BABA BANDA SINGH BAHADUR ENGINEERING COLLEGE

Department of Applied Sciences

Semester: Third**Subject:**Mathematics-III**Code:** BTAM-303-18

Topic: Laplace Transforms

1. State and prove First shifting theorem.

2. Using First shifting theorem prove that $L\{e^{at}\} = \frac{1}{s-a}$

3. If $L\{F(t)\} = f(s)$, prove that $L\{F(at)\} = \frac{1}{a} f\left(\frac{s}{a}\right)$.

4. Find the Laplace transform of the following functions:

   (a) $[t]$; $t > 0$ where $[t]$ stands for greatest integer function.

   (b) $\sqrt{t}e^{3t}$

   (c) $e^{-t}(\sin 2t - 2t\cos 2t)$

   (d) $\sin at.\sin bt$

5. If $L\left\{J_0\left(\sqrt{t}\right)\right\} = \frac{e^{-\frac{1}{4s}}}{s}$, then show that $L\left\{J_0\left(2\sqrt{t}\right)\right\} = \frac{1}{se^{\frac{1}{s}}}$.

6. Find the Laplace transform of $f(t)$ defined as:

   (a) $f(t) = \begin{cases} t & ; \quad 0 < t < 4 \\ 5 & ; \quad t > 4 \end{cases}$

   (b) $f(t) = \begin{cases} \cos t & ; \quad 0 < t < 2\pi \\ 0 & ; \quad t > 2\pi \end{cases}$

   (c) $f(t) = |t-1| + |t+1|, \quad t \geq 0.$

7. Find the inverse Laplace transform of the following functions:

(a) $\dfrac{1}{s(s-1)}$

(b) $\dfrac{s+1}{s^2+6s+25}$

(c) $\dfrac{21s-33}{(s-2)^3(s+1)}$

(d) $\dfrac{s+1}{s^2+2s}$

(e) $\dfrac{3s+1}{(s+1)^4}$

(f) $\dfrac{1}{(s+1)(s^2+2s+2)}$

8. Given that $L\left\{2\sqrt{\dfrac{t}{\pi}}\right\} = \dfrac{1}{s^{\frac{3}{2}}}$, show that $L\left\{\dfrac{1}{\sqrt{\pi t}}\right\} = \dfrac{1}{\sqrt{s}}$.

9. Show that $L^{-1}\left\{\tan^{-1}\dfrac{2}{s}\right\} = \dfrac{\sin 2t}{t}$.

10. Find the Laplace transform of the following functions:

(a) $\dfrac{e^{-at}-e^{-bt}}{t}$

(b) $\dfrac{\sin kt}{t}$

(c) $\dfrac{1-e^{-t}}{t}$

(d) $\displaystyle\int_0^t \frac{\sin t}{t}dt$

(e) $\displaystyle\int_0^t e^{-t}\cos t\ dt$

11. Evaluate the following:

(a) $\displaystyle\int_0^\infty \frac{e^{-2t}\sin^2 t}{t}.dt$

(b) $\int_0^{\infty} te^{-3t} \sin t \; dt$

(c) $\int_0^{\infty} \frac{e^{-t} - e^{-3t}}{t} \; dt$

12. Find the inverse Laplace transform of

(a) $\log\left(\frac{s+1}{s-1}\right)$

(b) $\log\left(\frac{s(s+1)}{s^2+4}\right)$

(c) $\cot^{-1}\left(\frac{s}{a}\right)$

(d) $\tan^{-1}\left(\frac{2}{s^2}\right)$

13. Using convolution theorem evaluate:

(a) $L^{-1}\left\{\frac{s^2}{\left(s^2+4\right)^2}\right\}$

(b) $L^{-1}\left\{\frac{s^2}{\left(s^2+a\right)\left(s^2+b\right)}\right\}$

(c) $L^{-1}\left\{\frac{1}{s\left(s^2-a^2\right)}\right\}$

(d) $L^{-1}\left\{\frac{s}{\left(s^2+a^2\right)^2}\right\}$

14. Show that $L^{-1}\left\{\frac{1}{s}\cos\frac{1}{s}\right\} = 1 - \frac{t^2}{(2!)^2} + \frac{t^4}{(4!)^2} - \frac{t^6}{(6!)^2} + \ldots\ldots\ldots\ldots\ldots$.

15. State Second Shifting theorem and use it to find the Laplace transform of

$$f(t) = \begin{cases} \cos t & ; \quad 0 < t < 2\pi \\ 0 & ; \quad t > 2\pi \end{cases}$$

16. Find the Laplace transform of Unit Step function.

17. Find the Laplace transform of Unit Impulse function.