Network Theorems

Objectives

- At the end of this topic, you should be able to:
  - apply the superposition theorem for circuit analysis
  - apply Thevenin's theorem to simplify the circuit for analysis
  - apply Norton's theorem to simplify the circuit for analysis
  - understand maximum power transfer and perform circuit conversion

Superposition Theorem

- The Superposition theorem states that if a linear system is driven by more than one independent power source, the total response is the sum of the individual responses. The following example will show the step of finding branches current using superpostion theorem

Refer to the Figure 1, determine the branches current using superposition theorem.

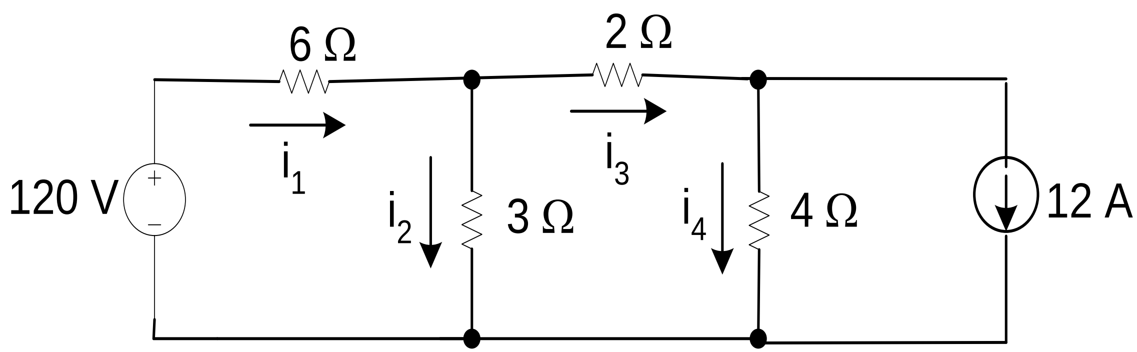


Figure 1

- Solution

- The application of the superposition theorem is shown in Figure 1, where it is used to calculate the branch current. We begin by calculating the branch current caused by the voltage source of 120 V. By substituting the ideal current with open circuit, we deactivate the current source, as shown in Figure 2.

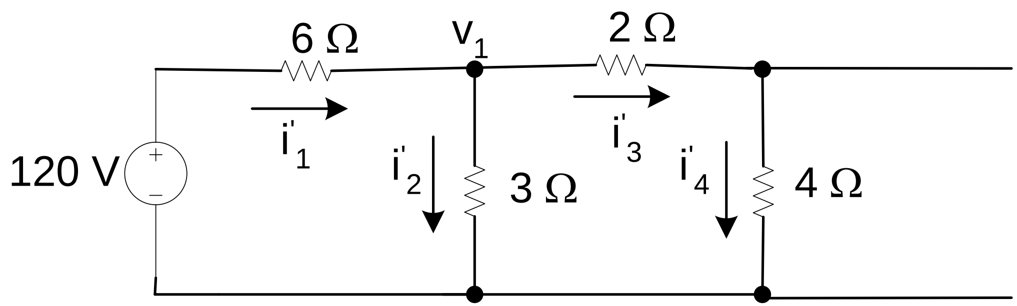


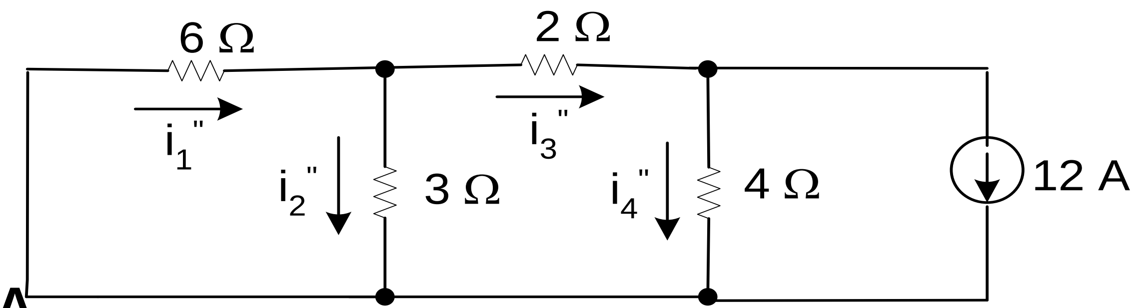
Figure 2

- To calculate the branch current, the node voltage across the 3Ω resistor must be known. Therefore

$$\frac{v_1 - 120}{6} + \frac{v_1}{3} + \frac{v_1}{2+4} = 0$$

where $v_1 = 30$ V

The equations for the current in each branch,



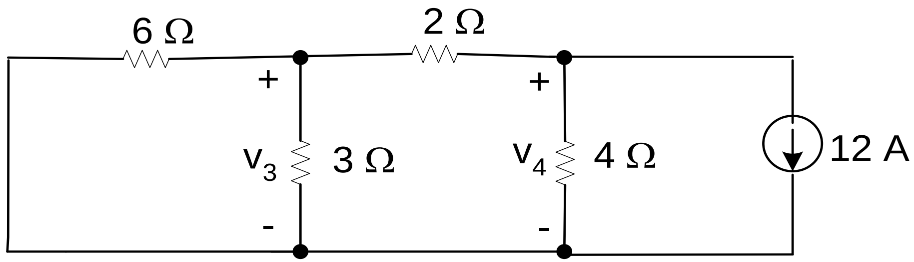
- $i'_1 = \dfrac{120 - 30}{6} = 15$ A

$i'_2 = \dfrac{30}{3} = 10$ A

$i'_3 = i'_4 = \dfrac{30}{6} = 5$ A

In order to calculate the current cause by the current source, we deactivate the ideal voltage source with a short circuit, as shown

- To determine the branch current, solve the node voltages across the 3Ω dan 4Ω resistors as shown in Figure 4



$$\frac{v_3}{3} + \frac{v_3}{6} + \frac{v_3 - v_4}{2} = 0$$

$$\frac{v_4 - v_3}{2} + \frac{v_4}{4} + 12 = 0$$

- The two node voltages are

- By solving these equations, we obtain
  - v_3 = -12 V
  - v_4 = -24 V

Now we can find the branches current,

$$i_1'' = \frac{-v_3}{6} = \frac{12}{6} = 2A$$

$$i_2'' = \frac{v_3}{3} = \frac{-12}{3} = -4A$$

$$i_3'' = \frac{v_3 - v_4}{2} = \frac{-12+24}{2} = 6A$$

$$i_4'' = \frac{v_4}{4} = \frac{-24}{4} = -6A$$

To find the actual current of the circuit, add the currents due to both the current and voltage source,

$$i_1 = i'_1 + i''_1 = 15 + 2 = 17\,A$$

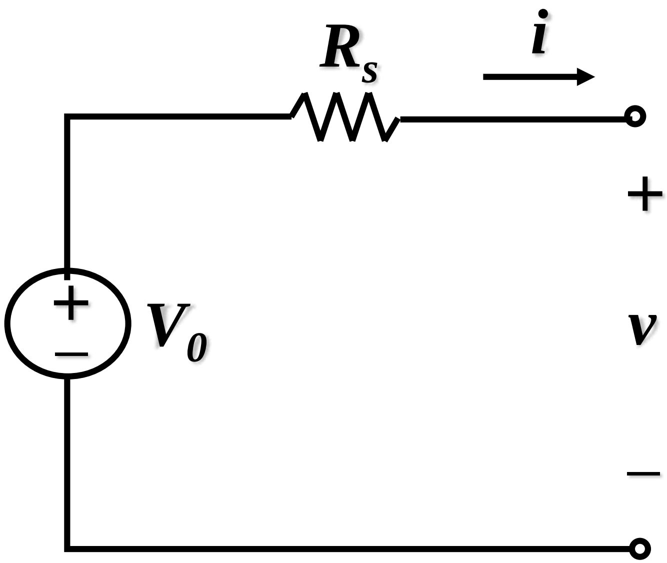
$$i_2 = i'_2 + i''_2 = 10 - 4 = 6\,A$$

$$i_3 = i'_3 + i''_3 = 5 + 6 = 11\,A$$

$$i_4 = i'_4 + i''_4 = 5 - 6 = -1\,A$$

Thevenin and Norton Equivalent Circuits

M. Leon Thévenin (1857-1926), published his famous theorem in 1883.



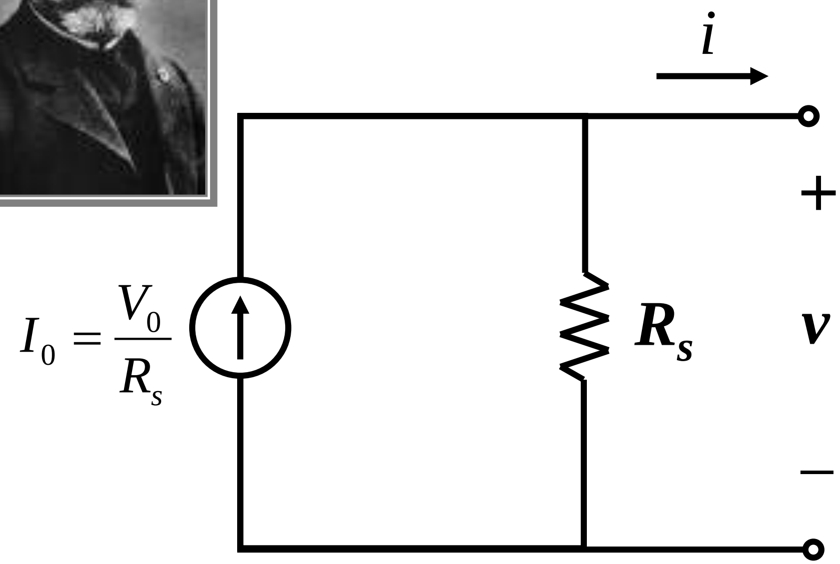


Fig.2.17 (a) Thevenin equivalent circuit ; (b) Norton equivalent circuit

$$v = V_0 - R_s\, i$$

$$i = I_0 - \frac{v}{R_s}$$

The equivalence of these two circuits is a special case of the *Thevenin and Norton Theorem*

Thevenin & Norton Equivalent Circuits

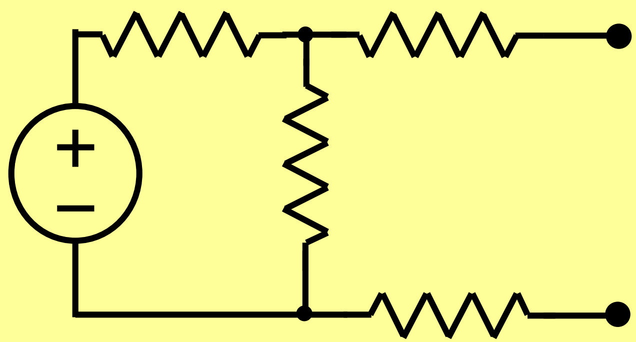
- *Thevenin's Theorem states that it is possible to simplify any linear circuit, no matter how complex, to an equivalent circuit with just a single voltage source and series resistance connected to a load.*

  A series combination of Thevenin equivalent voltage source V_0 and Thevenin equivalent resistance R_s

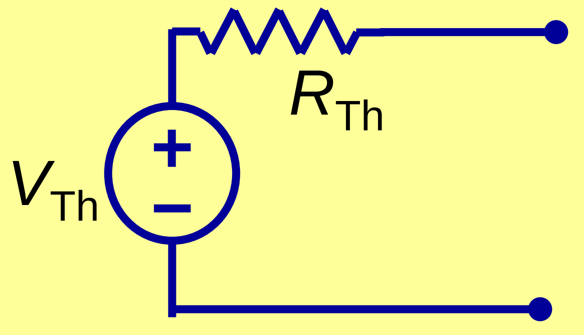
- Norton's Theorem states that it is possible to simplify any linear circuit, no matter how complex, to an equivalent circuit with just a single current source and parallel resistance connected to a load. Norton form:

  A parallel combination of Norton equivalent current source I_0 and Norton equivalent resistance R_s

Thévenin's Theorem: A **resistive circuit** can be represented by **one voltage source** and **one resistor**:

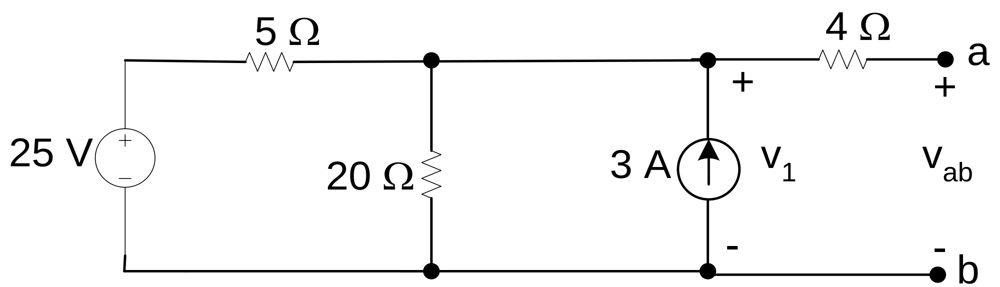


Resistive Circuit



Thévenin Equivalent Circuit

- Example
Refer to the Figure 6, find the Thevenin equivalent circuit.



- Solution

- In order to find the Thevenin equivalent circuit for the circuit shown in Figure 6, calculate the open circuit voltage, vab. Note that when the a, b terminals are open, there is no current flow to 4Ω resistor. Therefore, the voltage vab is the same as the voltage across the 3A current source, labeled v_1.
- To find the voltage v_1, solve the equations for the singular node voltage. By choosing the bottom right node as the reference node,

$$\frac{v_1 - 25}{5} + \frac{v_1}{20} - 3 = 0$$

- By solving the equation, v1 = 32 V. Therefore, the Thevenin voltage Vth for the circuit is 32 V.
- The next step is to short circuit the terminals and find the short circuit current for the circuit shown in Figure 7. Note that the current is in the same direction as the falling voltage at the terminal.

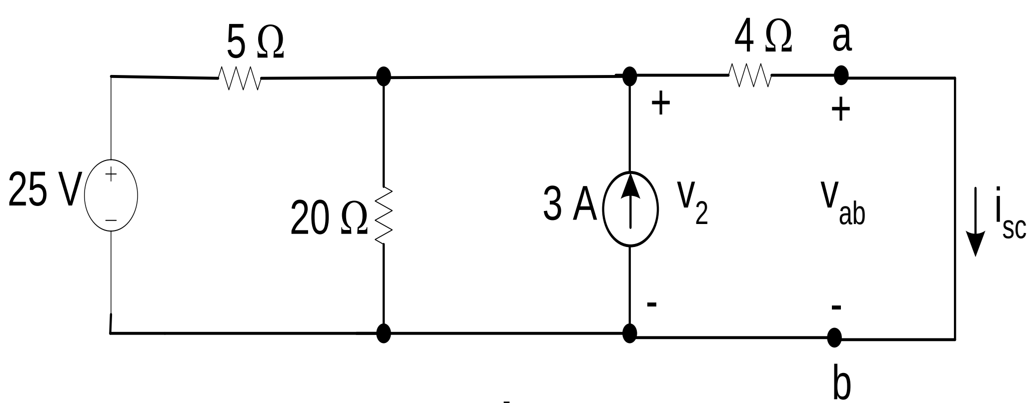


Figure 7

Current i_{sc} can be found if v_2 is known. By using the bottom right node as the reference node, the equationfor v_2 becomes

By solving the above equation, v_2 = 16 V. Therefore, the short circuit current i_{sc} is

$$\frac{v_2 - 25}{5} + \frac{v_2}{20} - 3 + \frac{v_2}{4} = 0$$

$$i_{sc} = \frac{16}{4} = 4A$$

The Thevenin resistance R_{Th} is

$$R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{32}{4} = 8\Omega$$

Figure 8 shows the Thevenin equivalent circuit for the Figure 6.

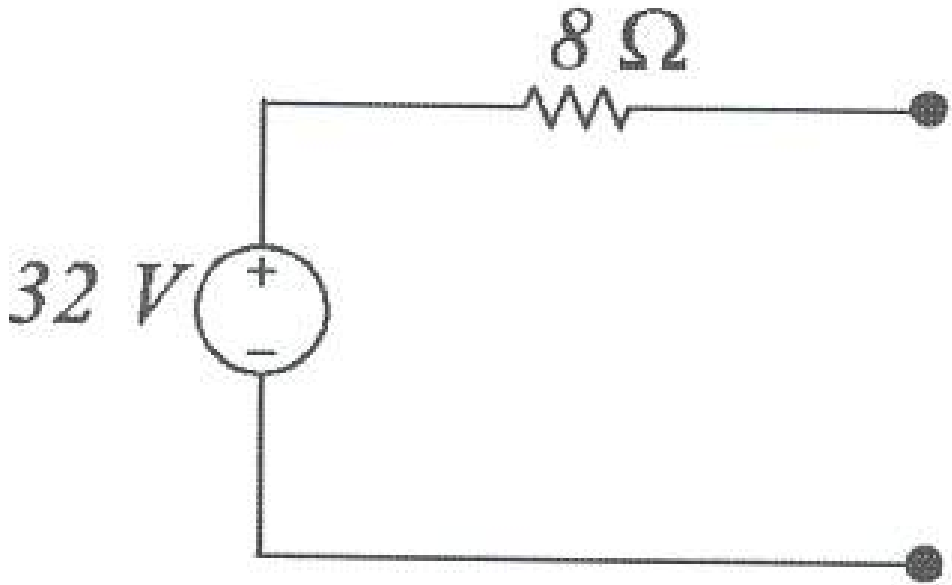


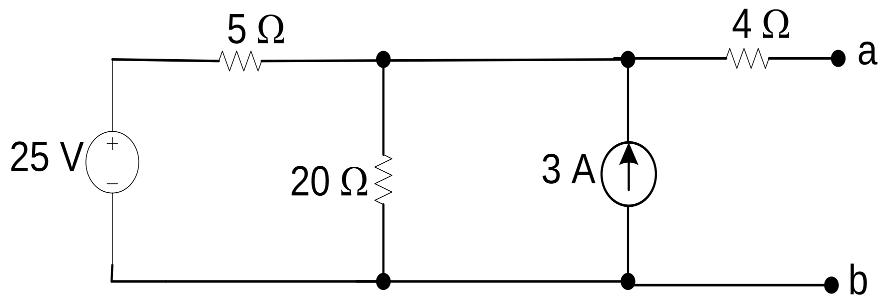
Figure 8

Norton’s Theorem

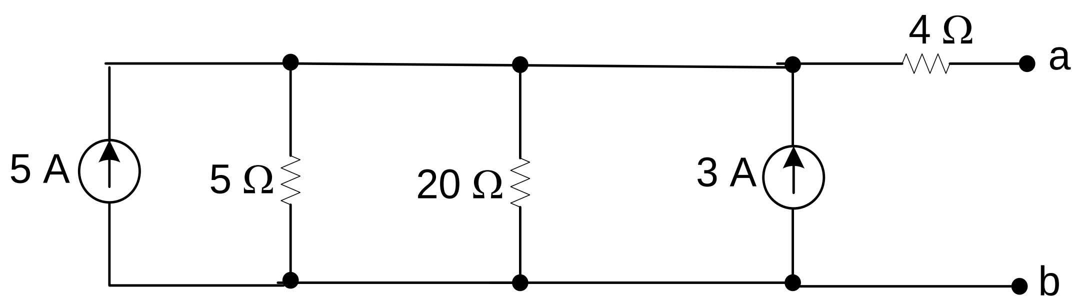
- The Norton equivalent circuit contains an independent current source which is parallel to the Norton equivalent resistance. It can be derived from the Thevenin equivalent circuit by using source transformation. Therefore, the Norton current is equivalent to the short circuit current at the terminal being studied, and Norton resistance is equivalent to Thevenin resistance.

- Example 3

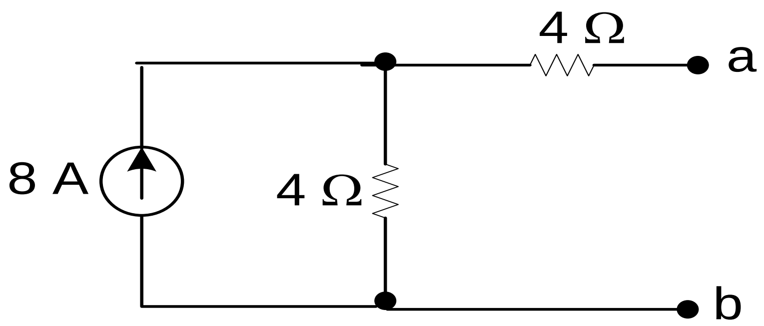
Derive the Thevenin and Norton equivalent circuits of Figure 6.



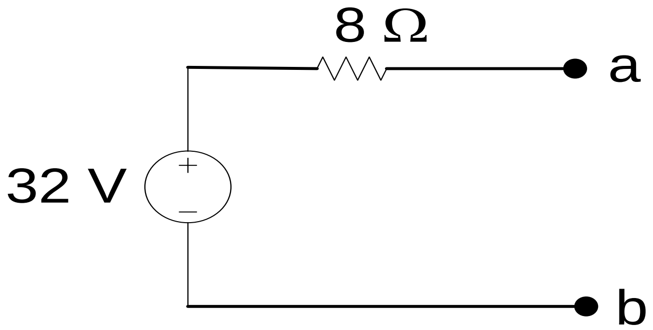
- Solution
- Step 1: Source transformation (The 25V voltage source is converted to a 5 A current source.)



Step 2: Combination of parallel source and parallel resistance



Step 3: Source transformation (combined serial resistance to produce the Thevenin equivalent circuit.)



- Step 4: Source transformation (To produce the Norton equivalent circuit. The current source is 4A (I = V/R = 32 V/8 Ω))

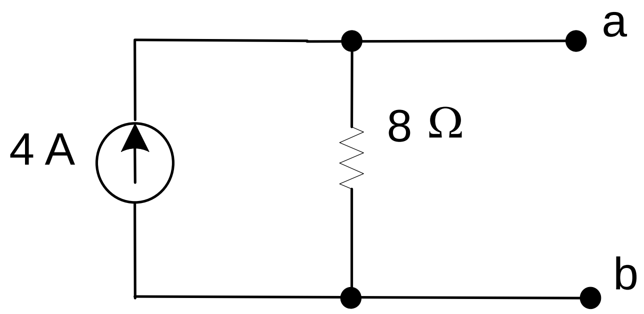


Figure 9 Steps in deriving Thevenin and Norton equivalent circuits.

Maximum Power Transfer

- Maximum power transfer can be illustrated by Figure 10. Assume that a resistance network contains independent and dependent sources, and terminals a and b to which the resistance RL is connected. Then determine the value of RL that allows the delivery of maximum power to the load resistor.

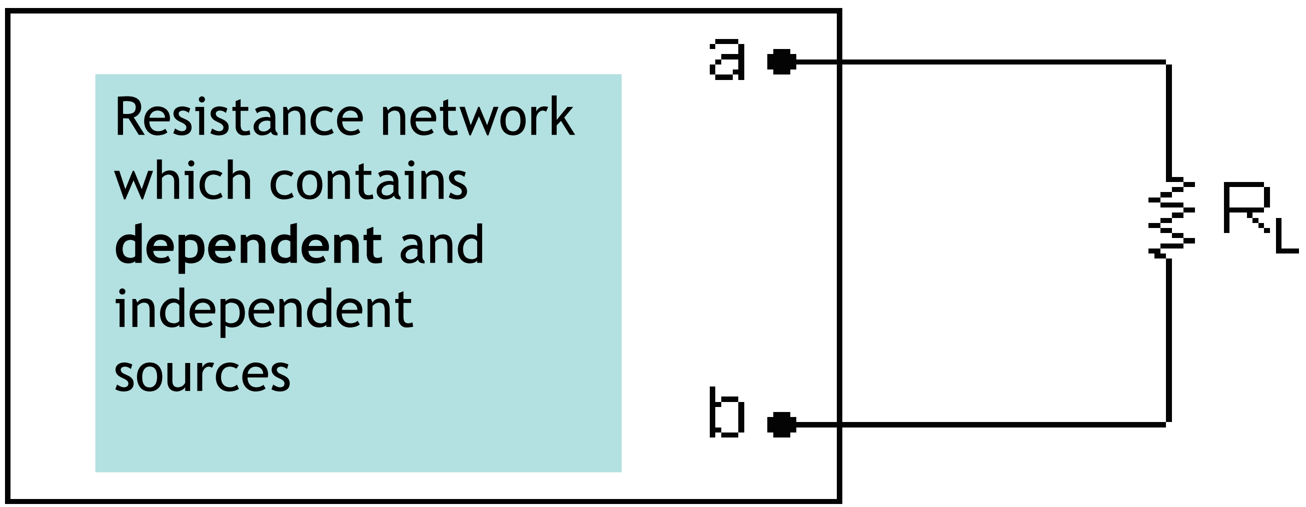

Resistance network which contains **dependent** and independent sources
a
b
R_L

Figure 10

- Maximum power transfer happens when the load resistance R_L is equal to the Thevenin equivalent resistance, R_{Th}. To find the maximum power delivered to R_L,

$$p_{max} = \frac{V_{Th}{}^2 R_L}{(2R_L)^2} = \frac{V_{Th}{}^2}{4R_L}$$

Circuit Transformation

- The configuration of circuit connection can be changed to make the calculation easier. There are TWO type of transformations which are Delta (Δ) to star connection (Y) and vice versa.

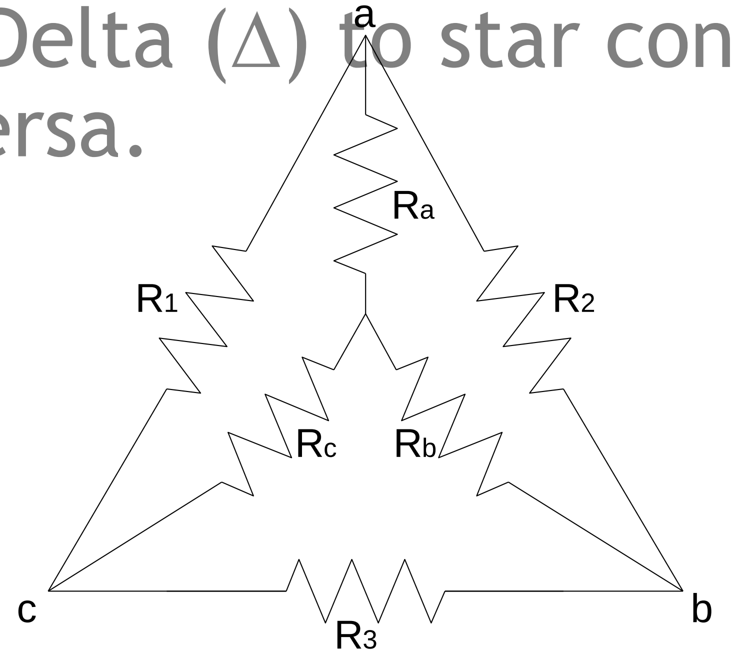


Figure 12 Delta and Star Circuit Connection

- Delta (Δ) to star (Y) transformation:

$$\mathrm{R}_{\mathrm{a}} = \frac{R_1 R_2}{\mathrm{R}_1 + R_2 + R_3}$$

$$\mathrm{R}_{\mathrm{b}} = \frac{R_2 R_3}{\mathrm{R}_1 + R_2 + R_3}$$

$$\mathrm{R}_{\mathrm{c}} = \frac{R_1 R_3}{\mathrm{R}_1 + R_2 + R_3}$$

- Star (Y) to Delta (Δ) transformation:

$$R_1 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$

$$R_2 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}$$

$$R_3 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}$$

- *Thank You*