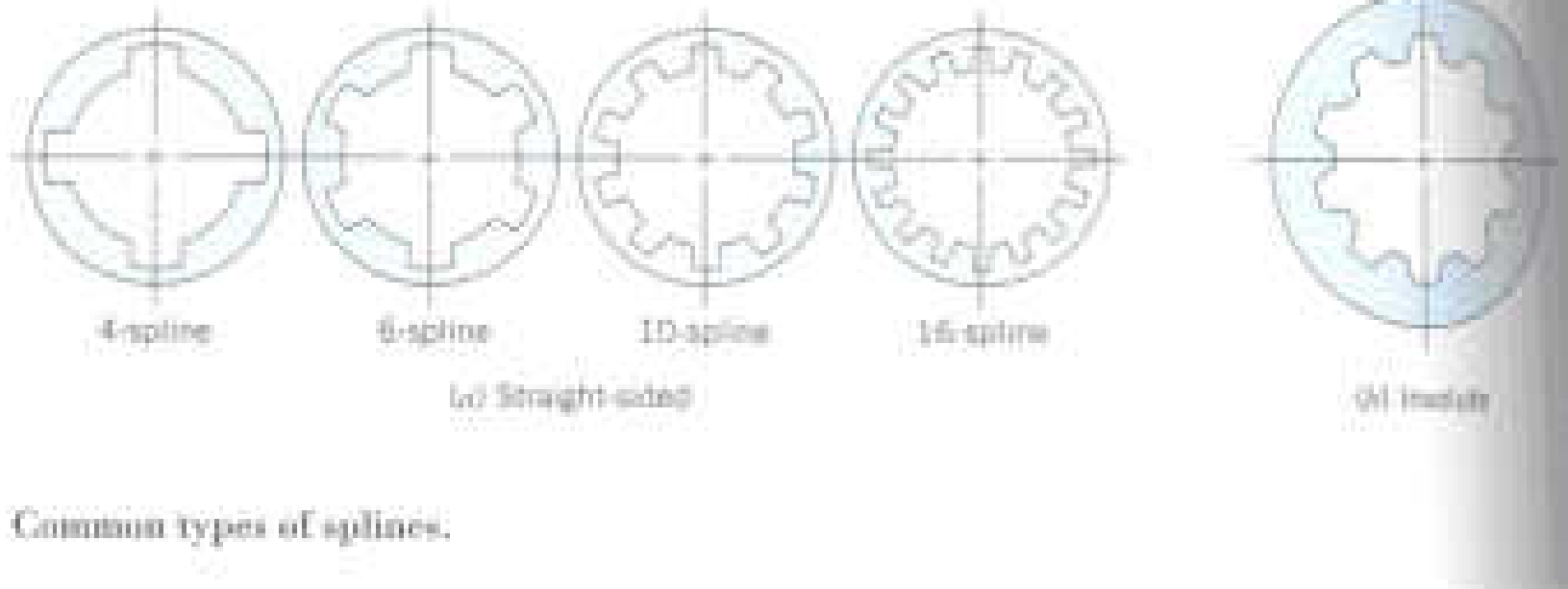


(b) Tapered round pin



(c) Split tubular spring pin

# Integrated splines in hubs and shafts allow axial motion and transmits torque



Common types of splines.

All keys, pins and splines give rise to stress concentration in the hub and shaft