Block diagrams

&

Signal flow graphs

Poles and Zeros and Transfer Functions

Transfer Function: A transfer function is defined as the ratio of the Laplace transform of the output to the input with all initial conditions equal to zero. Transfer functions are defined only for linear time invariant systems.

Considerations: Transfer functions can usually be expressed as the ratio of two polynomials in the complex variable, s.

Factorization: A transfer function can be factored into the following form.

$$G(s) = \frac{K(s+z_1)(s+z_2)\ldots(s+z_m)}{(s+p_1)(s+p_2)\ \ldots\ (s+p_n)}$$

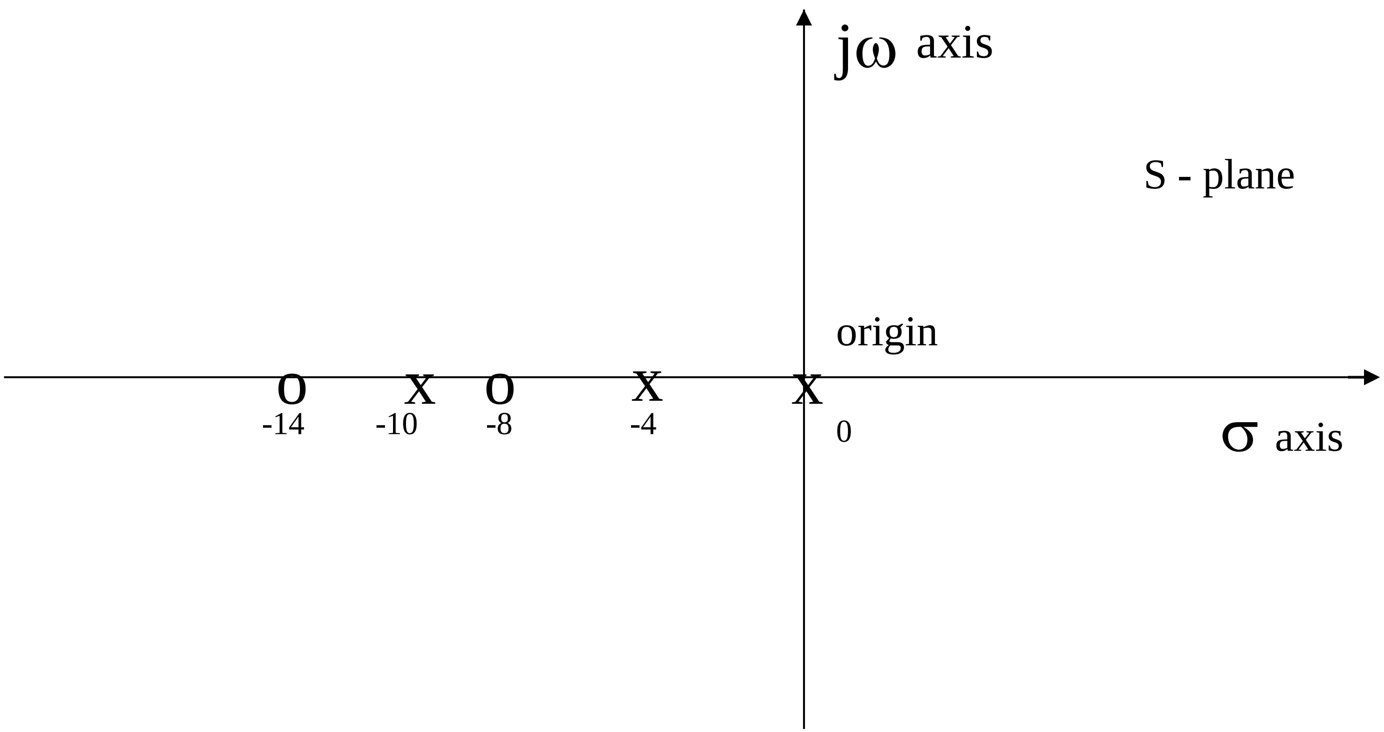
The roots of the numeator polynomial are called zeros.

The roots of the denominator polynomial are called poles.

Poles, Zeros and the S-Plane

<u>An Example</u>: You are given the following transfer function. Show the poles and zeros in the s-plane.

$$G(s) = \frac{(s+8)(s+14)}{s(s+4)(s+10)}$$



Poles, Zeros and Bode Plots

Characterization: Considering the transfer function of the previous slide. We note that we have 4 different types of terms in the previous general form: These are:

$$K_B, \quad \frac{1}{s}, \quad \frac{1}{(s/p+1)}, \quad (s/z+1)$$

Expressing in dB: Given the tranfer function:

$$G(jw) = \frac{K_B(jw/z+1)}{(jw)(jw/p+1)}$$

$$20\log|G(jw| = 20\log K_B + 20\log|(jw/z+1)| - 20\log|jw| - 20\log|jw/p+1|$$

Poles, Zeros and Bode Plots

Mechanics: We have 4 distinct terms to consider:

$20\log K_B$

$20\log|(jw/z + 1)|$

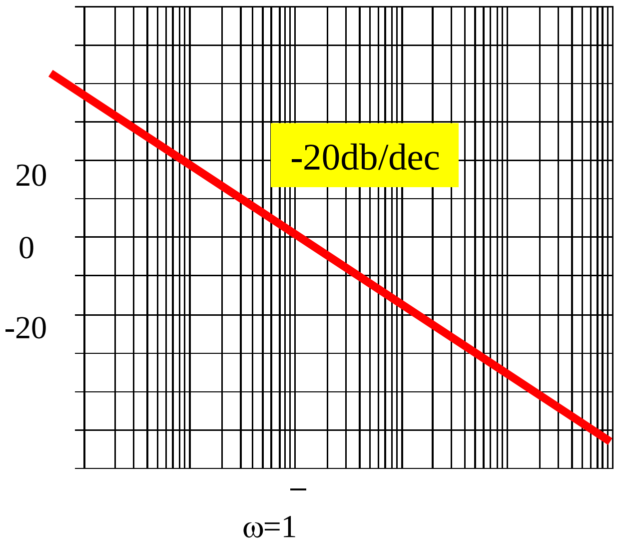
$-20\log|jw|$

$-20\log|(jw/p + 1)|$

Poles, Zeros and Bode Plots

Mechanics: The gain term, $20\log K_B$, is just so many dB and this is a straight line on Bode paper, independent of omega (radian frequency).

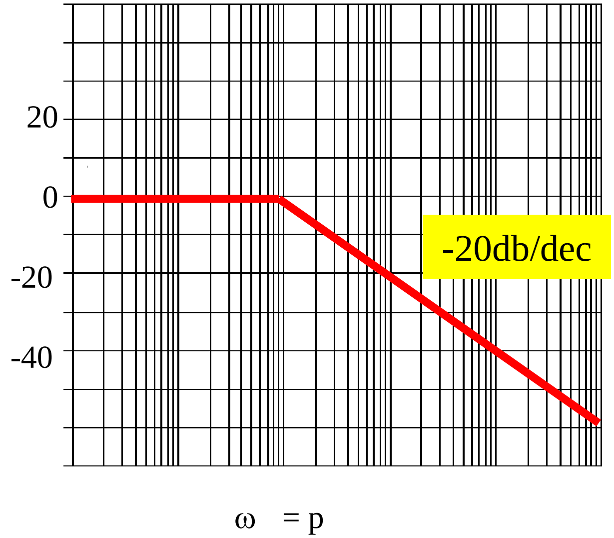
The term, $-20\log|jw| = -20\log w$, when plotted on semi-log paper is a straight line sloping at -20dB/decade. It has a magnitude of 0 at w = 1.



Poles, Zeros and Bode Plots

<u>Mechanics:</u>

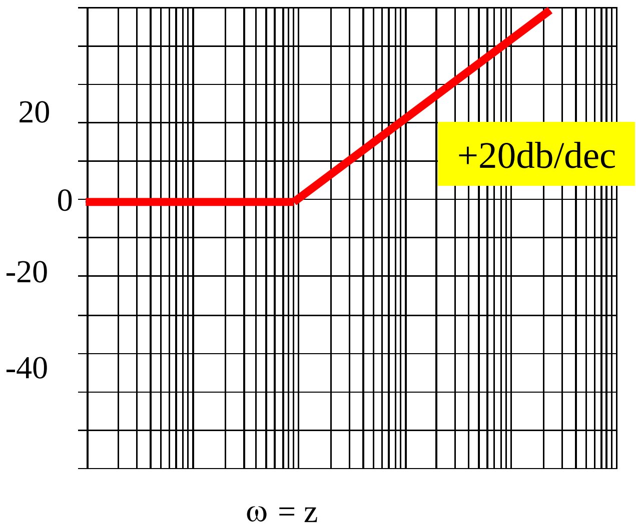
The term, - 20log|(jw/p + 1), is drawn with the following approximation: If w < p we use the approximation that –20log|(jw/p + 1)| = 0 dB, a flat line on the Bode. If w > p we use the approximation of –20log(w/p), which slopes at -20dB/dec starting at w = p. Illustrated below. It is easy to show that the plot has an error of -3dB at w = p and – 1 dB at w = p/2 and w = 2p. One can easily make these corrections if it is appropriate.

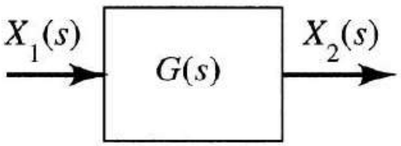


Poles, Zeros and Bode Plots

Mechanics:

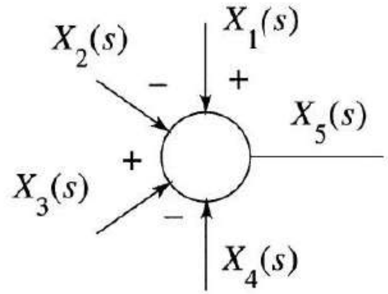
When we have a term of $20\log|(jw/z + 1)|$ we approximate it be a straight line of slop 0 dB/dec when $w < z$. We approximate it as $20\log(w/z)$ when $w > z$, which is a straight line on Bode paper with a slope of + 20dB/dec. Illustrated below.





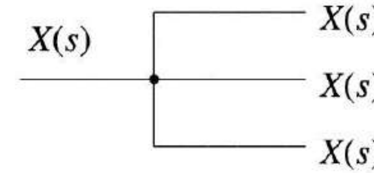
$X_2(s) = G(s)X_1(s)$

(a)
block

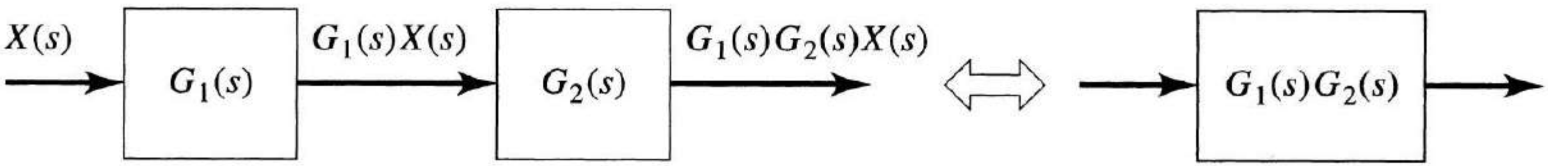


$X_5(s) = X_1(s) - X_2(s) + X_3(s) - X_4(s)$

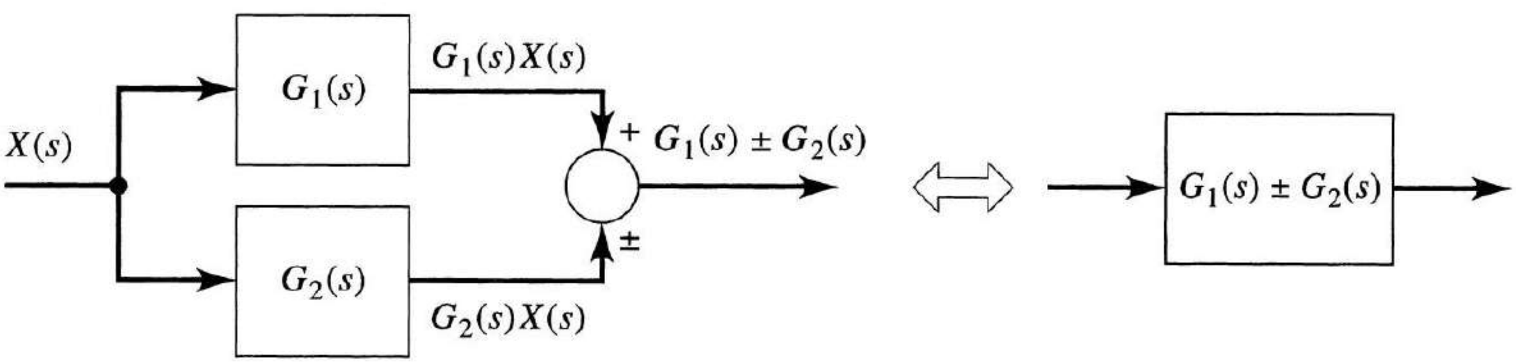
(b) ***summer***



(c)
pickoff point



(a)



(b)

Nyquist Plot or Polar Plot

- Nyquist Plots were invented by Nyquist - who worked at Bell Laboratories, the premiere technical organization in the U.S. at the time.

- Nyquist Plots are a way of showing frequency responses of linear systems.

- There are several ways of displaying frequency response data, including Bode' plots and Nyquist plots.

- Bode' plots use frequency as the horizontal axis and use two separate plots to display amplitude and phase of the frequency response.

- Nyquist plots display both amplitude and phase angle on a single plot, using frequency as a parameter in the plot.

- Nyquist plots have properties that allow you to see whether a system is stable or unstable.

Nyquist Plot

- A Nyquist plot is a polar plot of the frequency response function of a linear system.

- That means a Nyquist plot is a plot of the transfer function, $G(s)$ with $s = j\boldsymbol{\omega}$. That means you want to plot $G(j\boldsymbol{\omega})$.

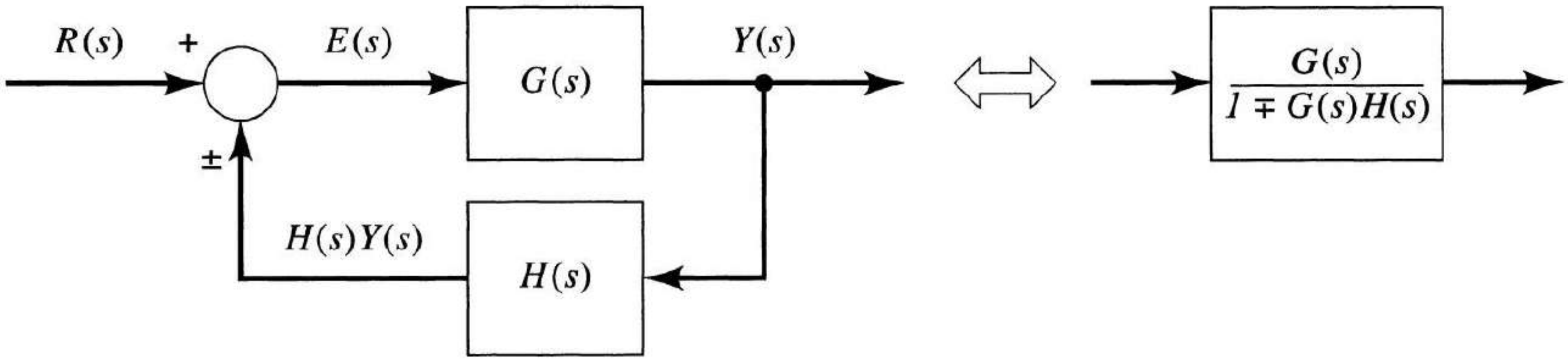
- $G(j\boldsymbol{\omega})$ is a complex number for any angular frequency, $\boldsymbol{\omega}$, so the plot is a plot of complex numbers.

- The complex number, $G(j\boldsymbol{\omega})$, depends upon frequency, so frequency will be a parameter if you plot the imaginary part of $G(j\boldsymbol{\omega})$ against the real part of $G(j\boldsymbol{\omega})$.

Sketch the Polar plot of Frequency Response

To sketch the polar plot of $G(j\omega)$ for the entire range of frequency ω, i.e., from 0 to infinity, there are four key points that usually need to be known:

1) The start of plot where $\omega = 0$,
2) The end of plot where $\omega = \infty$,
3) Where the plot crosses the real axis, i.e., $\mathrm{Im}(G(j\omega)) = 0$, and
4) Where the plot crosses the imaginary axis, i.e., $\mathrm{Re}(G(j\omega)) = 0$.

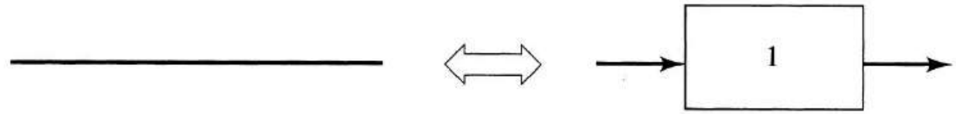


$$Y(s) = G(s)E(s)$$

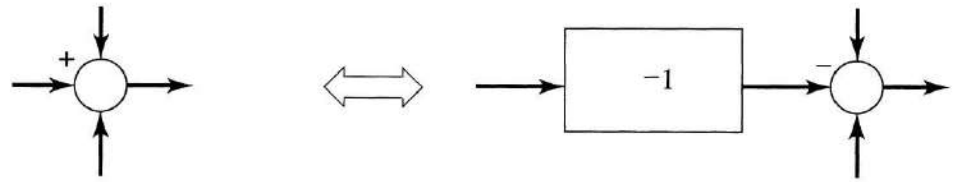
$$E(s) = R(s) \pm H(s)Y(s)$$

$$Y(s) = G(s)[R(s) \pm H(s)Y(s)] = G(s)R(s) \pm G(s)H(s)Y(s)$$

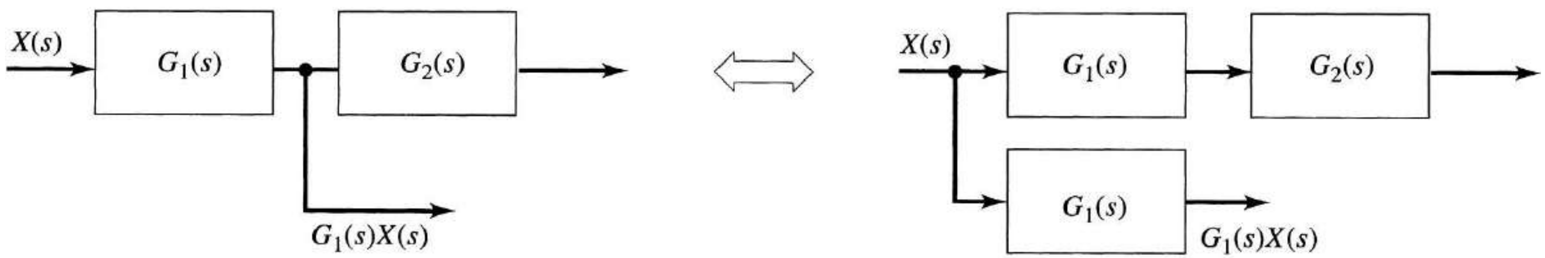
$$T(s) = \frac{Y(s)}{R(s)} = \frac{G(s)}{1 \mp G(s)H(s)}$$



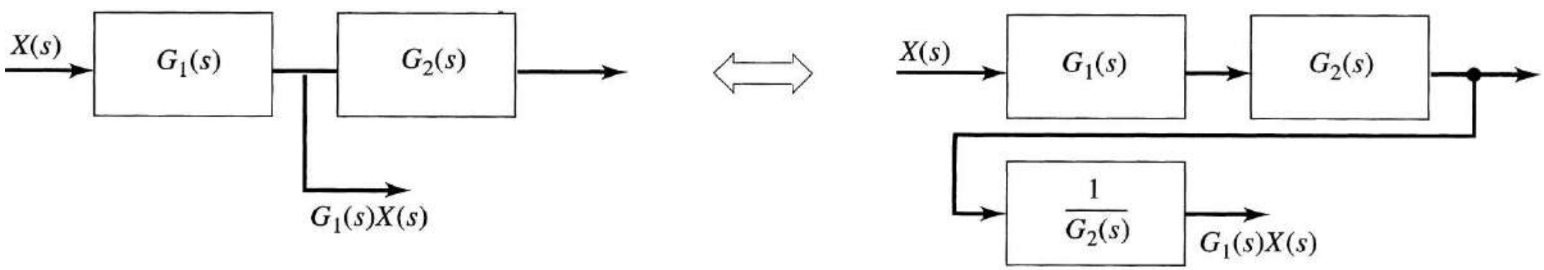
(a) Insertion or removal of unity gain



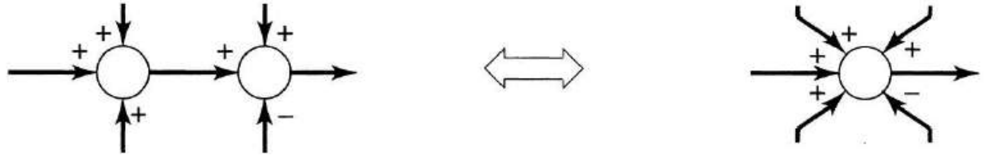
(b) Changing a summer sign



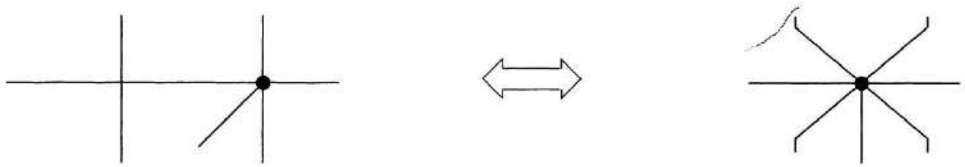
(c) Moving a pickoff point back



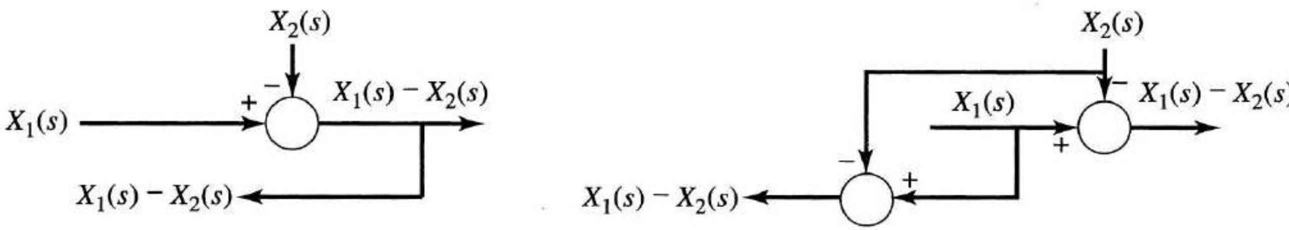
(d) Moving a pickoff point forward



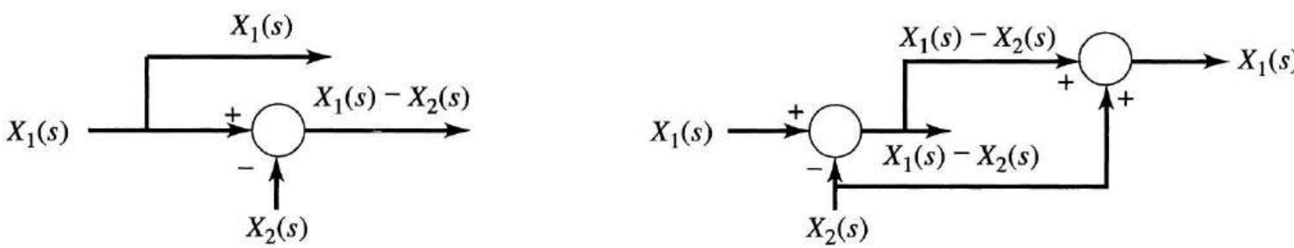
(*e*) Combining or expanding summations



(*f*) Combining or expanding junctions

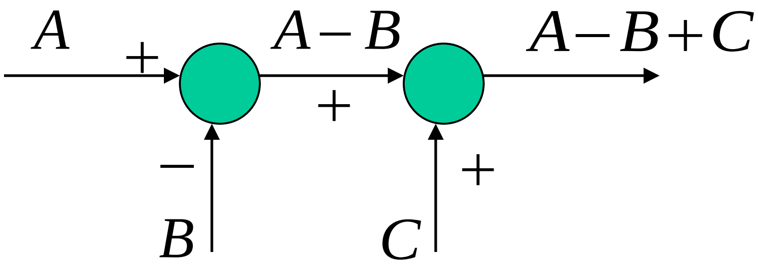


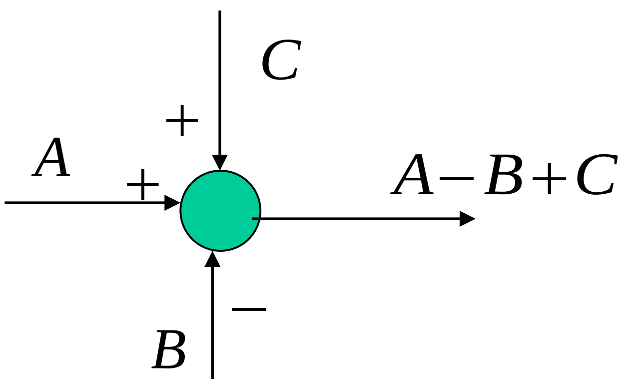
(*g*) Moving a pickoff point behind a summation



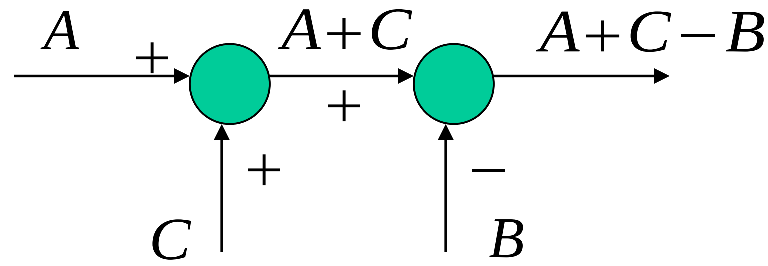
(*h*) Moving a pickoff point forward of a summation

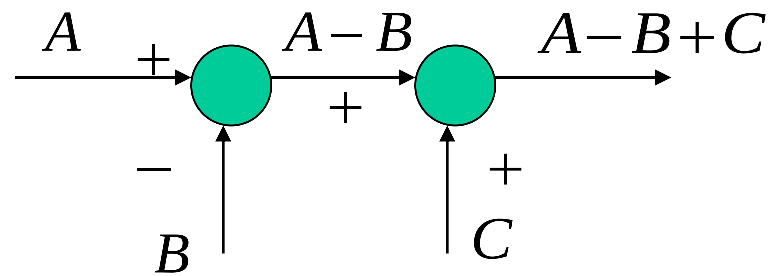
original



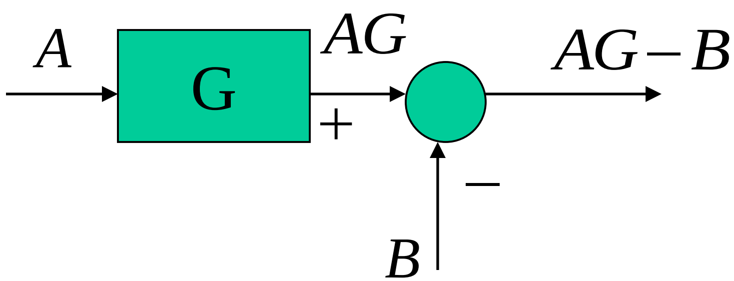


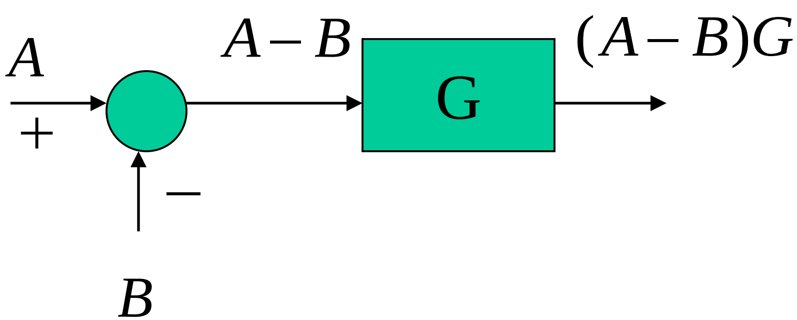
equivalent



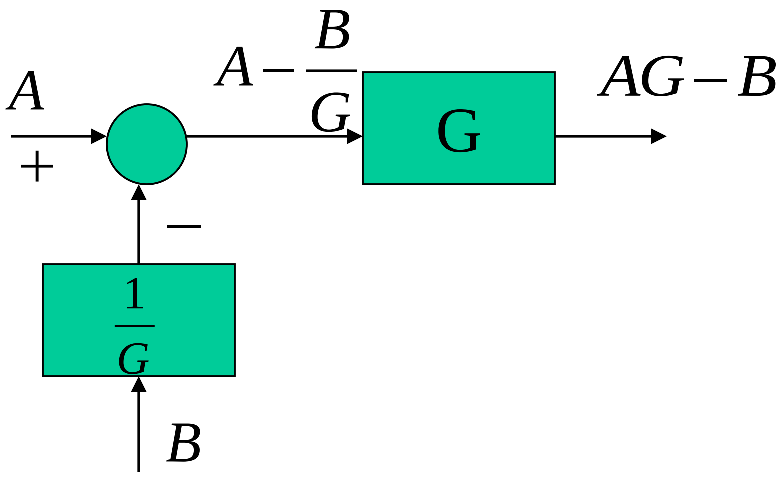


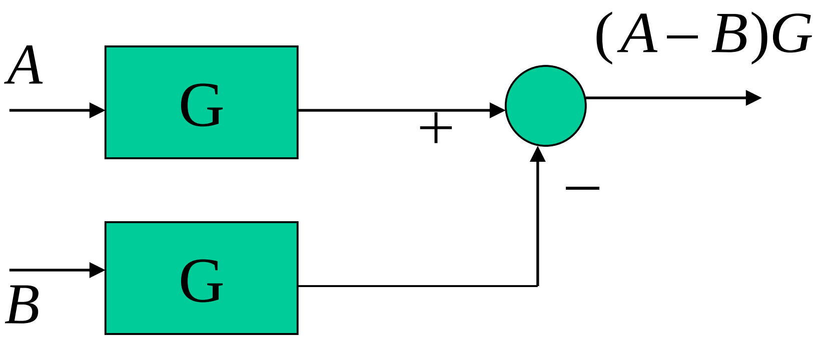
original





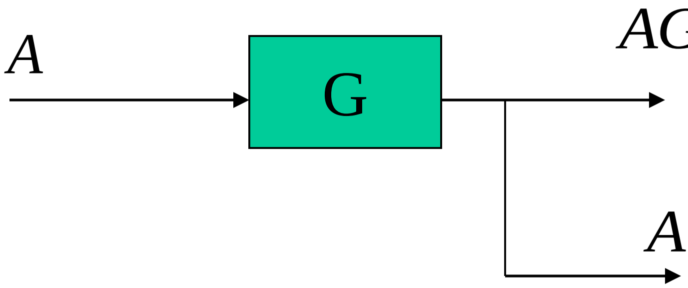
equivalent

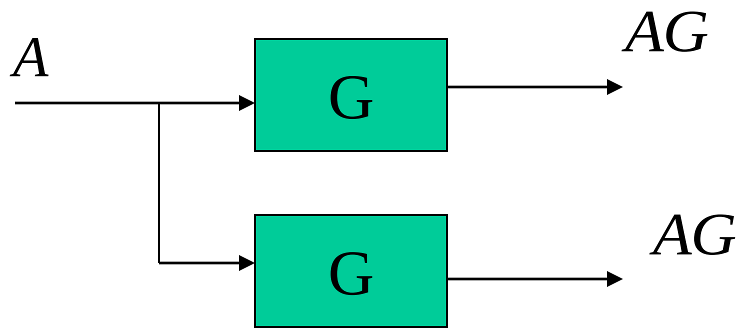


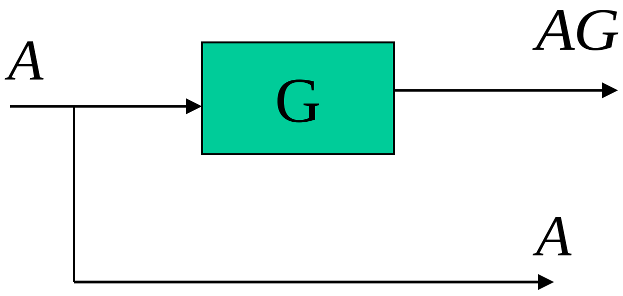


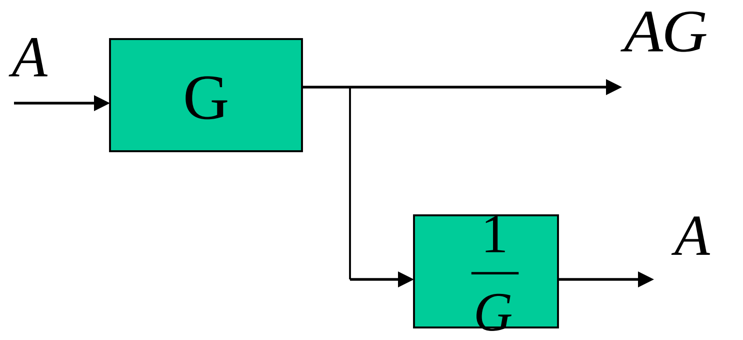
original

equivalent

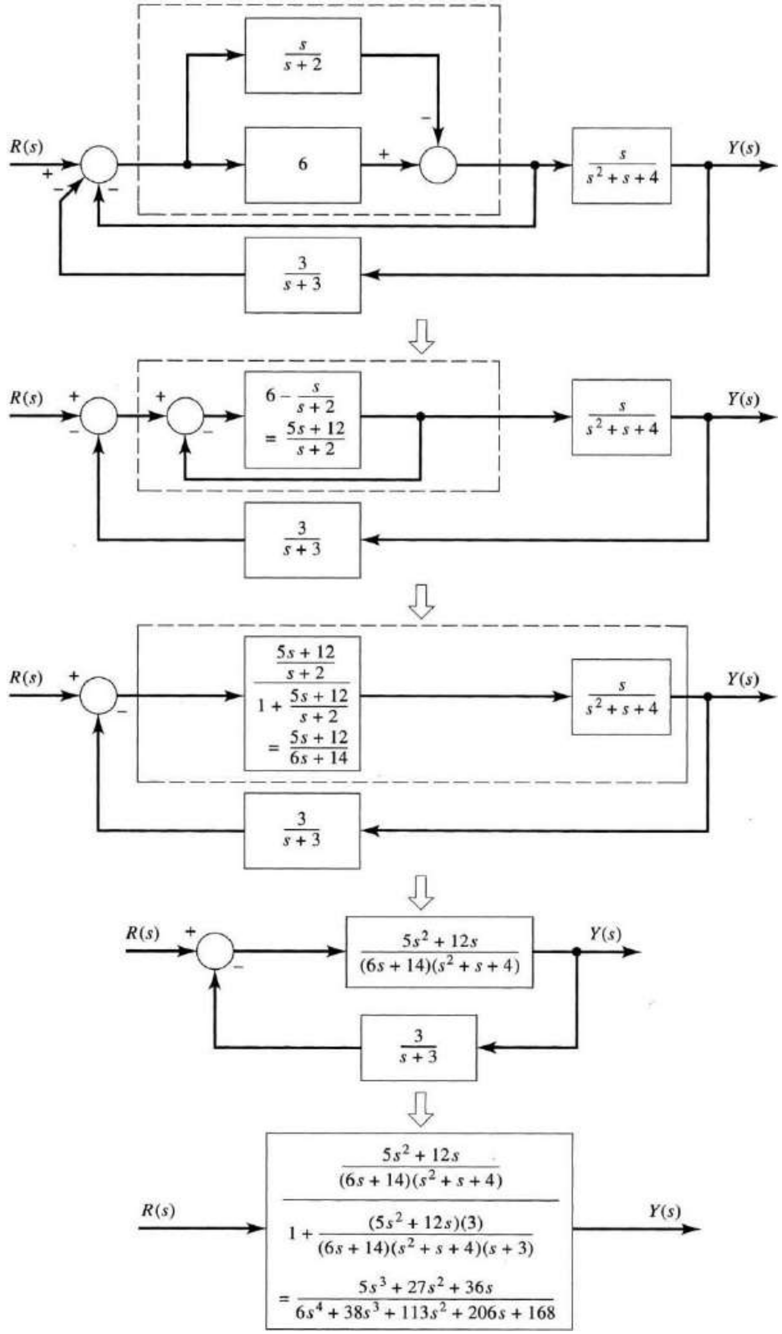






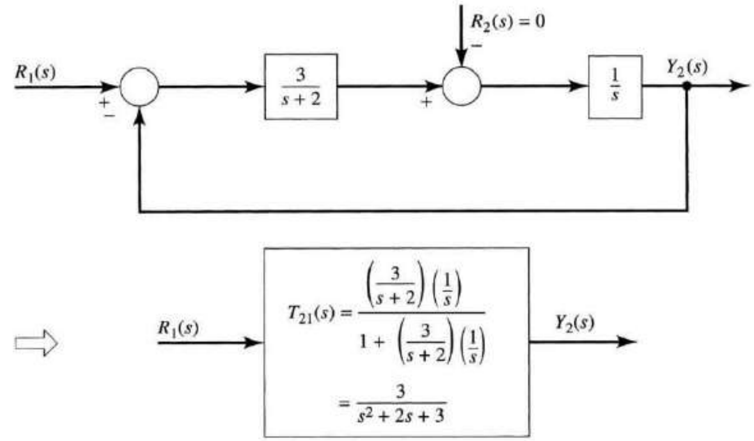


Example 1



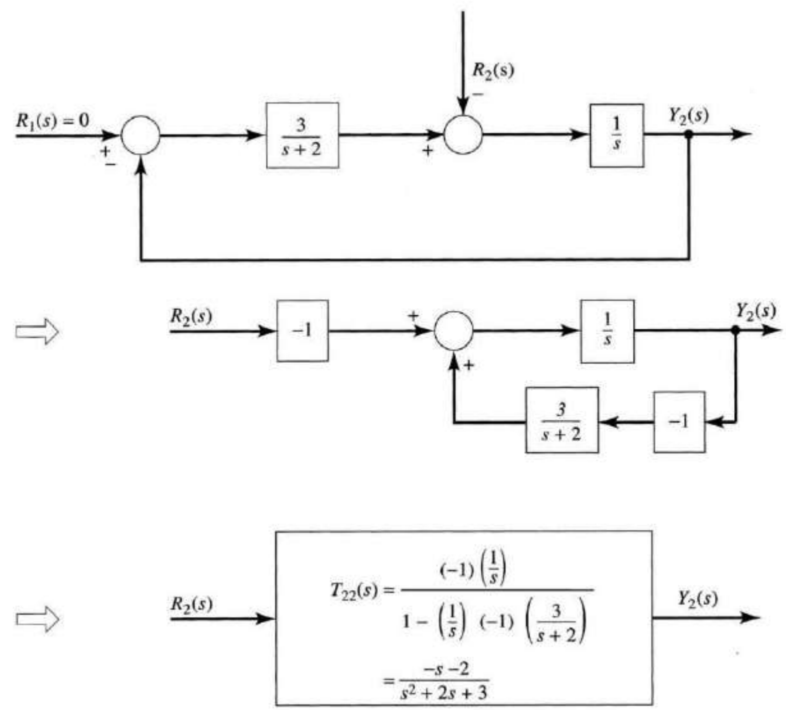
R(s)
Y(s)
s/(s + 2)
6
s/(s^2 + s + 4)
3/(s + 3)
6 − s/(s + 2) = (5s + 12)/(s + 2)
((5s + 12)/(s + 2)) / (1 + (5s + 12)/(s + 2)) = (5s + 12)/(6s + 14)
(5s^2 + 12s)/((6s + 14)(s^2 + s + 4))
((5s^2 + 12s)/((6s + 14)(s^2 + s + 4))) / (1 + (5s^2 + 12s)(3)/((6s + 14)(s^2 + s + 4)(s + 3)))
= (5s^3 + 27s^2 + 36s)/(6s^4 + 38s^3 + 113s^2 + 206s + 168)

Example 2



$R_2(s) = 0$
$R_1(s)$
$\frac{3}{s+2}$
$\frac{1}{s}$
$Y_2(s)$
$R_1(s)$
$T_{21}(s) = \frac{\left(\frac{3}{s+2}\right)\left(\frac{1}{s}\right)}{1+\left(\frac{3}{s+2}\right)\left(\frac{1}{s}\right)} = \frac{3}{s^2+2s+3}$
$Y_2(s)$

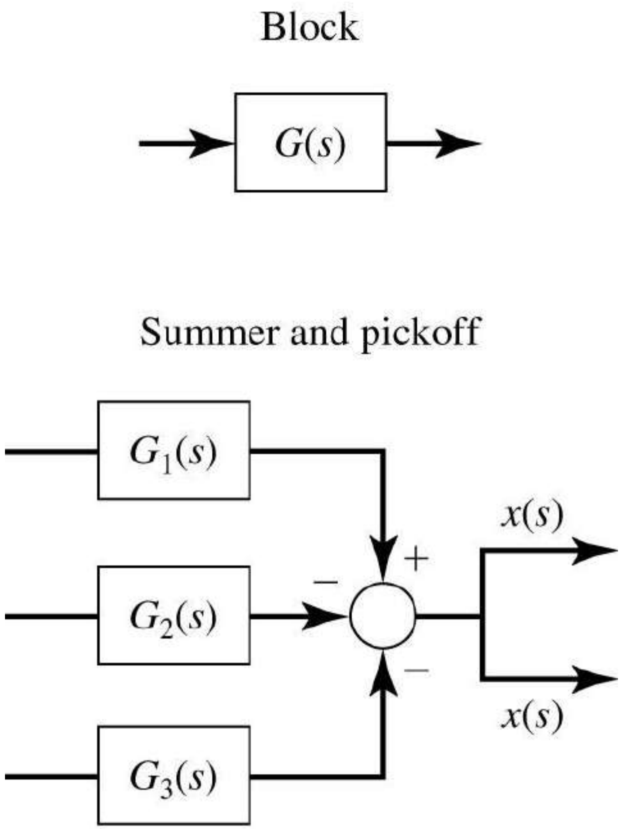
(a)



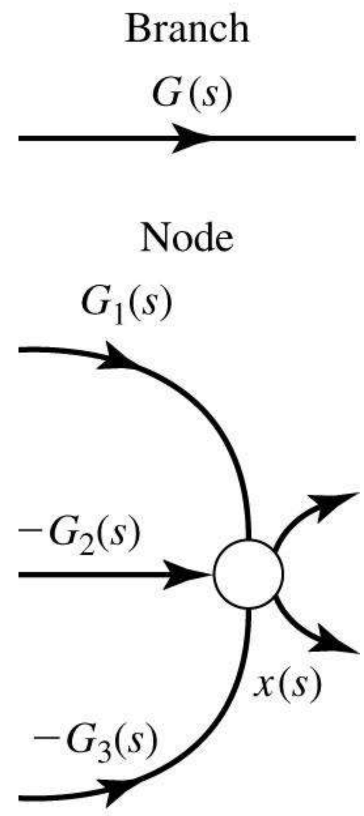
$R_2(s)$
$R_1(s) = 0$
$\frac{3}{s+2}$
$\frac{1}{s}$
$Y_2(s)$
$R_2(s)$
-1
$\frac{1}{s}$
$Y_2(s)$
$\frac{3}{s+2}$
-1
$R_2(s)$
$T_{22}(s) = \frac{(-1)\left(\frac{1}{s}\right)}{1-\left(\frac{1}{s}\right)(-1)\left(\frac{3}{s+2}\right)} = \frac{-s-2}{s^2+2s+3}$
$Y_2(s)$

(b)

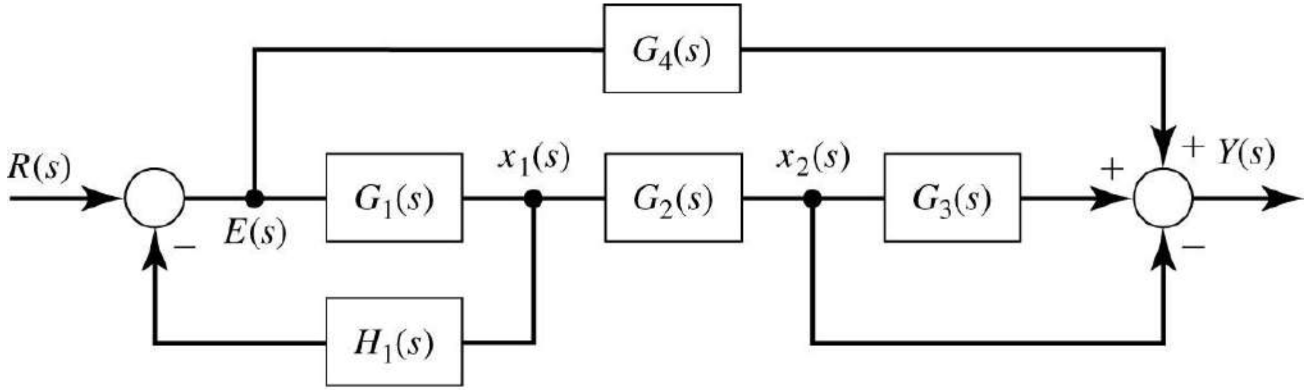
Signal flow graphs



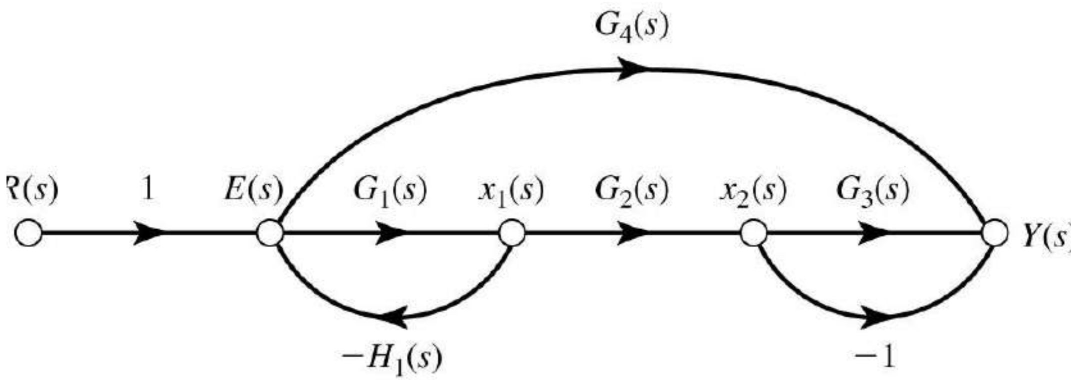
(a)



(b)


$G_4(s)$
$R(s)$
$E(s)$
$G_1(s)$
$x_1(s)$
$G_2(s)$
$x_2(s)$
$G_3(s)$
$H_1(s)$
$+$
$+$
$-$
$-$
$Y(s)$

(a)


$G_4(s)$
$R(s)$
1
$E(s)$
$G_1(s)$
$x_1(s)$
$G_2(s)$
$x_2(s)$
$G_3(s)$
$Y(s)$
$-H_1(s)$
-1

(b)

Mason's Rule

Mason's gain rule is as follows: the transfer function of a system with signal-input, signal-output flow graphs is

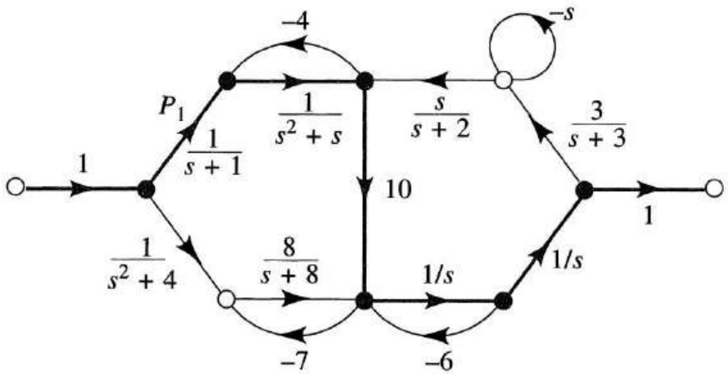
$$T(s) = \frac{p_1\Delta_1 + p_2\Delta_2 + p_3\Delta_3 + \cdots}{\Delta}$$

*Δ=1***-***(sum of all loop gains)+(sum of products of gains of all combinations if 2 nontouching loops)***-** *(sum of products of gains of all combinations if 3 nontouching loops)+…*

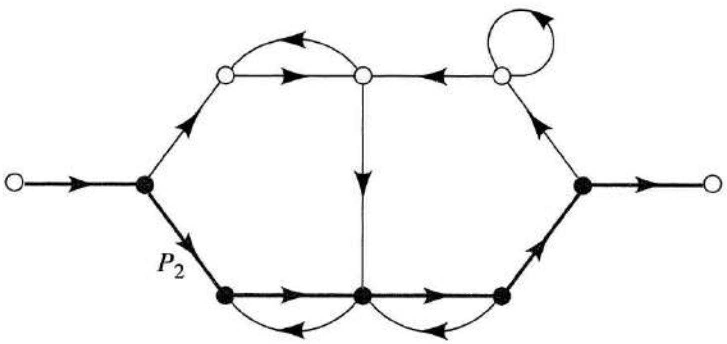
A ***path*** *is any succession of branches, from input to output, in the direction of the arrows, that does not pass any node more than once.*

A ***loop*** *is any closed succession of branches in the direction of the arrows that does not pass any node more than once.*

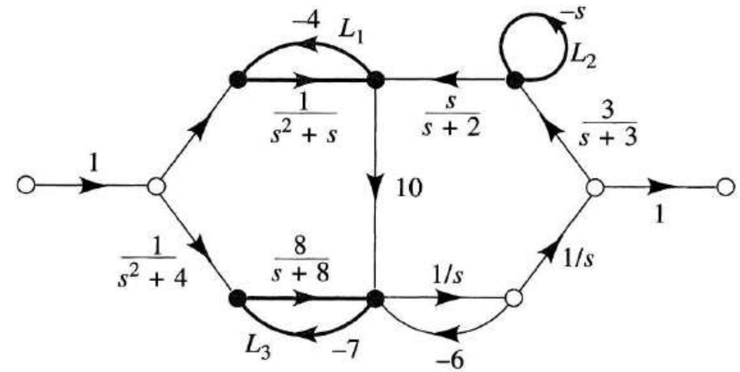
Example 3



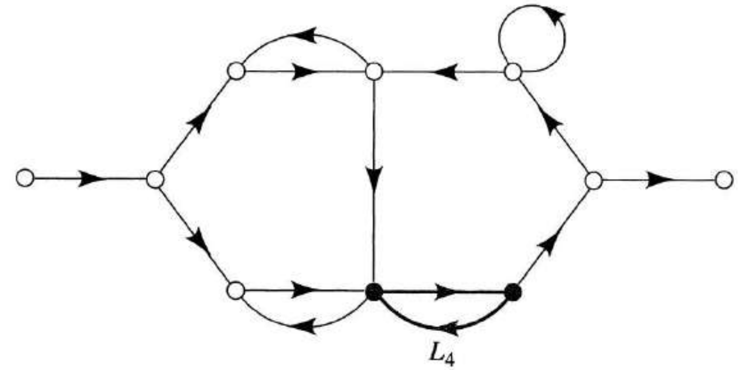
1
−4
−s
P_1
$\frac{1}{s+1}$
$\frac{1}{s^2+s}$
$\frac{s}{s+2}$
$\frac{3}{s+3}$
10
1
$\frac{1}{s^2+4}$
$\frac{8}{s+8}$
$1/s$
$1/s$
−7
−6



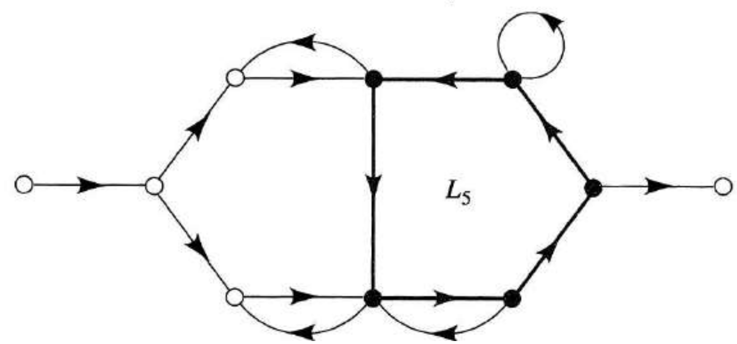
P_2



−4
L_1
−s
L_2
1
$\frac{1}{s^2+s}$
$\frac{s}{s+2}$
$\frac{3}{s+3}$
10
1
$\frac{1}{s^2+4}$
$\frac{8}{s+8}$
$1/s$
$1/s$
L_3
−7
−6



L_4



L_5

Example 4

find $\dfrac{y_5}{y_3}$