Chapter 1

Introduction to Fluid Mechanics

Definition

- **Mechanics** is the oldest physical science that deals with both stationary and moving bodies under the influence of forces.
- The branch of mechanics that deals with bodies at rest is called **statics**, while the branch that deals with bodies in motion is called **dynamics**.
- The subcategory **fluid mechanics** is defined as the science that deals with the behavior of fluids at rest (*fluid statics)* or in motion (*fluid dynamics), and the interaction of fluids with solids* or other fluids at the boundaries.
- The study of fluids at rest is called **fluid statics**.

Definition

- The study of f1uids in motion, where pressure forces are not considered, is called **fluid kinematics** and if the pressure forces are also considered for the fluids in motion. that branch of science is called **fluid dynamics**.
- Fluid mechanics itself is also divided into several categories.
- The study of the motion of fluids that are practically incompressible (such as liquids, especially water, and gases at low speeds) is usually referred to as **hydrodynamics.**
- A subcategory of hydrodynamics is **hydraulics,** which deals with liquid flows in pipes and open channels.

Definition

- **Gas dynamics** deals with the flow of fluids that undergo significant density changes, such as the flow of gases through nozzles at high speeds.
- The category **aerodynamics** deals with the flow of gases (especially air) over bodies such as aircraft, rockets, and automobiles at high or low speeds.
- Some other specialized categories such as **meteorology, oceanography, and hydrology** deal with naturally occurring flows.

What is a Fluid?

- A substance exists in three primary phases: solid, liquid, and gas. A substance in the liquid or gas phase is referred to as a **fluid.**
- Distinction between a solid and a fluid is made on the basis of the substance's ability to resist an applied shear (or tangential) stress that tends to change its shape.
- A solid can resist an applied shear stress by deforming, whereas a fluid deforms continuously under the influence of shear stress, no matter how small.
- In solids stress is proportional to *strain, but in fluids stress is proportional* to *strain rate*.

What is a Fluid?

- *When a constant shear force is applied, a solid eventually* stops deforming, at some fixed strain angle, whereas a fluid never stops deforming and approaches a certain rate of strain.

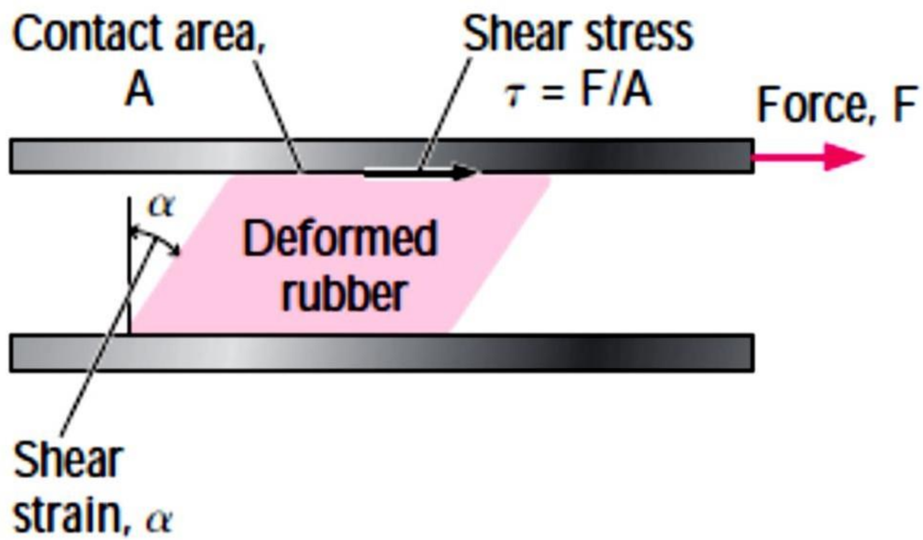
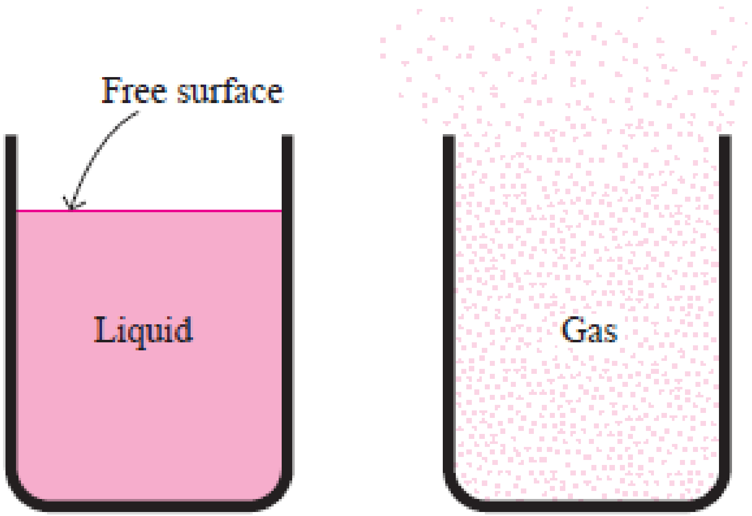


Figure.
Deformation of a rubber eraser placed between two parallel plates under the influence of a shear force.

What is a Fluid?

- In a liquid, molecules can move relative to each other, but the volume remains relatively constant because of the strong cohesive forces between the molecules.
- As a result, a liquid takes the shape of the container it is in, and it forms a free surface in a larger container in a gravitational field.
- A gas, on the other hand, expands until it encounters the walls of the container and fills the entire available space.
- This is because the gas molecules are widely spaced, and the cohesive forces between them are very small.
- Unlike liquids, gases cannot form a free surface



What is a Fluid?

- Differences between liquid and gases

Liquid	Gases
Difficult to compress and often regarded as incompressible	Easily to compress – changes of volume is large, cannot normally be neglected and are related to temperature
Occupies a fixed volume and will take the shape of the container	No fixed volume, it changes volume to expand to fill the containing vessels
A free surface is formed if the volume of container is greater than the liquid.	Completely fill the vessel so that no free surface is formed.

Application areas of Fluid Mechanics

- Mechanics of fluids is extremely important in many areas of engineering and science. Examples are:
- **Biomechanics**
  - Blood flow through arteries and veins
  - Airflow in the lungs
  - Flow of cerebral fluid
- **Households**
  - Piping systems for cold water, natural gas, and sewage
  - Piping and ducting network of heating and air-conditioning systems
  - refrigerator, vacuum cleaner, dish washer, washing machine, water meter, natural gas meter, air conditioner, radiator, etc.
- **Meteorology and Ocean Engineering**
  - Movements of air currents and water currents

Application areas of Fluid Mechanics

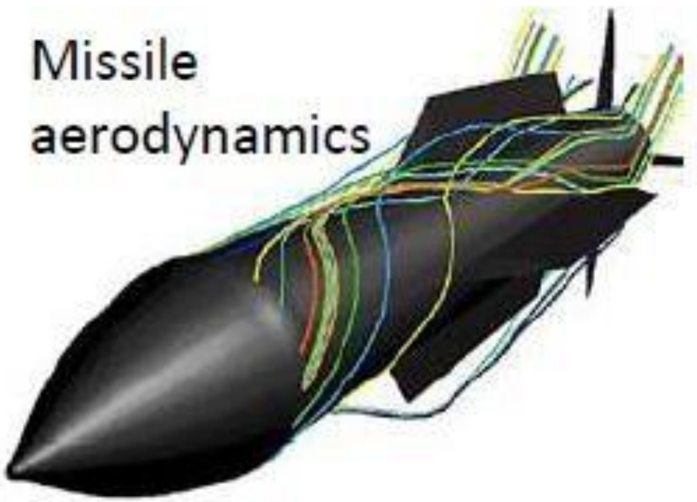
- **Mechanical Engineering**
  - Design of pumps, turbines, air-conditioning equipment, pollution-control equipment, etc.
  - Design and analysis of aircraft, boats, submarines, rockets, jet engines, wind turbines, biomedical devices, the cooling of electronic components, and the transportation of water, crude oil, and natural gas.
- **Civil Engineering**
  - Transport of river sediments
  - Pollution of air and water
  - Design of piping systems
  - Flood control systems
- **Chemical Engineering**
  - Design of chemical processing equipment

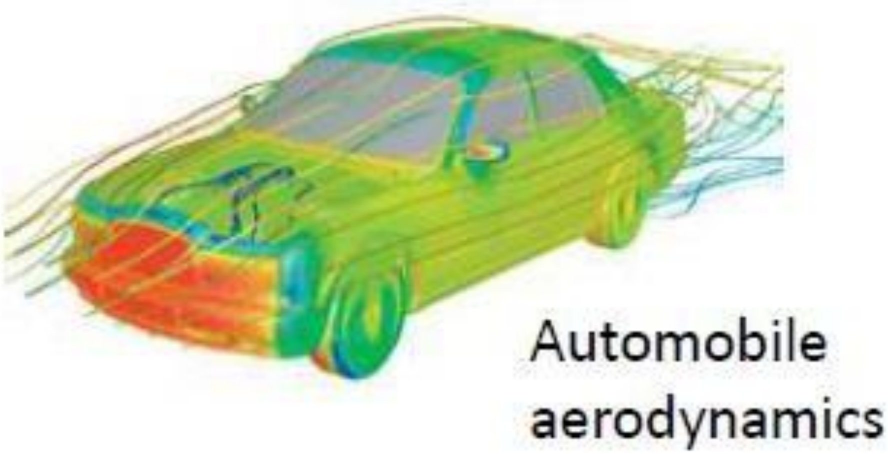
Application areas of Fluid Mechanics

- **Turbomachines:** pump, turbine, fan, blower, propeller, etc.
- **Military:** Missile, aircraft, ship, underwater vehicle, dispersion of chemical agents, etc.
- **Automobile:** IC engine, air conditioning, fuel flow, external aerodynamics, etc.
- **Medicine:** Heart assist device, artificial heart valve, Lab-on-a-Chip device, glucose monitor, controlled drug delivery, etc.
- **Electronics:** Convective cooling of generated heat.
- **Energy:** Combuster, burner, boiler, gas, hydro and wind turbine, etc.
- **Oil and Gas:** Pipeline, pump, valve, offshore rig, oil spill cleanup, etc.
- Almost everything in our world is either in contact with a fluid or is itself a fluid.

Application areas of Fluid Mechanics

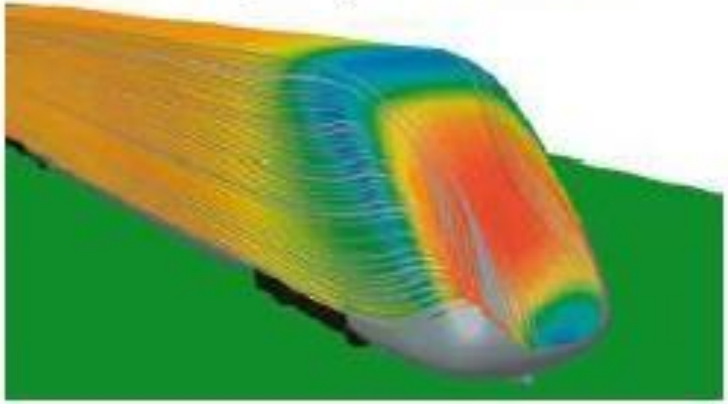
- The number of fluid engineering applications is enormous: breathing, blood flow, swimming, pumps, fans, turbines, airplanes, ships, rivers, windmills, pipes, missiles, icebergs, engines, filters, jets, and sprinklers, to name a few.
- When you think about it, almost everything on this planet either is a fluid or moves within or near a fluid.



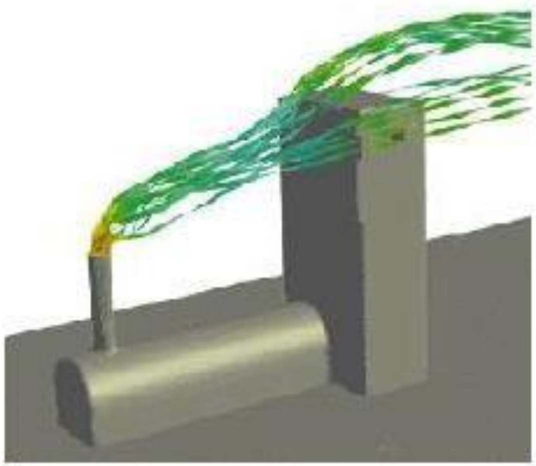


Application areas of Fluid Mechanics

High speed train

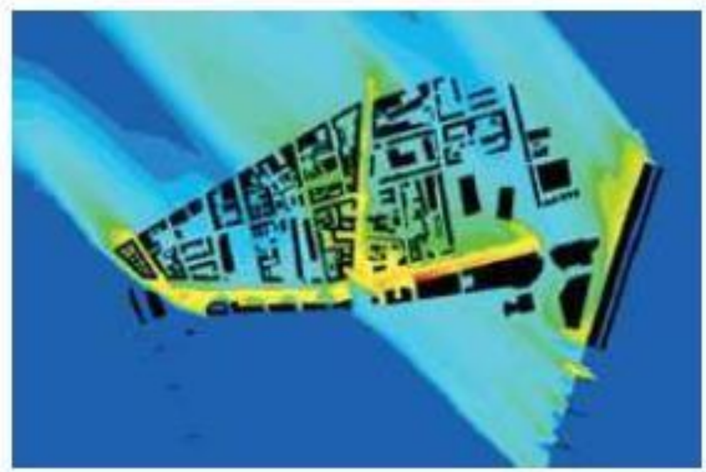


Wind turbines

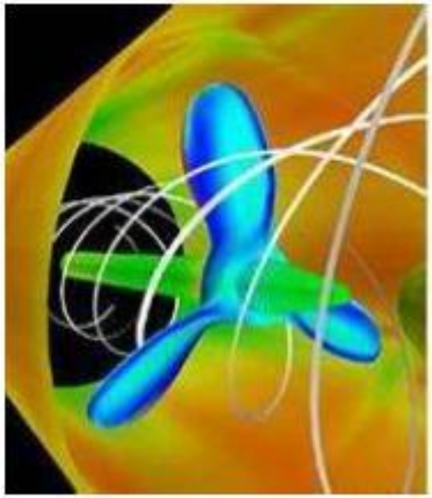


Smoke from a stack

Pollutant dispersion over a city



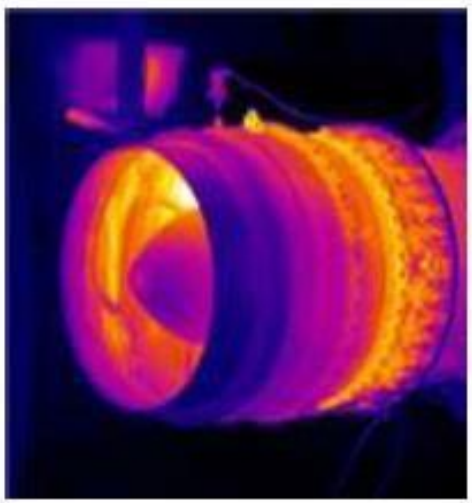
Application areas of Fluid Mechanics



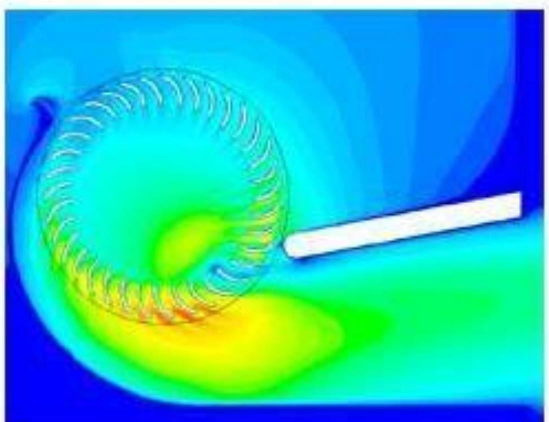
Propeller

Fan

Centrifugal Pump



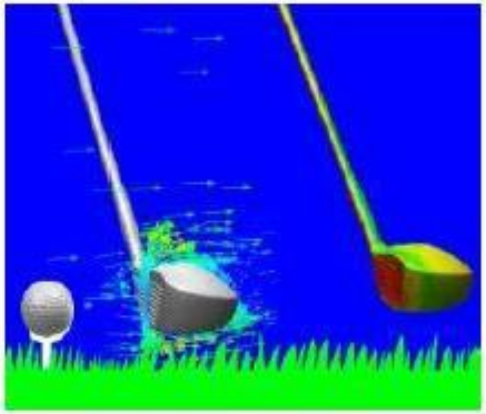
Jet Engine
Propulsion

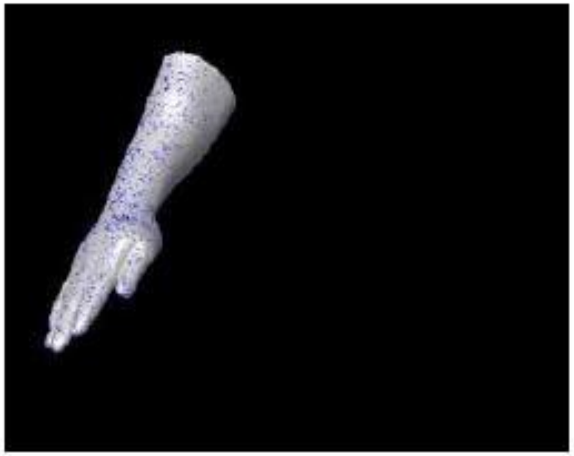


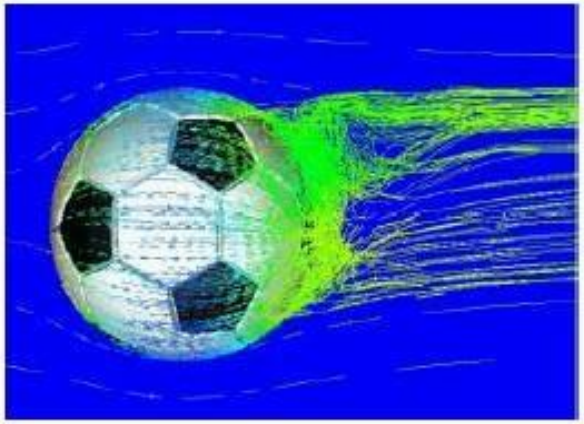
Crossflow fan

Application areas of Fluid Mechanics

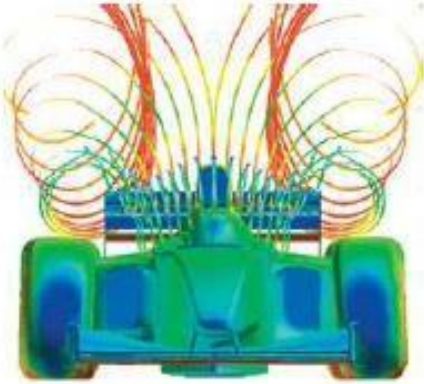

Ski Jumping


Golf


Swimming


Football


Cycling


Indy Car Racing

Classification of Fluid Flows

- There is a wide variety of fluid flow problems encountered in practice, and it is usually convenient to classify them on the basis of some common characteristics to make it feasible to study them in groups.

Viscous versus Inviscid Regions of Flow

- When two fluid layers move relative to each other, a friction force develops between them and the slower layer tries to slow down the faster layer.
- This internal resistance to flow is quantified by the fluid property *viscosity,* which is a measure of internal stickiness of the fluid.
- Viscosity is caused by cohesive forces between the molecules in liquids and by molecular collisions in gases.

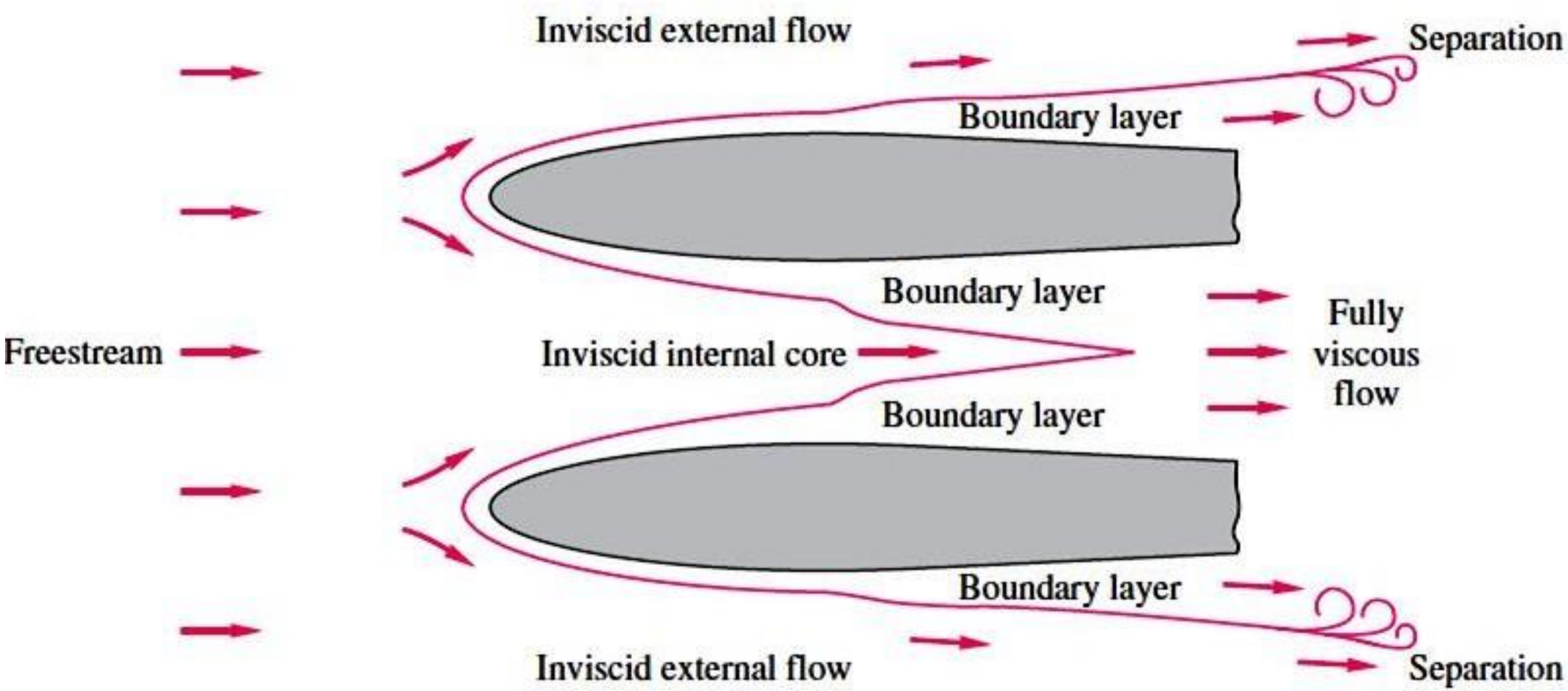
Classification of Fluid Flows

Viscous versus Inviscid Regions of Flow…

- There is no fluid with zero viscosity, and thus all fluid flows involve viscous effects to some degree.
- Flows in which the frictional effects are significant are called **viscous flows.**
- However, in many flows of practical interest, there are *regions (typically regions not close to solid surfaces)* where viscous forces are negligibly small compared to inertial or pressure forces.
- Neglecting the viscous terms in such **inviscid** flow regions greatly simplifies the analysis without much loss in accuracy.

Classification of Fluid Flows

Viscous versus Inviscid Regions of Flow



Classification of Fluid Flows

Internal versus External Flow

- A fluid flow is classified as being internal or external, depending on whether the fluid is forced to flow in a confined channel or over a surface.
- The flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe is **external flow.**
- The flow in a pipe or duct is **internal flow** if the fluid is completely bounded by solid surfaces.
- Water flow in a pipe, for example, is internal flow, and airflow over a ball or over an exposed pipe during a windy day is external flow .

Classification of Fluid Flows

Compressible versus Incompressible Flow

- A flow is classified as being compressible or incompressible, depending on the level of variation of density during flow.
- Incompressibility is an approximation, and a flow is said to be **incompressible** if the density remains nearly constant throughout.
- Therefore, the volume of every portion of fluid remains unchanged over the course of its motion when the flow (or the fluid) is incompressible.
- The densities of liquids are essentially constant, and thus the flow of liquids is typically incompressible. Therefore, liquids are usually referred to as *incompressible substances*.

Classification of Fluid Flows

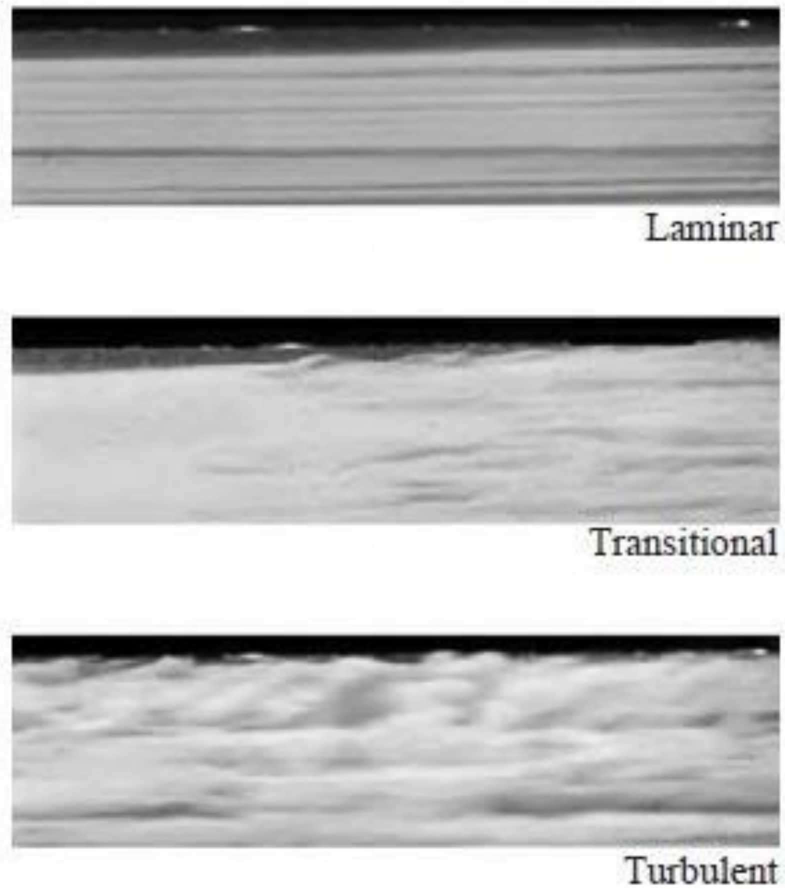
Compressible versus Incompressible Flow…

- A pressure of 210 atm, for example, causes the density of liquid water at 1 atm to change by just 1 percent.
- Gases, on the other hand, are highly compressible. A pressure change of just 0.01 atm, for example, causes a change of 1 percent in the density of atmospheric air.
- Gas flows can often be approximated as incompressible if the density changes are under about 5 percent.
- The compressibility effects of air can be neglected at speeds under about 100 m/s.

Classification of Fluid Flows

Laminar versus Turbulent Flow

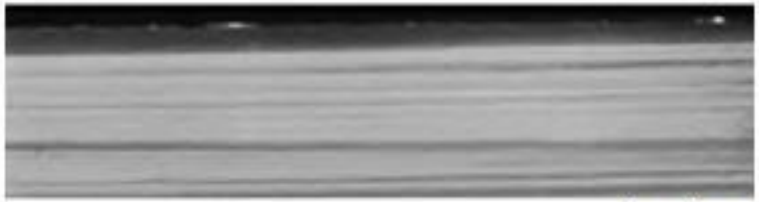
- Some flows are smooth and orderly while others are rather chaotic.
- The highly ordered fluid motion characterized by smooth layers of fluid is called **laminar.**
- The flow of high-viscosity fluids such as oils at low velocities is typically laminar.
- The highly disordered fluid motion that typically occurs at high velocities and is characterized by velocity fluctuations is called **turbulent .**



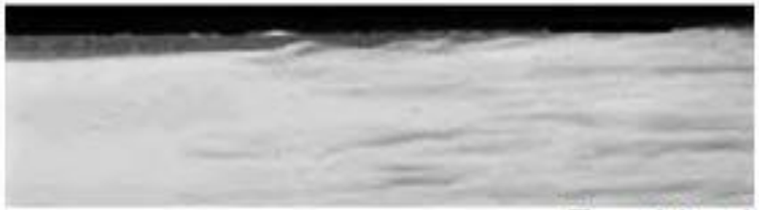
Classification of Fluid Flows

Laminar versus Turbulent Flow

- The flow of low-viscosity fluids such as air at high velocities is typically turbulent.
- A flow that alternates between being laminar and turbulent is called **transitional.**



Laminar



Transitional

Turbulent

Classification of Fluid Flows

Natural (or Unforced) versus Forced Flow

- A fluid flow is said to be natural or forced, depending on how the fluid motion is initiated.
- In **forced flow,** a fluid is forced to flow over a surface or in a pipe by external means such as a **pump or a fan**.
- In **natural flows,** any fluid motion is due to natural means such as the buoyancy effect, which manifests itself as the rise of the warmer (and thus lighter) fluid and the fall of cooler (and thus denser) fluid .
- In solar hot-water systems, for example, the thermosiphoning effect is commonly used to replace pumps by placing the water tank sufficiently above the solar collectors.

Classification of Fluid Flows

Steady versus Unsteady Flow

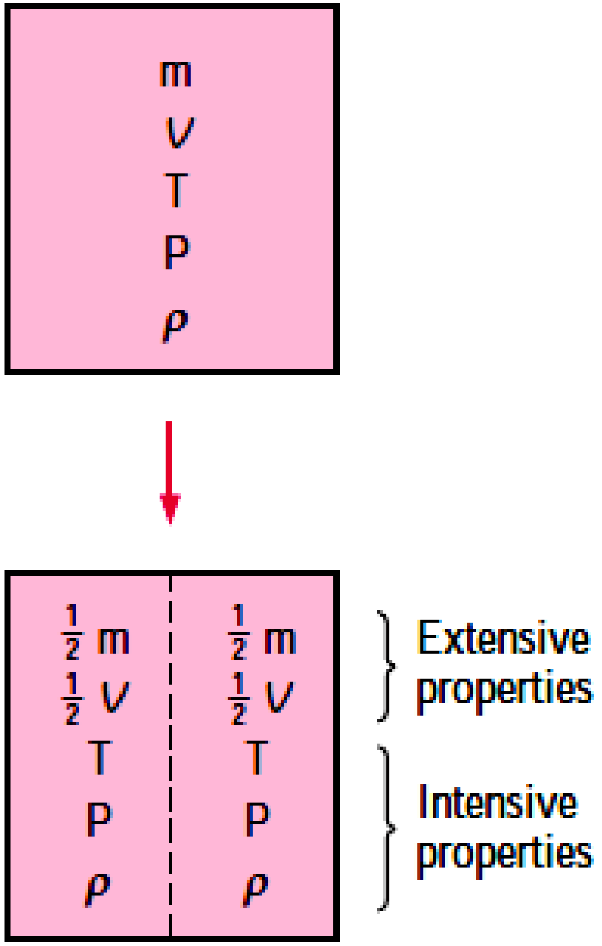
- The terms *steady and uniform are used frequently in engineering, and thus it* is important to have a clear understanding of their meanings.
- The term steady implies ***no change at a point with time*****.**
- The opposite of steady is **unsteady.**
- The term **uniform** implies ***no change with location*** *over a specified* region.

Properties of Fluids

- Any characteristic of a system is called a **property.**
- Some familiar properties are pressure *P*, temperature T, volume V, and mass m.
- Other less familiar properties include viscosity, thermal conductivity, modulus of elasticity, thermal expansion coefficient, electric resistivity, and even velocity and elevation.
- Properties are considered to be either ***intensive or extensive***.
- ***Intensive*** properties are those that are independent of the mass of a system, such as temperature, pressure, and density.
- **Extensive** properties are those whose values depend on the size—or extent—of the system. Total mass, total volume *V*, and total momentum are some examples of extensive properties.

Properties of Fluids

- An easy way to determine whether a property is intensive or extensive is to divide the system into two equal parts with an imaginary partition.
- Each part will have the same value of intensive properties as the original system, but half the value of the extensive properties.



Properties of Fluids

Density or Mass Density

- Density or mass density of a fluid is defined as the ratio of the mass of a f1uid to its volume. Thus mass per unit volume of a fluid is called density. It is denoted the symbol ρ (rho). The unit of mass density in SI unit is kg per cubic meter, i.e ., kg/m^3.
- The density of liquids may be considered as constant while that of gases changes with the variation of pressure and temperature.
- Mathematically mass density is written as.

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}$$

- The value of density of water is 1 gm/cm^3 or 1000 kg/m^3.

Properties of Fluids

Density or Mass Density

- The density of a substance, in general, depends on temperature and pressure.
- The density of most gases is proportional to pressure and inversely proportional to temperature.
- Liquids and solids, on the other hand, are essentially incompressible substances, and the variation of their density with pressure is usually negligible.

Properties of Fluids

Specific weight or Weight Density

- Specific weight or weight density of a fluid is the ratio between the weight of a fluid to its volume.
- Thus weight per unit volume of a fluid is called weight density and it is denoted by the symbol w.
- Mathematically,

$$w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{(\text{Mass of fluid}) \text{ x Acceleration due to gravity}}{\text{Volume of fluid}}$$

$$= \frac{\text{Mass of fluid x g}}{\text{Volume of fluid}}$$

$$= \rho \text{ x g}$$

$$w = \rho g$$

Properties of Fluids

Specific Volume

- Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume.
- Mathematically, it is expressed as

$$\text{Specific volume} = \frac{\text{Volume of fluid}}{\text{Mass of fluid}} = \frac{1}{\dfrac{\text{Mass of fluid}}{\text{Volume}}} = \frac{1}{\rho}$$

- Thus specific volume is the reciprocal of mass density. It is expressed as m^3/kg.
- It is commonly applied to gases.

Properties of Fluids

Specific Gravity.

- Specific gravity is defined as the ratio of the weight density (or density) of a fluid to the weight density (or density) of a standard fluid.
- For liquids, the standard fluid is taken water and for gases, the standard fluid is taken air. Specific gravity is also called relative density. It is dimensionless quantity and is denoted by the symbol *S*.

$$S(\text{for liquids}) = \frac{\text{Weight density (density) of liquid}}{\text{Weight density (density) of water}}$$

$$S(\text{for gases}) = \frac{\text{Weight density (density) of gas}}{\text{Weight density (density) of air}}$$

$$\text{Thus weight density of a liquid} = S \text{ x Weight density of water}$$

$$= S \text{ x } 1000 \text{ x } 9.81 \text{N/m}^3$$

$$\text{Thus density of a liquid} = S \text{ x Density of water}$$

$$= S \text{ x } 1000 \text{kg/m}^3$$

Properties of Fluids

Specific Gravity.

- If the specific gravity of a fluid is known, then the density of the fluid will be equal to specific gravity of fluid multiplied by the density of water.
- For example the specific gravity of mercury is 13.6, hence density of mercury = 13.6 x 1000 = 13600 kg/m^3.

Specific gravities of some substances at 0°C

Substance	SG
Water	1.0
Blood	1.05
Seawater	1.025
Gasoline	0.7
Ethyl alcohol	0.79
Mercury	13.6
Wood	0.3–0.9
Gold	19.2
Bones	1.7–2.0
Ice	0.92
Air (at 1 atm)	0.0013

Properties of Fluids

Example 1.

Calculate the specific weight, density and specific gravity of one liter of a liquid which weighs 7 N.

Solution. Given :

$$\text{Volume} = 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \quad \left(\because \ 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \text{ or } 1 \text{ litre} = 1000 \text{ cm}^3 \right)$$

$$\text{Weight} = 7 \text{ N}$$

(*i*) Specific weight (*w*)

$$= \frac{\text{Weight}}{\text{Volume}} = \frac{7 \text{ N}}{\left(\frac{1}{1000}\right) \text{m}^3} = \mathbf{7000 \ N/m^3. \ Ans.}$$

(*ii*) Density (ρ)

$$= \frac{w}{g} = \frac{7000}{9.81} \text{ kg/m}^3 = \mathbf{713.5 \ kg/m^3. \ Ans.}$$

(*iii*) Specific gravity

$$= \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{713.5}{1000} \quad \{ \because \ \text{Density of water} = 1000 \text{ kg/m}^3 \}$$

$$= \mathbf{0.7135. \ Ans.}$$

Example 2. *Calculate the density, specific weight and weight of one liter of petrol of specific gravity = 0.7*

Solution. Given : Volume = 1 litre = $1 \times 1000 \text{ cm}^3 = \frac{1000}{10^6} \text{ m}^3 = 0.001 \text{ m}^3$

Sp. gravity $S = 0.7$

(*i*) *Density* (ρ)

Density (ρ) $= S \times 1000 \text{ kg/m}^3 = 0.7 \times 1000 = \mathbf{700 \text{ kg/m}^3}$. **Ans.**

(*ii*) *Specific weight* (*w*)

$$w = \rho \times g = 700 \times 9.81 \text{ N/m}^3 = \mathbf{6867 \text{ N/m}^3}. \textbf{ Ans.}$$

(*iii*) *Weight* (*W*)

We know that specific weight $= \dfrac{\text{Weight}}{\text{Volume}}$

or $$w = \frac{W}{0.001} \text{ or } 6867 = \frac{W}{0.001}$$

∴ $$W = 6867 \times 0.001 = \mathbf{6.867 \text{ N}}. \textbf{ Ans.}$$

Properties of Fluids

Viscosity

- Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.
- When two layers of a fluid, a distance *'dy' apart move one* over the other at different velocities say u and u+ *du as shown in Fig. 1.1 , the viscosity together with* relative velocity causes a shear stress acting between the fluid layers:

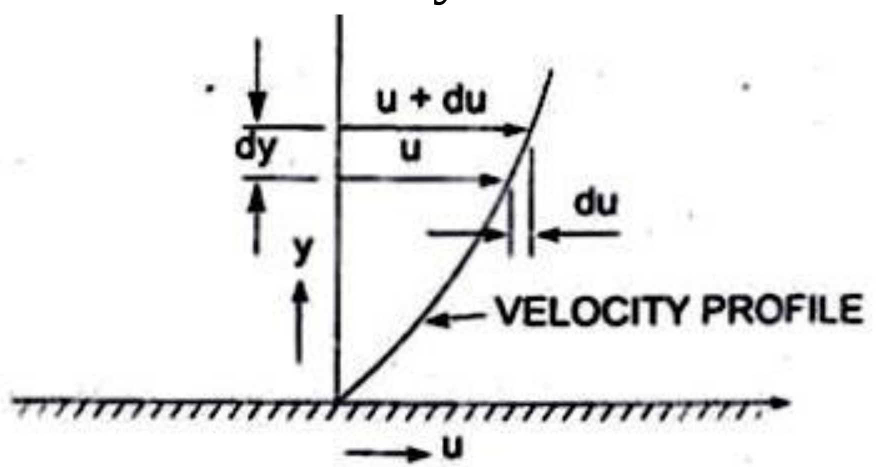


Fig. 1.1 *Velocity variation near a solid boundary.*

Properties of Fluids

Viscosity

- The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer.
- This shear stress is proportional to the rate of change of velocity with respect to *y. It is denoted by symbol* τ called Tau.
- Mathematically,

$$\tau \propto \frac{du}{dy}$$

- or

$$\boxed{\tau = \mu \frac{du}{dy} \qquad (1.2)}$$

Properties of Fluids

- where μ (called m*u) is the constant of proportionality and is known as the coefficient of dynamic viscosity* or only viscosity.
- $\frac{du}{dy}$ *represents the rate of shear strain or rate of shear deformation or velocity gradient.*
- *From equation (1.2) we have*

$$\boxed{\mu = \frac{\tau}{\frac{du}{dy}} \qquad (1.3)}$$

- Thus viscosity is also defined as the shear stress required to produce unit rate of shear strain.

Properties of Fluids

Unit of Viscosity.

- The unit of viscosity is obtained by putting the dimension of the quantities in equation (1.3)

$$\mu = \frac{\text{Shear stress}}{\dfrac{\text{Change of velocity}}{\text{Change of distance}}} = \frac{\text{Force/Area}}{\left(\dfrac{\text{Length}}{\text{Time}}\right) \times \dfrac{1}{\text{Length}}}$$

$$= \frac{\text{Force/(length)}^2}{\dfrac{1}{\text{Time}}} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2}$$

$$\text{SI unit of viscosity} = \frac{\text{Newton second}}{\text{m}^2} = \frac{\text{Ns}}{\text{m}^2}$$

Properties of Fluids

Kinematic Viscosity.

- It is defined as the ratio between the dynamic viscosity and density of fluid.lt is denoted by the Greek symbol (ν) called 'nu' . Thus, mathematically,

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho}$$

- The SI unit of kinematic viscosity is m^2/s.

Newton's Law of Viscosity.

- It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co-efficient viscosity. Mathematically, it is expressed as given by equation (1 . 2).

Properties of Fluids

- Fluids which obey the above relation are known as Newtonian fluids and the fluids which do not obey the above relation are called Non-newtonian fluids.

Variation of Viscosity with Temperature

- Temperature affects the viscosity.
- The viscosity of liquids decreases with the increase of temperature while the viscosity of gases increases with increase of temperature. This is due to reason that the viscous forces in a fluid are due to cohesive forces and *molecular momentum transfer.*
- *In liquids the cohesive forces predominates the molecular momentum* transfer due to closely packed molecules and with the increase in temperature, the cohesive forces decreases with the result of decreasing viscosity.

Properties of Fluids

- But in the case of gases the cohesive force are small and molecular momentum transfer predominates. With the increase in temperature, molecular momentum transfer increases and hence viscosity increases. The relation between viscosity and temperature for liquids and gases are:

(i) For liquids, $\mu = \mu_o \left(\frac{1}{1 + \alpha t + \beta t^2} \right)$

where $\mu =$ Viscosity of liquid at t^oC, in poise $1 \text{ poise} = \frac{1}{10} \frac{Ns}{m^2}$

$\mu_o =$ Viscosity of liquid at 0^oC, in poise

$\alpha, \beta =$ are constants for the liquid

For water, $\mu_o = 1.79 \times 10^{-3}$ poise, $\alpha = 0.03368$ and $\beta = 0.000221$

(ii) For a gas, $\mu = \mu_o + \alpha t - \beta t^2$

where for air $\mu_o = 0.000017$, $\alpha = 0.000000056$, $\beta = 0.1189 \times 10^{-9}$

Types of Fluids

1. Ideal Fluid. A fluid, which is incompressible and is having no viscosity, is known as an ideal fluid. Ideal fluid is only an imaginary fluid as all the fluids, which exist, have some viscosity.
2. Real fluid. A fluid, which possesses viscosity, is known as real fluid. All the fluids: in actual practice, are real fluids.
3. Newtonian Fluid. A real fluid, in which the shear stress is directly, proportional to the rate of shear strain (or velocity gradient), is known as a Newtonian fluid.
4. Non-Newtonian fluid. A real fluid, in which shear stress is not proportional to the rate of shear strain (or velocity gradient), known as a Non-Newtonian fluid.

Types of Fluids

5. Ideal Plastic Fluid.

A fluid, in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain (or velocity gradient), is known as ideal plastic fluid.

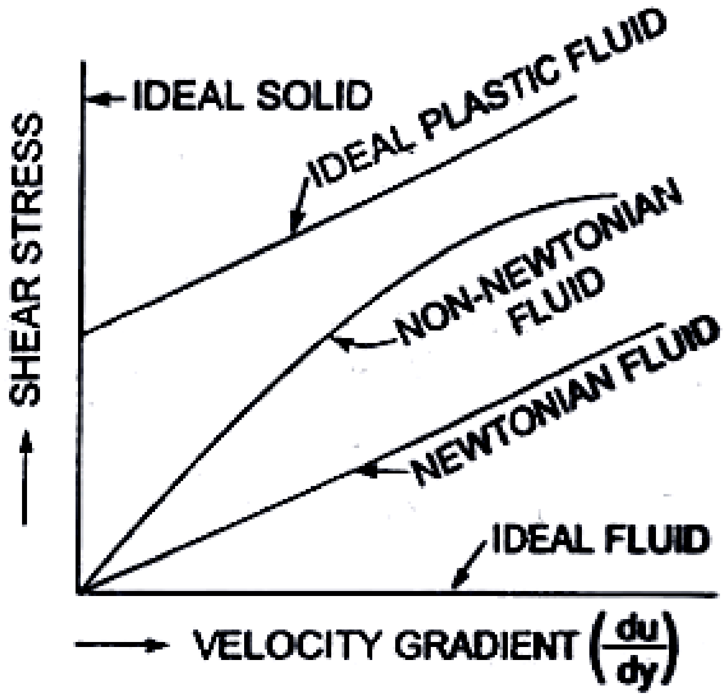


Fig. 1.2 *Types of fluids.*

Example 3

If the velocity distribution over a plate is given by

$$u = \frac{2}{3}y - y^2$$

in which u is velocity in metre per second at a distance y metre above the plate, determine the shear stress at y = 0 and y= 0.15 m. Take dynamic viscosity of fluid as 8.63 poises.

Solution. Given : $u = \frac{2}{3}y - y^2 \quad \therefore \quad \frac{du}{dy} = \frac{2}{3} - 2y$

$$\left(\frac{du}{dy}\right)_{\text{at } y=0} \text{ or } \left(\frac{du}{dy}\right)_{y=0} = \frac{2}{3} - 2(0) = \frac{2}{3} = 0.667$$

Also $\left(\frac{du}{dy}\right)_{\text{at } y=0.15}$ or $\left(\frac{du}{dy}\right)_{y=0.15} = \frac{2}{3} - 2 \times .15 = .667 - .30 = 0.367$

Value of $\mu = 8.63$ poise $= \frac{8.63}{10}$ SI units $= 0.863$ N s/m^2

Now shear stress is given by equation (1.2) as $\tau = \mu \frac{du}{dy}$.

(*i*) Shear stress at $y = 0$ is given by

$$\tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0} = 0.863 \times 0.667 = \mathbf{0.5756\ N/m^2}. \textbf{ Ans.}$$

(*ii*) Shear stress at $y = 0.15$ m is given by

$$(\tau)_{y=0.15} = \mu \left(\frac{du}{dy}\right)_{y=0.15} = 0.863 \times 0.367 = \mathbf{0.3167\ N/m^2}. \textbf{ Ans.}$$

Example 4

Calculate the dynamic viscosity of an oil, which is used for lubrication between a square plate of size 0.8 m x 0.8 m and an inclined plane with angle of inclination 30^o as shown in Fig. 1.4. The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of oil film is 1.5 mm.

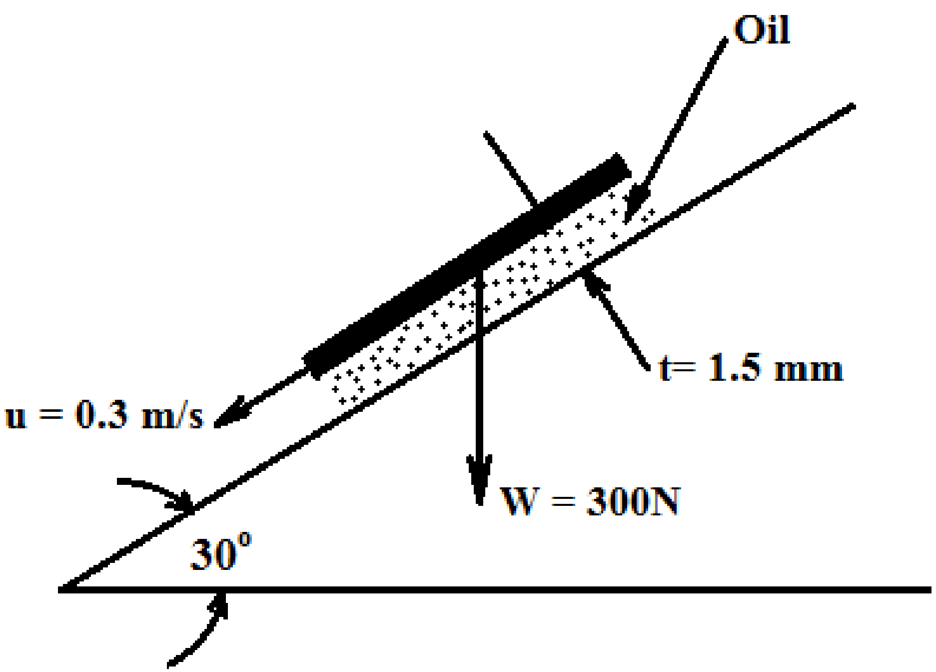


Fig.1.4

Solution. Given :

Area of plate, $A = 0.8 \times 0.8 = 0.64 \text{ m}^2$

Angle of plane, $\theta = 30°$

Weight of plate, $W = 300 \text{ N}$

Velocity of plate, $u = 0.3 \text{ m/s}$

Thickness of oil film, $t = dy = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$

Let the viscosity of fluid between plate and inclined plane is μ.

Component of weight W, along the plane $= W \cos 60° = 300 \cos 60° = 150 \text{ N}$

Thus the shear force, F, on the bottom surface of the plate $= 150 \text{ N}$

and shear stress, $$\tau = \frac{F}{\text{Area}} = \frac{150}{0.64} \text{ N/m}^2$$

Now using equation (1.2), we have

$$\tau = \mu \frac{du}{dy}$$

where du = change of velocity $= u - 0 = u = 0.3 \text{ m/s}$

$$dy = t = 1.5 \times 10^{-3} \text{ m}$$

$$\therefore \quad \frac{150}{0.64} = \mu \frac{0.3}{1.5 \times 10^{-3}}$$

$$\therefore \quad \mu = \frac{150 \times 1.5 \times 10^{-3}}{0.64 \times 0.3} = 1.17 \text{ N s/m}^2 = 1.17 \times 10 = 11.7 \text{ poise.}$$ **Ans.**

Example 5

The space between two square flat parallel plates is filled with oil. Each side of the plate is 60 cm. The thickness of the oil film is 12.5 mm. The upper plate, which moves at 2.5 metre per sec requires a force of 98.1 N to maintain the speed. Determine : ·

i. the dynamic viscosity of the oil, and

ii. the kinematic viscosity of the oil if the specific gravity of the oil is 0.95.

Solution. Given:

Each side of a square plate = 60 cm = 0.6 m

Area $A = 0.6 \times 0.6 = 0.36\ m^2$

Thickness of oil film $dy = 12.5\ mm = 12.5 \times 10^{-3}\ m$

Velocity of upper plate $u = 2.5\ m/s$

$\therefore$ Change of velocity between plates, $du = 2.5$ m/sec

Force required on upper plate, $F = 98.1$ N

$\therefore$ Shear stress, $$\tau = \frac{\text{Force}}{\text{Area}} = \frac{F}{A} = \frac{98.1\text{ N}}{0.36\text{ m}^2}$$

(*i*) Let μ = Dynamic viscosity of oil

Using equation (1.2), $$\tau = \mu\frac{du}{dy} \text{ or } \frac{98.1}{0.36} = \mu \times \frac{2.5}{12.5\times10^{-3}}$$

$\therefore$ $$\mu = \frac{98.1}{0.36}\times\frac{12.5\times10^{-3}}{2.5} = 1.3635\ \frac{\text{Ns}}{\text{m}^2} \quad \textbf{Ans.}$$

(*ii*) Sp. gr. of oil, S = 0.95

Let ν = kinematic viscosity of oil

Using equation (1.1 A),

Mass density of oil, $\rho = S \times 1000 = 0.95 \times 1000 = 950$ kg/m^3

Using the relation, $\nu = \frac{\mu}{\rho}$, we get $$\nu = \frac{1.3635\left(\frac{\text{Ns}}{\text{m}^2}\right)}{950} = .001435\ \text{m}^2/\text{sec} \quad \textbf{Ans.}$$

Thermodynamic Properties

- Fluids consist of liquids or gases. But gases are compressible fluids and hence thermodynamic properties play an important role.
- With the change of pressure and temperature, the gases undergo large variation in density.
- The relationship between pressure (absolute), specific volume and temperature (absolute) of a gas is given by the equation of state as

$$\boxed{p\forall = RT \text{ or } \frac{p}{\rho} = RT}$$

where p = Absolute pressure of a gas in N/m^2

$\forall$ = Specific volume = $\frac{1}{\rho}$

R = Gas constant

T = Absolute temperature in °K

ρ = Density of a gas.

Thermodynamic Properties

- The value of gas constant R is $R = 287 \dfrac{J}{kg.K}$

- **Isothermal Process**. If the changes in density occurs at constant temperature, then the process is called isothermal and relationship between pressure (p) and density (ρ) is given by

$$\frac{p}{\rho} = \text{constant}$$

- **Adiabatic Process.** If the change in density occurs with no heat exchange to and from the gas, the process is called adiabatic. And if no heat is generated within the gas due to friction, the relationship between pressure and density is given by

$$\frac{p}{\rho^k} = \text{constant}$$

Thermodynamic Properties

- where k = *Ratio of specific heat of a gas at constant pressure and constant volume.*
- k = 1.4 for air

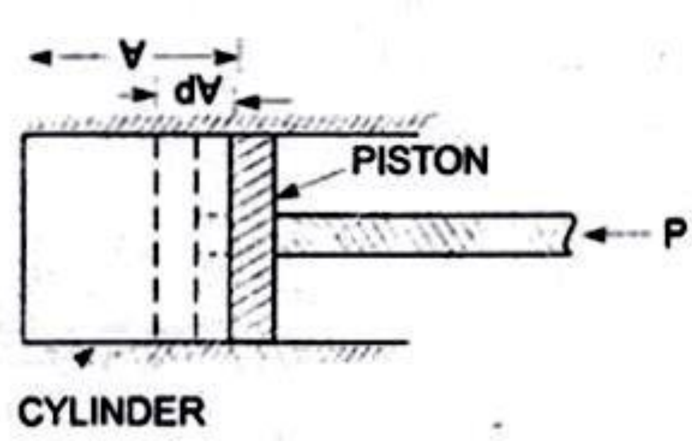
Compressibility and Bulk Modulus

- Compressibility is the reciprocal of the bulk modulus of elasticity, K which is defined as the ratio of compressive stress to volumetric strain.
- Consider a cylinder fitted with a piston as shown in the Fig.
- Let V= *Volume of a gas enclosed in the cylinder*

  p =*Pressure of gas when volume is* V
- Let the pressure is increased *to* $p+dp$, *the volume of gas* decreases from V *to* $V - dV$.
- Then increase in pressure = dp
- Decrease in volume = dV
- Volumetric strain = - dV/V

Compressibility and Bulk Modulus

- - ve sign means the volume decreases with increase of pressure.



$$\therefore \text{Bulk modules} \quad K = \frac{\text{Increase of pressure}}{\text{Volumetric strain}}$$

$$= \frac{dp}{\frac{-dV}{V}} = -\frac{dp}{dV}V$$

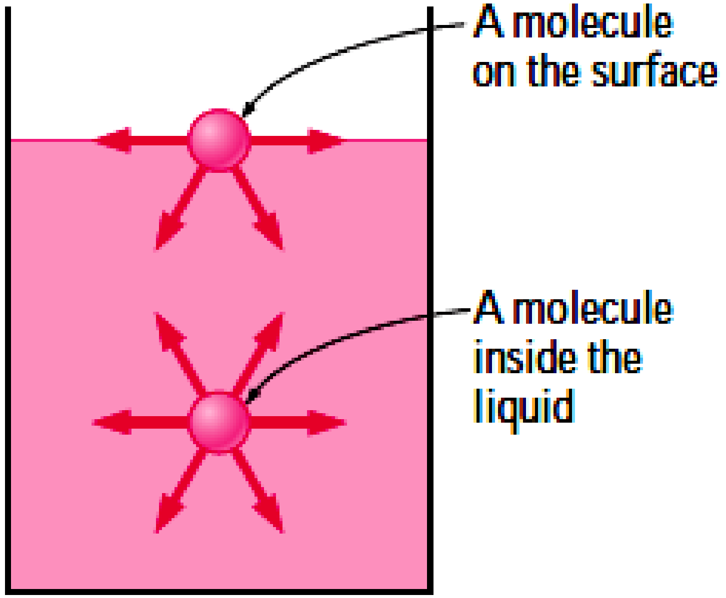
- Compressibility is given by = 1/K

Surface Tension and Capillarity

- Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension.
- Surface tension is created due to the unbalanced cohesive forces acting on the liquid molecules at the fluid surface.
- Molecules in the interior of the fluid mass are surrounded by molecules that are attracted to each other equally.
- However, molecules along the surface are subjected to a net force toward the interior.
- The apparent physical consequence of this unbalanced force along the surface is to create the hypothetical skin or membrane.

Surface Tension and Capillarity

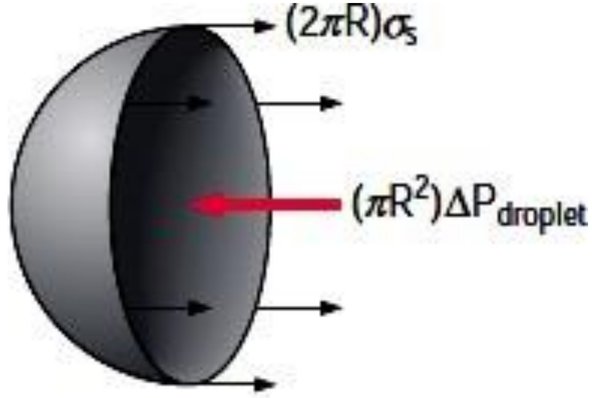
- A tensile force may be considered to be acting in the plane of the surface along any line in the surface.
- The intensity of the molecular attraction per unit length along any line in the surface is called the ***surface tension.***
- It is denoted by Greek letter σ (called sigma).
- The SI unit is N/m.



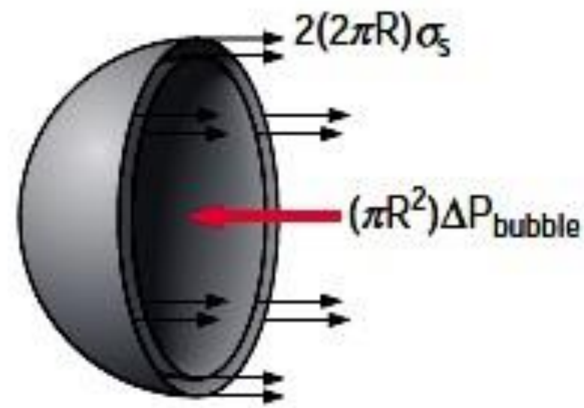
Surface Tension and Capillarity

Surface Tension on liquid Droplet and Bubble

- Consider a small spherical droplet of a liquid of radius *‘R’. On the entire surface of the droplet, the tensile force due to surface tension will be acting.*
- Let σ = surface tension of the liquid
- ΔP= Pressure intensity inside the droplet (in excess of the outside pressure intensity)
- *R= Radius of droplet.*
- Let the droplet is cut into two halves. The forces acting on one half (say left half) will be



(a) Half a droplet



(b) Half a bubble

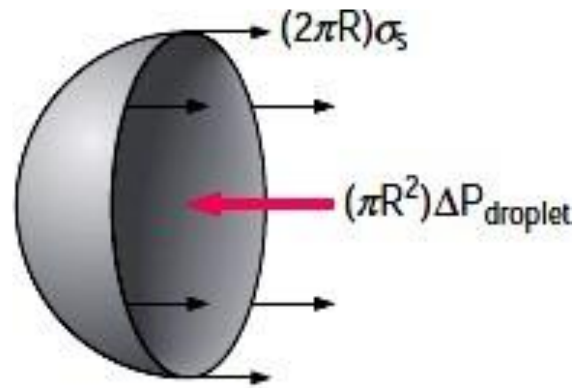
Surface Tension and Capillarity

- (i) tensile force due to surface tension acting around the circumference of the cut portion as shown and this is equal to

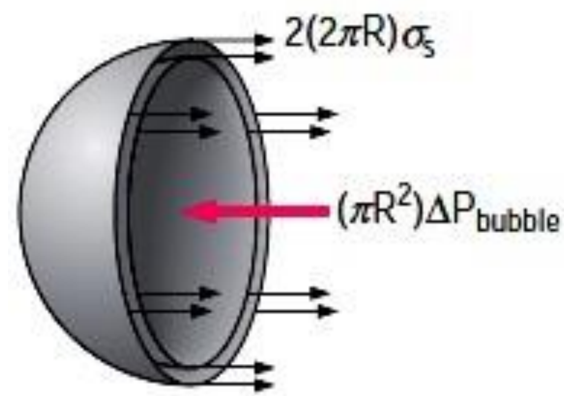
  $= \sigma \text{ x Circumference}$

  $= \sigma \text{ x } 2\pi R$

- (ii) pressure force on the area $(\pi/4)d^2$ and
- $= \Delta P \; x \; \pi R^2 \; as \; shown$



(a) Half a droplet



(b) Half a bubble

Surface Tension and Capillarity

- These two forces will be equal and opposite under equilibrium conditions, *i.e.*,

$$\text{Droplet:} \qquad (2\pi R)\sigma_s = (\pi R^2)\Delta P_{\text{droplet}} \rightarrow \Delta P_{\text{droplet}} = P_i - P_o = \frac{2\sigma_s}{R}$$

$$\text{Bubble:} \qquad 2(2\pi R)\sigma_s = (\pi R^2)\Delta P_{\text{bubble}} \rightarrow \Delta P_{\text{bubble}} = P_i - P_o = \frac{4\sigma_s}{R}$$

- A hollow bubble like a soap bubble in air has two surfaces in contact with air, one inside and other outside. Thus two surfaces are subjected surface tension.

Surface Tension……. Example 1

- Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is 2.5 N/m^2 above atmospheric pressure.

Solution. Given :

Dia. of bubble, $d = 40 \text{ mm} = 40 \times 10^{-3}$ m

Pressure in excess of outside, $p = 2.5$ N/m^2

For a soap bubble, using equation (1.15), we get

$$p = \frac{8\sigma}{d} \quad \text{or} \quad 2.5 = \frac{8 \times \sigma}{40 \times 10^{-3}}$$

$$\sigma = \frac{2.5 \times 40 \times 10^{-3}}{8} \text{ N/m} = \mathbf{0.0125\ N/m.\ Ans.}$$

Surface Tension……. Example 2

- The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/cm² (atmospheric pressure). Calculate the pressure within the droplet if surface tension is given as 0.0725 N/m of water.

Solution. Given :

Dia. of droplet, $d = 0.04 \text{ mm} = .04 \times 10^{-3} \text{ m}$

Pressure outside the droplet $= 10.32 \text{ N/cm}^2 = 10.32 \times 10^4 \text{ N/m}^2$

Surface tension, $\sigma = 0.0725 \text{ N/m}$

The pressure inside the droplet, in excess of outside pressure is given by

or $$p = \frac{4\sigma}{d} = \frac{4 \times 0.0725}{.04 \times 10^{-3}} = 7250 \text{ N/m}^2 = \frac{7250 \text{ N}}{10^4 \text{ cm}^2} = 0.725 \text{ N/cm}^2$$

$\therefore$ Pressure inside the droplet $= p +$ Pressure outside the droplet

$= 0.725 + 10.32 = \mathbf{11.045 \text{ N/cm}^2}$. **Ans.**

Surface Tension and Capillarity

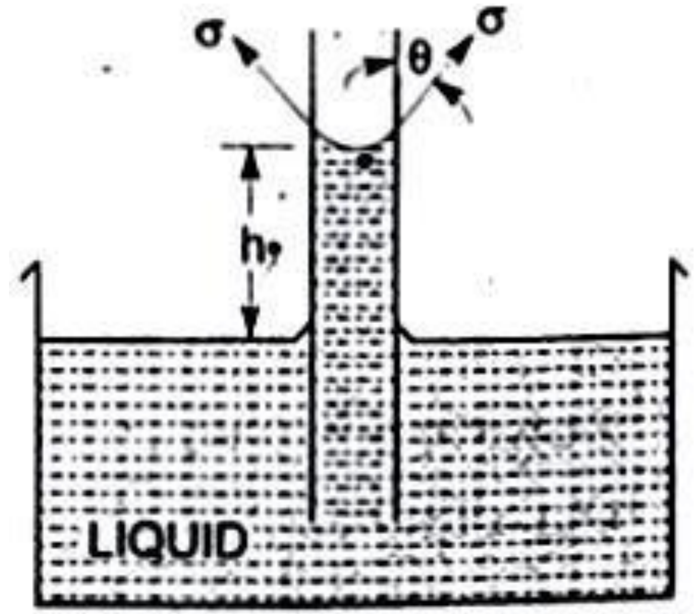
Capillarity

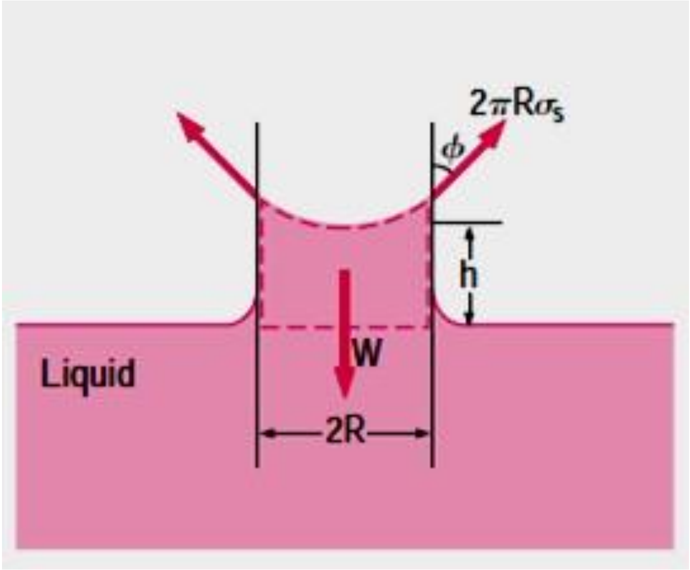
- Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid.
- The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression.
- The attraction (adhesion) between the wall of the tube and liquid molecules is strong enough to overcome the mutual attraction (cohesion) of the molecules and pull them up the wall. Hence, the liquid is said to *wet the solid surface.*
- It is expressed in terms of cm or mm of liquid. Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.

Surface Tension and Capillarity

Expression for Capillary Rise

- Consider a glass tube of small diameter ‘d’ opened at both ends and is inserted in a liquid, say water.
- The liquid will rise in the tube above the level of the liquid.
- Let h = the height of the liquid in the tube . Under a state of equilibrium, the weight of the liquid of height h is balanced by the force at the surface of the liquid in the tube. But the force at the surface of the liquid in the tube is due to surface tension.





Expression for Capillary Rise…

- Let σ = Surface tension of liquid

  θ = Angle of contact between the liquid and glass tube
- The weight of the liquid of height h in the tube

  = (Area of the tube x h) x ρ x g

$$= \frac{\pi}{4} d^2 \times h \times \rho \times g$$

where ρ = Density of liquid

Vertical component of the surface tensile force

$$= (\sigma \times \text{Circumference}) \times \cos\theta$$

$$= \sigma \times \pi d \times \cos\theta$$

For equilibrium, equating (1.17) and (1.18), we get

$$\frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \times \pi d \times \cos\theta$$

or

$$h = \frac{\sigma \times \pi d \times \cos\theta}{\frac{\pi}{4} d^2 \times \rho \times g} = \frac{4\sigma\cos\theta}{\rho \times g \times d}$$

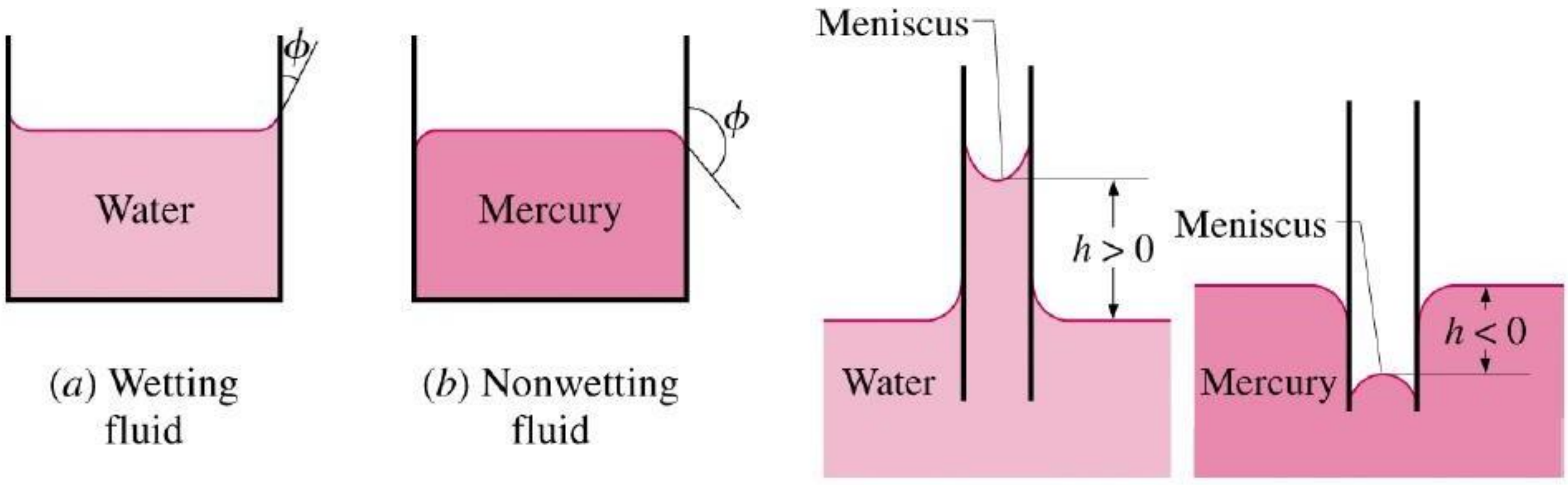
Expression for Capillary Rise…

- The value of θ between water and clean glass tube is approximately equal to zero and hence cos θ is equal to unity. Then rise of water is given by

$$\boxed{h = \frac{4\sigma}{\rho \times g \times d}}$$

- Contact angle depends on both the liquid and the solid.
- If θ is less than 90°, the liquid is said to "wet" the solid. However, if θ is greater than 90°, the liquid is repelled by the solid, and tries not to "wet" it.
- For example, water wets glass, but not wax. Mercury on the other hand does not wet glass.

Capillarity



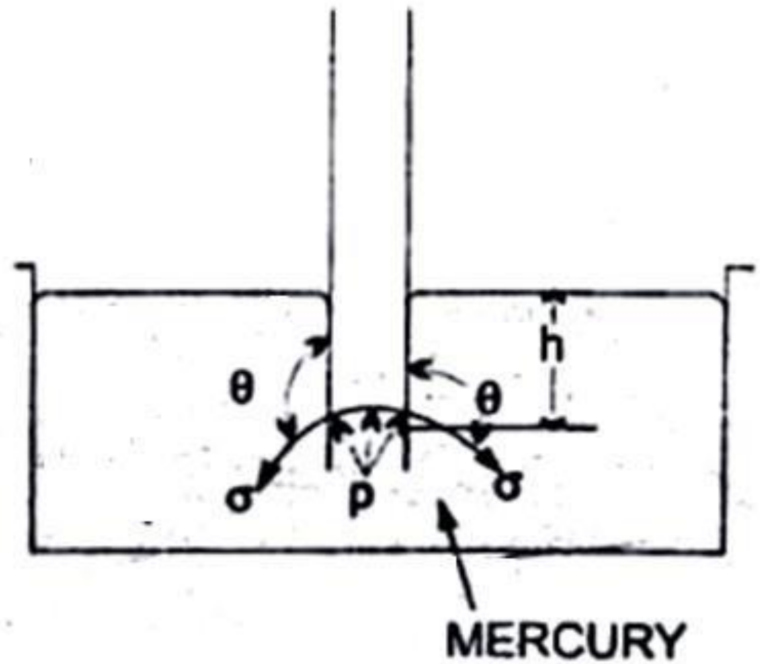
Expression for Capillary Fall

- If the glass tube is dipped in mercury, the revel of mercury in the tube will be lower than the general level of the outside liquid as shown above.

Capillarity

Expression for Capillary Fall

- Let *h = Height of depression in tube.*
- Then in equilibrium, two forces arc acting on the mercury inside the tube.
- First one is due to surface tension acting in the downward direction and is equal to σ x $\pi d \times \cos \theta$.
- Second force is due to hydrostatic force acting upward and is equal to intensity of pressure at a depth *'h' x Area*



Capillarity

Expression for Capillary Fall

$$= p \times \frac{\pi}{4} d^2 = \rho g \times h \times \frac{\pi}{4} d^2 \quad \{\because p = \rho g h\}$$

Equating the two, we get

$$\sigma \times \pi d \times \cos\theta = \rho g h \times \frac{\pi}{4} d^2$$

$$h = \frac{4\sigma \cos\theta}{\rho g d}$$

Value of θ for mercury and glass tube is 128^{o}

Capillarity…Example 1

- Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in (a) water and (b) mercury. Take surface tensions σ = 0.0725 N/m for water and σ = 0.52 N/m for mercury in contact with air. The specific gravity for mercury is given as 13.6 and angle of contact = 130°.

Solution. Given :

Dia. of tube,	$d = 2.5 \text{ mm} = 2.5 \times 10^{-3}$ m
Surface tenstion, σ for water	= 0.0725 N/m
σ for mercury	= 0.52 N/m
Sp. gr. of mercury	= 13.6

Capillarity…Example 1

$\therefore$ Density $= 13.6 \times 1000 \text{ kg/m}^3$.

(*a*) **Capillary rise for water ($\theta = 0$)**

Using equation (1.20), we get $h = \dfrac{4\sigma}{\rho \times g \times d} = \dfrac{4 \times 0.0725}{1000 \times 9.81 \times 2.5 \times 10^{-3}}$

$= .0118 \text{ m} = \mathbf{1.18 \text{ cm. Ans.}}$

(*b*) **For mercury**

Angle of constant between mercury and glass tube, $\theta = 130°$

Using equation (1.21), we get $h = \dfrac{4\sigma \cos\theta}{\rho \times g \times d} = \dfrac{4 \times 0.52 \times \cos 130°}{13.6 \times 1000 \times 9.81 \times 2.5 \times 10^{-3}}$

$= -.004 \text{ m} = \mathbf{-0.4 \text{ cm. Ans.}}$

The negative sign indicates the capillary depression.

Capillarity…Example 2

- Find out the minimum size of glass tube that can be used to measure water level if the capillary rise in the tube is to be restricted to 2 mm. Consider surface tension of water in contact with air as 0.073575 N/m.

Solution. Given :

Capillary rise, $h = 2.0 \text{ mm} = 2.0 \times 10^{-3}$ m

Surface tension, $\sigma = 0.073575$ N/m

Let dia. of tube $= d$

The angle θ for water $= 0$

The density for water, $\rho = 1000 \text{ kg/m}^3$

$$h = \frac{4\sigma}{\rho \times g \times d} \quad \text{or} \quad 2.0 \times 10^{-3} = \frac{4 \times 0.073575}{1000 \times 9.81 \times d}$$

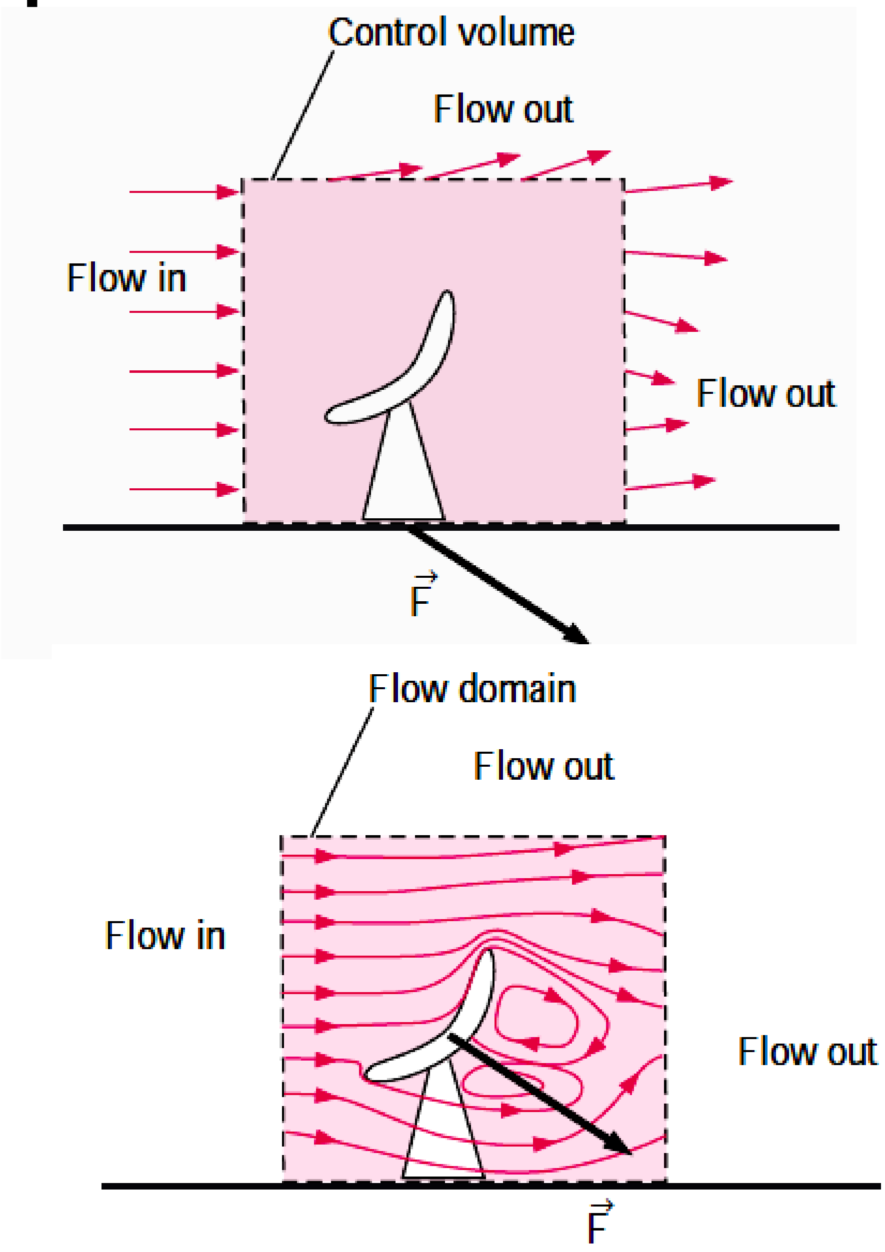
$$\therefore \quad d = \frac{4 \times 0.073575}{1000 \times 9.81 \times 2 \times 10^{-3}} = 0.015 \text{ m} = \mathbf{1.5\ cm.\ Ans.}$$

Thus minimum diameter of the tube should be 1.5 cm.

Flow Analysis Techniques

- In analyzing fluid motion, we might take one of two paths:

1. Seeking an estimate of gross effects (mass flow, induced force, energy change) over a finite region or control volume or
2. Seeking the point-by-point details of a flow pattern by analyzing an infinitesimal region of the flow.



Flow Analysis Techniques

- The control volume technique is useful when we are interested in the overall features of a flow, such as mass flow rate into and out of the control volume or net forces applied to bodies.
- Differential analysis, on the other hand, involves application of differential equations of fluid motion to *any and every point in the flow field over a region* called the flow domain.
- When solved, these differential equations yield details about the velocity, density, pressure, etc., at *every point* throughout the *entire flow domain.*

Flow Patterns

- Fluid mechanics is a highly visual subject. The patterns of flow can be visualized in a dozen different ways, and you can view these sketches or photographs and learn a great deal qualitatively and often quantitatively about the flow.
- Four basic types of line patterns are used to visualize flows:

1. A ***streamline*** is a line everywhere tangent to the velocity vector at a given instant.
2. A ***pathline*** is the actual path traversed by a given fluid particle.
3. A ***streakline*** is the locus of particles that have earlier passed through a prescribed point.
4. A ***timeline*** is a set of fluid particles that form a line at a given instant.

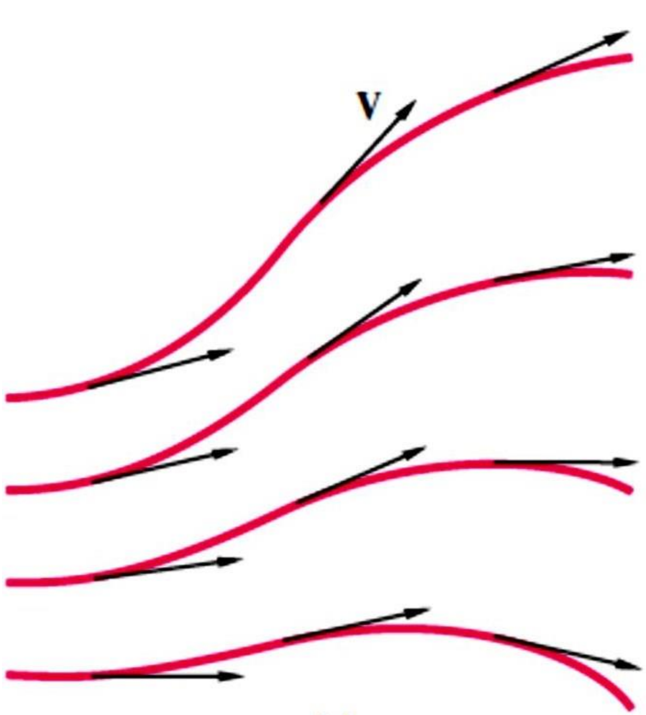
Flow Patterns

- The streamline is convenient to calculate mathematically, while the other three are easier to generate experimentally.
- Note that a streamline and a timeline are instantaneous lines, while the pathline and the streakline are generated by the passage of time.
- A ***streamline*** is a line that is everywhere tangent to the velocity field. If the flow is steady, nothing at a fixed point (including the velocity direction) changes with time, so the streamlines are fixed lines in space.
- For unsteady flows the streamlines may change shape with time.
- A pathline is the line traced out by a given particle as it flows from one point to another.

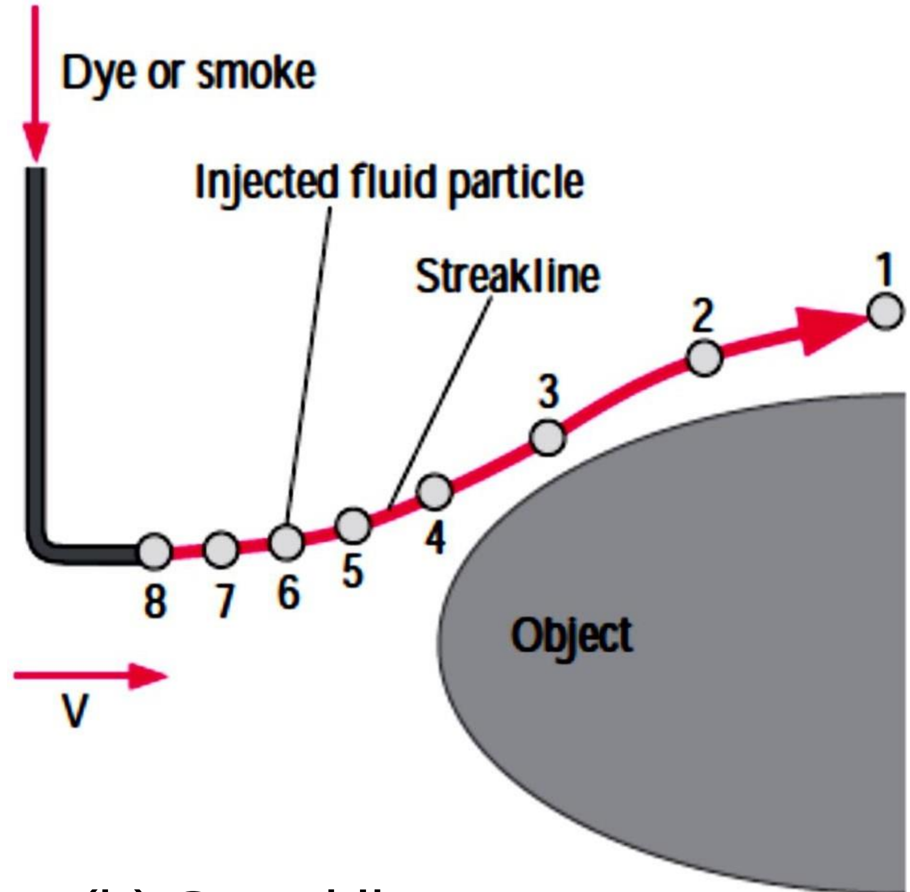
Flow Patterns

- A ***streakline*** consists of all particles in a flow that have previously passed through a common point. Streaklines are more of a laboratory tool than an analytical tool.
- They can be obtained by taking instantaneous photographs of marked particles that all passed through a given location in the flow field at some earlier time.
- Such a line can be produced by continuously injecting marked fluid (neutrally buoyant smoke in air, or dye in water) at a given location.
- If the flow is steady, each successively injected particle follows precisely behind the previous one forming a steady streakline that is exactly the same as the streamline through the injection point.

Flow Patterns



(a)

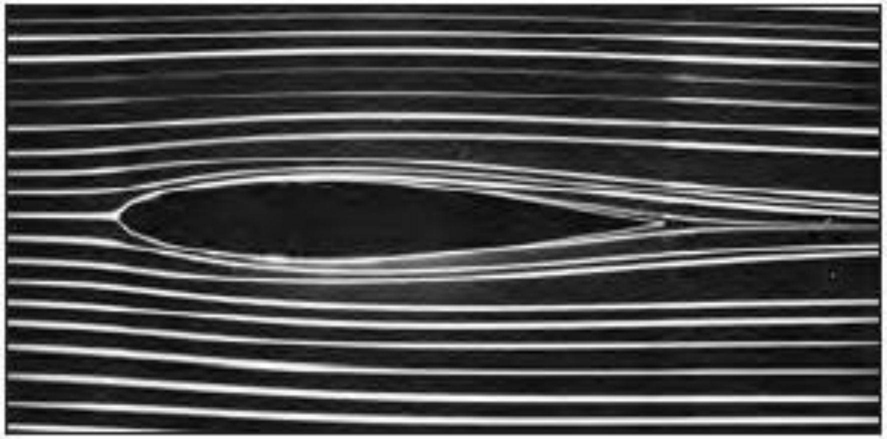


(a) Streamlines

(b) Streaklines

Flow Patterns

- Streaklines are often confused with streamlines or pathlines.
- While the three flow patterns are identical in steady flow, they can be quite different in unsteady flow.
- The main difference is that a streamline represents an *instantaneous* flow pattern at a given instant in time, while a streakline and a pathline are flow patterns that have some *age* and thus a *time history* associated with them.
- *If the flow is steady, streamlines, pathlines, and streaklines are identical*



Dimensions and Units

- Fluid mechanics deals with the measurement of many variables of many different types of units. Hence we need to be very careful to be consistent.

Dimensions and Base Units

- The *dimension of a measure is independent of any particular system of units. For* example, velocity may be inmetres per second or miles per hour, but dimensionally, it is always length per time, or $L/T = LT^{-1}$.
- The dimensions of the relevant base units of the Système International (SI) system are:

Dimensions and Units

Unit-Free		SI Units	
Dimension	**Symbol**	**Unit**	**Symbol**
Mass	M	kilogram	kg
Length	L	metre	m
Time	T	second	s
Temperature	θ	kelvin	K

Derived Units

Quantity	Dimension	SI Unit	
		Derived	**Base**
Velocity	LT^{-1}	m/s	$m\,s^{-1}$
Acceleration	LT^{-2}	m/s^2	$m\,s^{-2}$
Force	MLT^{-2}	Newton, N	$kg\,m\,s^{-2}$

Pressure Stress	$ML^{-1}T^{2}$	Pascal, Pa N/m^2	$kg\ m^{-1}\ s^{-2}$
Density	ML^{-3}	kg/m^3	$kg\ m^{-3}$
Specific weight	$ML^{-2}T^{-2}$	N/m^3	$kg\ m^{-2}\ s^{-2}$
Relative density	Ratio	Ratio	Ratio
Viscosity	$ML^{-1}T^{-1}$	Ns/m^2	$kg\ m^{-1}\ s^{-1}$
Energy (work)	$ML^{2}T^{-2}$	Joule, J Nm	$kg\ m^{2}\ s^{-2}$
Power	$ML^{2}T^{-3}$	Watt, W Nm/s	$kg\ m^{2}\ s^{-3}$

Note: The acceleration due to gravity will always be taken as 9.81 m/s^2.

Unit Table

Quantity	SI Unit	English Unit
Length (L)	Meter (*m*)	Foot (*ft*)
Mass (m)	Kilogram (*kg*)	Slug (slug) = $lb*sec^2/ft$
Time (T)	Second (*s*)	Second (*sec*)
Temperature (θ)	Celcius (^oC)	Farenheit (^oF)
Force	Newton $(N)=kg*m/s^2$	Pound (lb)

Dimensions and Units…

- 1 *Newton* – Force required to accelerate a 1 *kg* of mass to 1 m/s^2
- 1 *slug* – is the mass that accelerates at 1 ft/s^2 when acted upon by a force of 1 *lb*
- To remember units of a Newton use F=ma (Newton's 2nd Law)
  - $[F] = [m][a] = kg*m/s^2 = N$
- To remember units of a slug also use F=ma => m = F / a
- $[m] = [F] / [a] = lb / (ft / sec^2) = lb*sec^2 / ft$

End of Chapter 1

Next Lecture

Chapter 2: Fluid Statics

Fluids and Fluid Mechanics

Fluids

Continuous media that flow

Some new quantities and new units.

Density

$$\rho = \frac{\Delta m}{\Delta V}$$

Mass per unit volume.

Units are kg/m^3.

Pressure

$$P = \frac{\Delta F}{\Delta A}$$

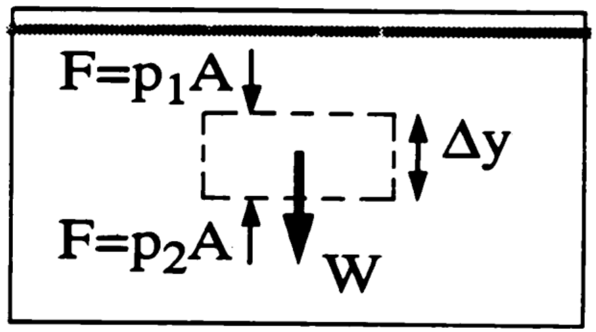
Force per unit area.

Units are $N/m^2 = Pa$.

$$1 \text{ atm} = 14.7 \text{ lb/in}^2 = 1.01\times10^5 \text{ Pa} = 101 \text{ kPa}$$

Pressure and Depth

Consider the weight of a block of water inside a swimming pool. Why doesn't it fall?



$$P_2 A = P_1 A + W$$

In terms of density, $W = \rho A \Delta y$ g and this is simply

$$P_2 = P_1 + \rho g \Delta y$$

This is commonly written in terms of depth *h*:

$$P = P_0 + \rho g h$$

P is called the "absolute" pressure
P_0 is usually just atmospheric pressure
$P - P_0$ is called the "gauge" pressure

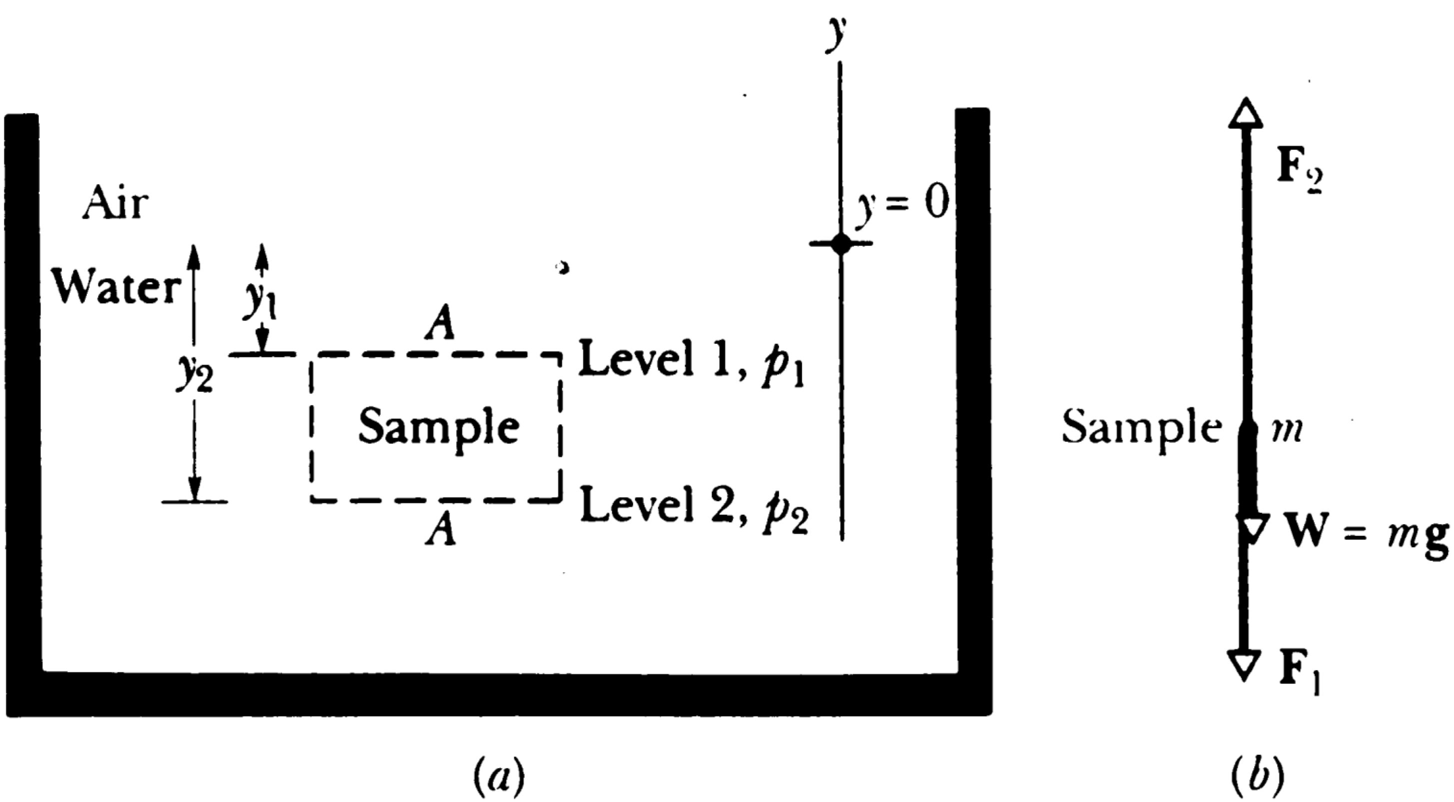
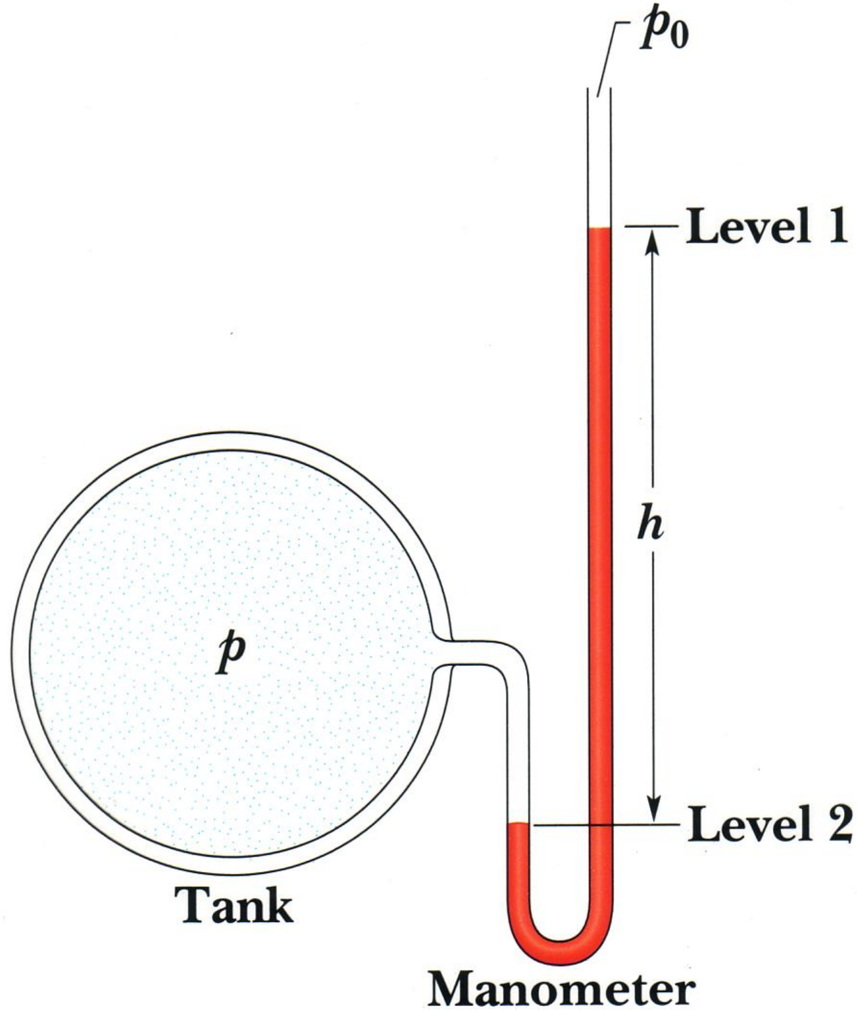


FIGURE 15-2 (*a*) A tank of water in which a sample of water is contained in an imaginary cylinder of horizontal base area A. (*b*) A free-body diagram of the water sample. The water in the sample is in static equilibrium, its weight being balanced by the net upward buoyant force that acts on it.



p_0
Level 1
h
p
Level 2
Tank
Manometer

Archimedes' Principle

A slug of water stays in place inside the pool because pressure balances all the forces on it.

If you replace the slug with some other material, those forces still act on the surface of the "slug".

⇒ *A body fully or partially immersed in a fluid is buoyed up by a force equal to the weight of the fluid that the body displaces.*

Archimedes' principle:

A body fully or partially immersed in a fluid is buoyed up by a force equal to the weight of the fluid that the body displaces.

PASCAL'S PRINCIPLE

A change in the pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of the containing vessel.

Bernoulli's Equation

Conservation of energy in a moving fluid

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

If the fluid is motionless, then v=0 and

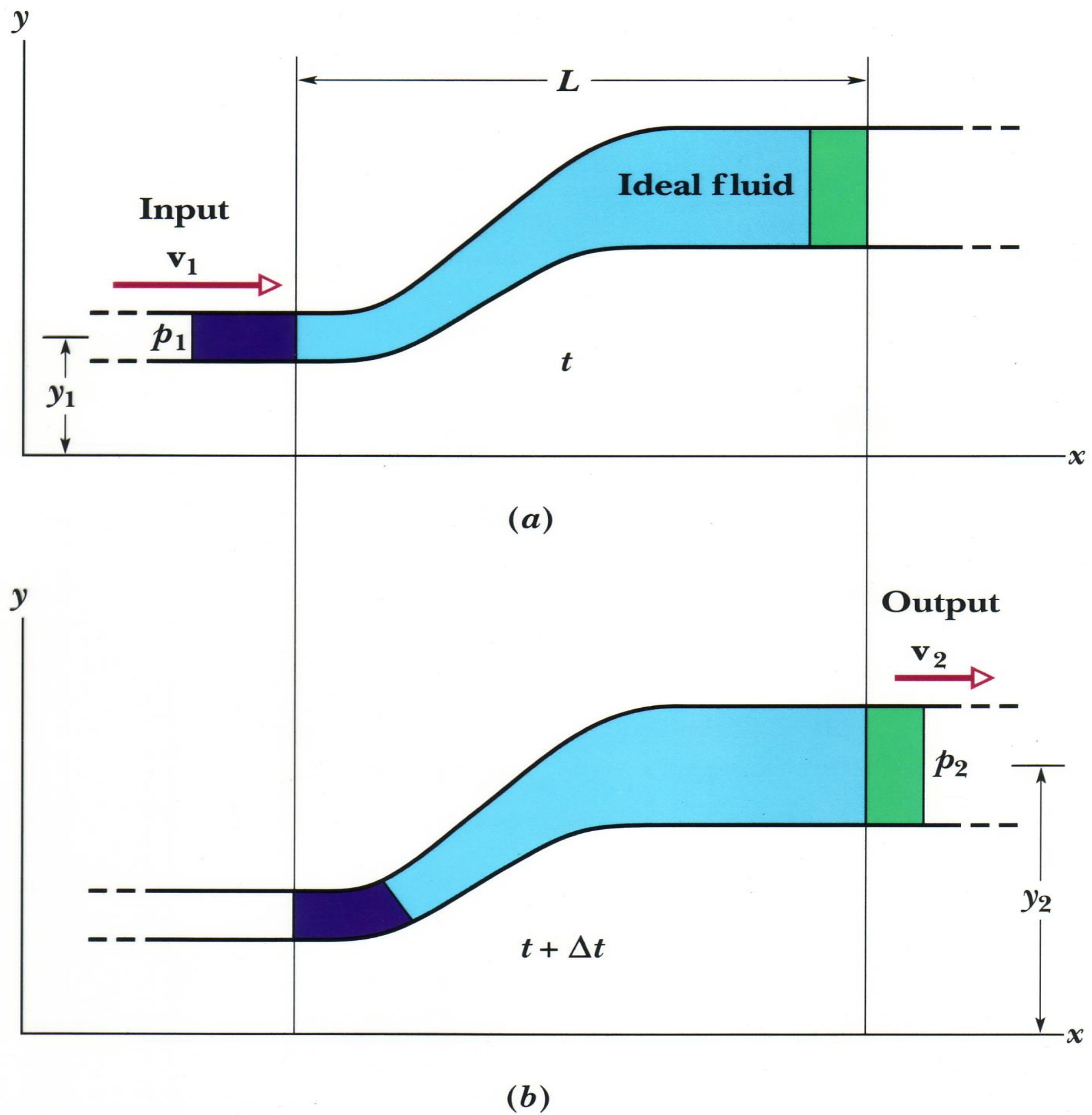
$$p + \rho g y = \text{constant}$$

which we've already seen.

If the fluid moves without changing it height,

$$p + \frac{1}{2}\rho v^2 = \text{constant}$$

This tells you how pressure changes with speed.


y
L
Ideal fluid
Input
v1
p1
y1
t
x
(a)
y
Output
v2
p2
y2
t + Δt
x
(b)

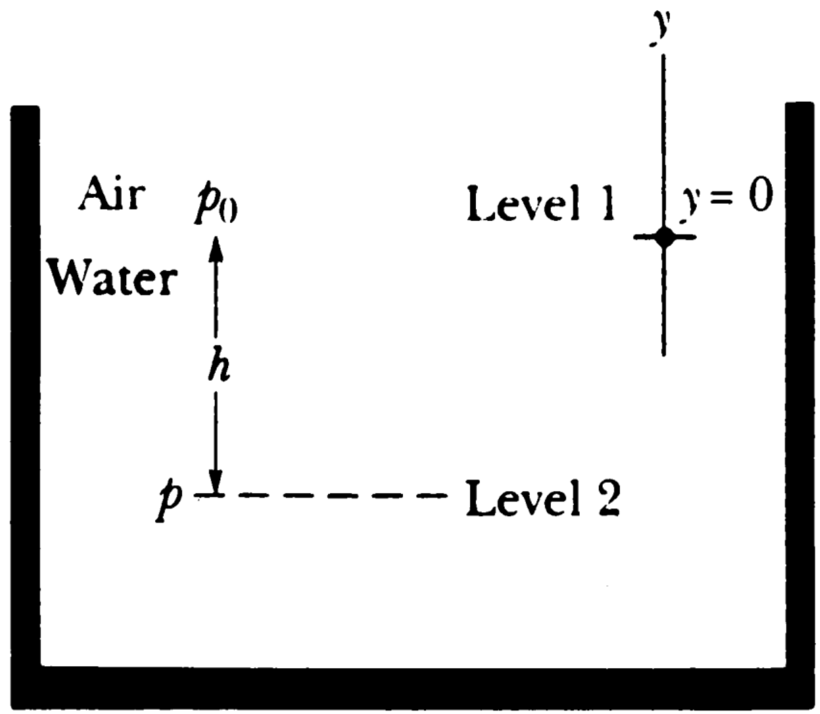
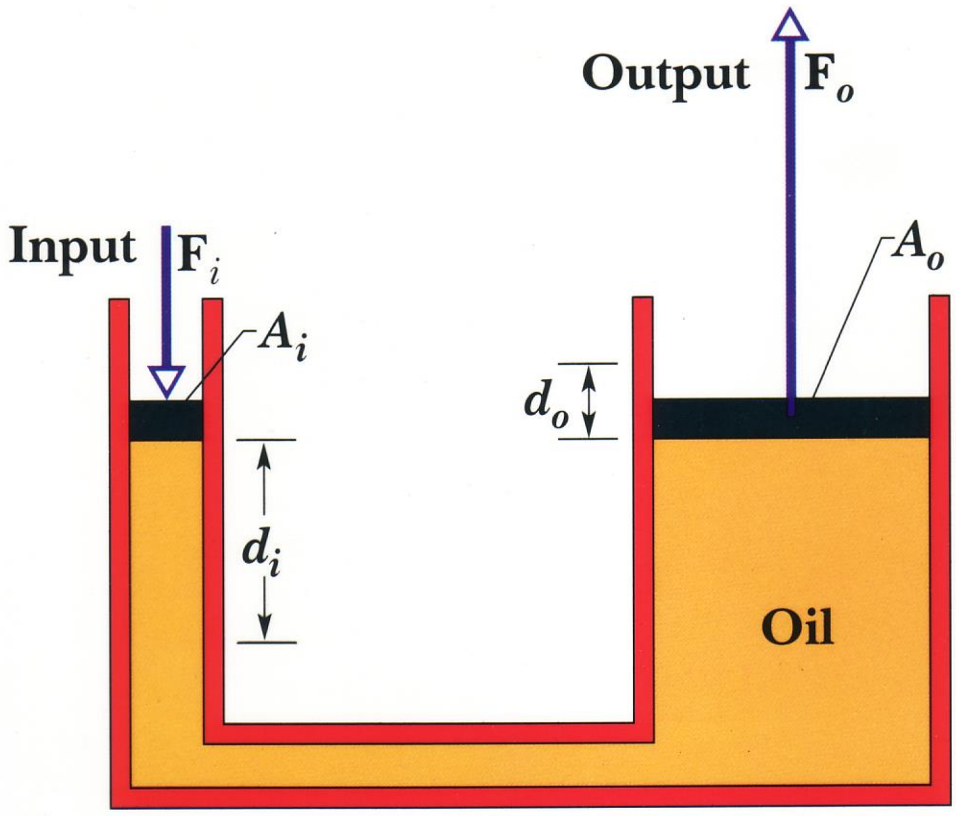


FIGURE 15-3 The pressure p increases with depth h below the water surface according to Eq. 15-5.

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2. \qquad (15\text{-}16)$$

We can also write this equation as

$$p + \tfrac{1}{2}\rho v^2 + \rho g y = \text{a constant}. \qquad (15\text{-}17)$$



Output
$\mathbf{F}_o$
Input
$\mathbf{F}_i$
A_o
A_i
d_o
d_i
Oil

TABLE 15-2 SOME PRESSURES

	PRESSURE (Pa)
Center of the Sun	2×10^{16}
Center of Earth	4×10^{11}
Highest sustained laboratory pressure	1.5×10^{10} (2×10^{11} Pa)
Deepest ocean trench (bottom)	1.1×10^{8}
Spike heels on a dance floor	1×10^{6}
Automobile tire[a]	2×10^{5}
Atmosphere at sea level	1.0×10^{5}
Normal blood pressure[a,b]	1.6×10^{4}
Best laboratory vacuum	10^{-12}

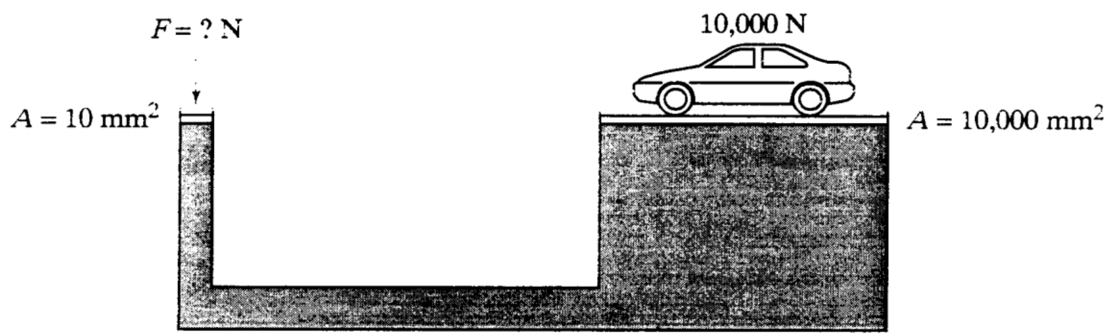
[a] Pressure in excess of atmospheric pressure.

[b] The systolic pressure, corresponding to 120 torr on the physician's pressure gauge.

TABLE 15-1 SOME DENSITIES

MATERIAL OR OBJECT	DENSITY (kg/m^3)
Interstellar space	10^{-20}
Best laboratory vacuum	10^{-17}
Air: 20°C and 1 atm	1.21
20°C and 50 atm	60.5
Styrofoam	1×10^2
Water: 20°C and 1 atm	0.998×10^3
20°C and 50 atm	1.000×10^3
Seawater: 20°C and 1 atm	1.024×10^3
Whole blood	1.060×10^3
Ice	0.917×10^3
Iron	7.9×10^3
Mercury	13.6×10^3
Earth: average	5.5×10^3
core	9.5×10^3
crust	2.8×10^3
Sun: average	1.4×10^3
core	1.6×10^5
White dwarf star (core)	10^{10}
Uranium nucleus	3×10^{17}
Neutron star (core)	10^{18}
Black hole (1 solar mass)	10^{19}

A container is filled with oil and fitted on both ends with pistons. The area of the left piston is 10 mm^2; that of the right piston 10,000 mm^2. What force must be exerted on the left piston to keep the 10,000-N car on the right at the same height?

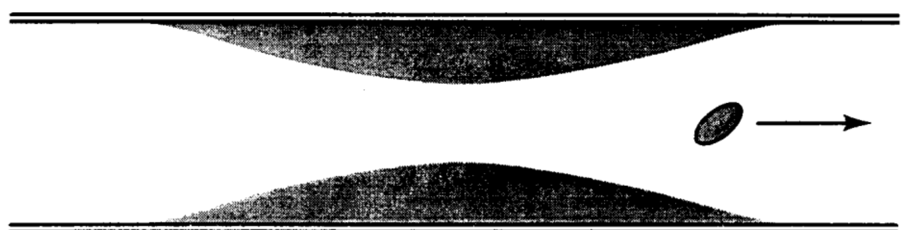


1. 10 N
2. 100 N
3. 10,000 N
4. 10^6 N
5. 10^8 N
6. insufficient information

When a hole is made in the side of a container holding water, water flows out and follows a parabolic trajectory. If the container is dropped in free fall, the water flow

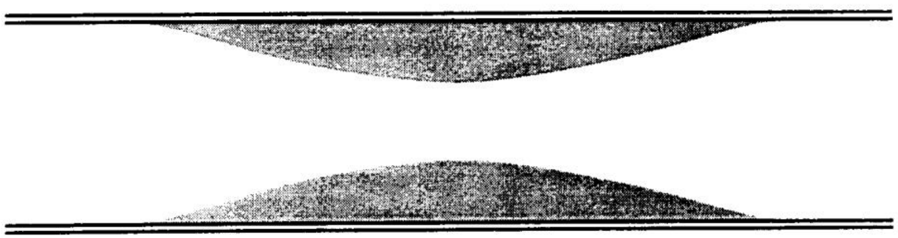
1. diminishes.
2. stops altogether.
3. goes out in a straight line.
4. curves upward.

A blood platelet drifts along with the flow of blood through an artery that is partially blocked by deposits. As the platelet moves from the narrow region to the wider region, it experiences



1. an increase in pressure.
2. no change in pressure.
3. a decrease in pressure.

Blood flows through a coronary artery that is partially blocked by deposits along the artery wall. Through which part of the artery is the flow speed largest?



1. The narrow part.
2. The wide part.
3. The flux is the same in both parts.

Two cups are filled to the same level with water. One of the two cups has ice cubes floating in it. Which weighs more?

1. The cup without ice cubes.
2. The glass with ice cubes.
3. The two weigh the same.

Bernoulli
Along a Streamline

$$-\nabla p = \rho \mathbf{a} + \rho g \hat{\mathbf{k}}$$

$$-\frac{\partial p}{\partial s} = \rho a_s + \rho g \frac{dz}{ds}$$

Separate acceleration due to gravity. Coordinate system may be in any orientation!

k is vertical, s is in direction of flow, n is normal.

Component of g in s direction

Note: No shear forces! Therefore flow must be frictionless.

Steady state (no change in p wrt time)

Bernoulli
Along a Streamline

$$-\frac{\partial p}{\partial s} = \rho a_s + \gamma \frac{dz}{ds}$$

Can we eliminate the partial derivative?

Chain rule

$$a_s = \frac{dV}{dt} = \overbrace{\frac{\partial V}{\partial s}\frac{ds}{dt}}^{\text{Chain rule}} = \frac{\partial V}{\partial s} V$$

Write acceleration as derivative wrt s

0 (n is constant along streamline, *dn*=0)

$$dp = \frac{\partial p}{\partial s} ds + \frac{\partial p}{\partial n} dn$$

$$\therefore dp/ds = \partial p/\partial s \quad \text{and} \quad dV/ds = \partial V/\partial s$$

$$-\frac{dp}{ds} = \rho V \frac{dV}{ds} + \gamma \frac{dz}{ds}$$

Integrate F=ma Along a Streamline

$$-\frac{dp}{ds} = \rho V \frac{dV}{ds} + \gamma \frac{dz}{ds}$$

Eliminate *ds*

$$dp + \rho V dV + \gamma dz = 0$$

Now let's integrate…

But density is a function of pressure.

$$\int \frac{dp}{\rho} + \int V dV + g \int dz = 0$$

$$\int \frac{dp}{\rho} + \frac{1}{2} V^2 + gz = C_p$$

If density is constant…

$$\boxed{p + \frac{1}{2} \rho V^2 + \gamma z = C_{p'}}$$

Bernoulli Equation

- Assumptions needed for Bernoulli Equation
  - Frictionless
  - Steady
  - Constant density (incompressible)
  - Along a streamline

- Eliminate the constant in the Bernoulli equation?

  Apply at two points along a streamline.

- Bernoulli equation does not include
  - Mechanical energy to thermal energy
  - Heat transfer, Shaft Work

Bernoulli Equation

The Bernoulli Equation is a statement of the conservation of Mechanical Energy

$$\underbrace{\frac{p}{\rho} + gz}_{\text{p.e.}} + \underbrace{\frac{1}{2}V^2}_{\text{k.e.}} = C_p$$

$$\frac{p}{\gamma} + z + \frac{V^2}{2g} = C_{p''}$$

$\dfrac{p}{\gamma} =$ Pressure head

$z =$ Elevation head

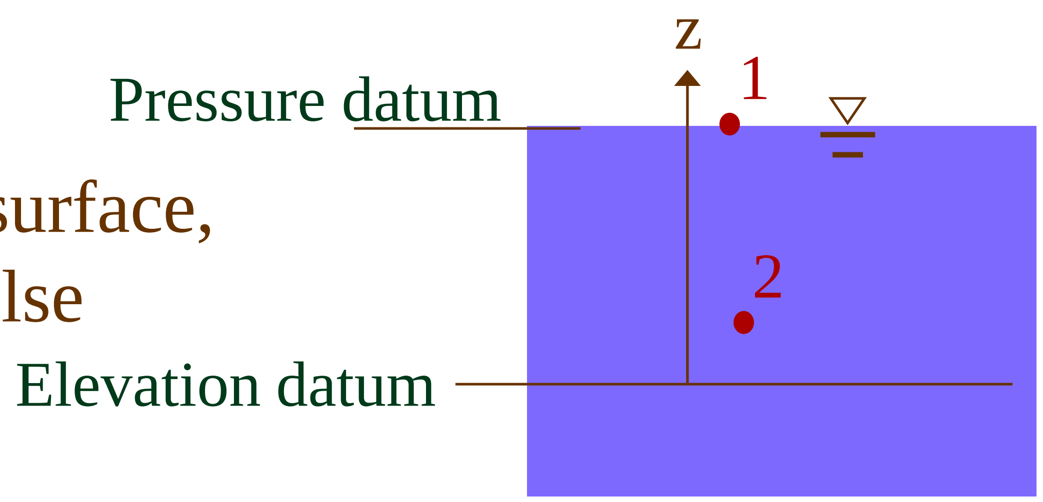
$\dfrac{V^2}{2g} =$ Velocity head

$\dfrac{p}{\gamma} + z =$ Piezometric head — Hydraulic Grade Line

$\dfrac{p}{\gamma} + z + \dfrac{V^2}{2g} =$ Total head — Energy Grade Line

Bernoulli Equation: Simple Case ($V = 0$)

- Reservoir ($V = 0$)
  - Put one point on the surface, one point anywhere else



$$\frac{p}{\gamma} + z + \cancel{\frac{V^2}{2g}} = C_{p"}$$

$$\cancel{\frac{p_1}{\gamma}} + z_1 = \frac{p_2}{\gamma} + z_2$$

We didn't **<u>cross</u>** any streamlines so this analysis is okay!

$$z_1 - z_2 = \frac{p_2}{g}$$

<u>Same as we found using statics</u>

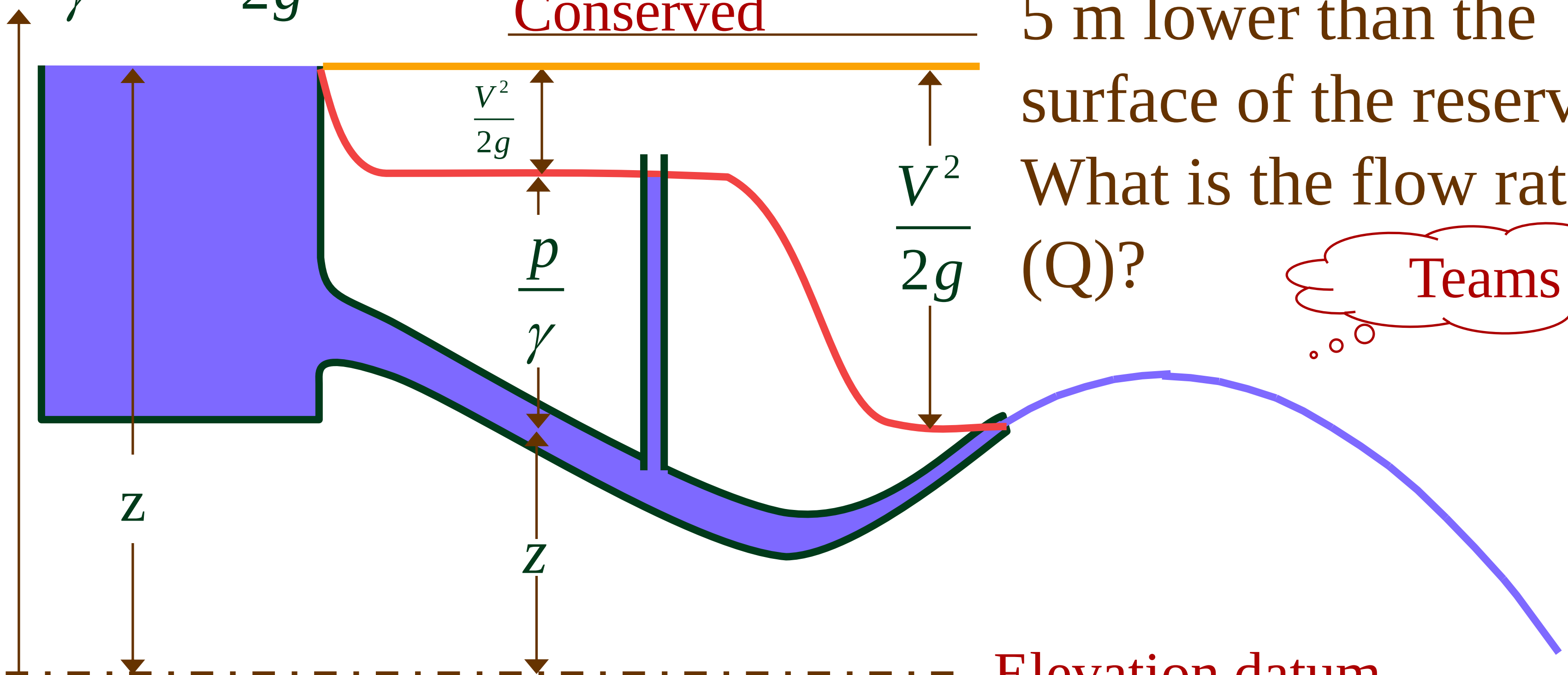
Hydraulic and Energy Grade Lines (neglecting Mechanical energy losses for now)

$$\frac{p}{\gamma} + z + \frac{V^2}{2g} = C_{p''}$$

Mechanical Energy Conserved

The 2 cm diameter jet is 5 m lower than the surface of the reservoir. What is the flow rate (Q)?

Teams



Elevation datum

Pressure datum? Atmospheric pressure

Bernoulli Equation: Simple Case (p = 0 or constant)

- What is an example of a fluid experiencing a change in elevation, but remaining at a constant pressure? __Free jet__

$$\cancel{\frac{p_1}{\gamma}} + z_1 + \frac{V_1^2}{2g} = \cancel{\frac{p_2}{\gamma}} + z_2 + \frac{V_2^2}{2g}$$

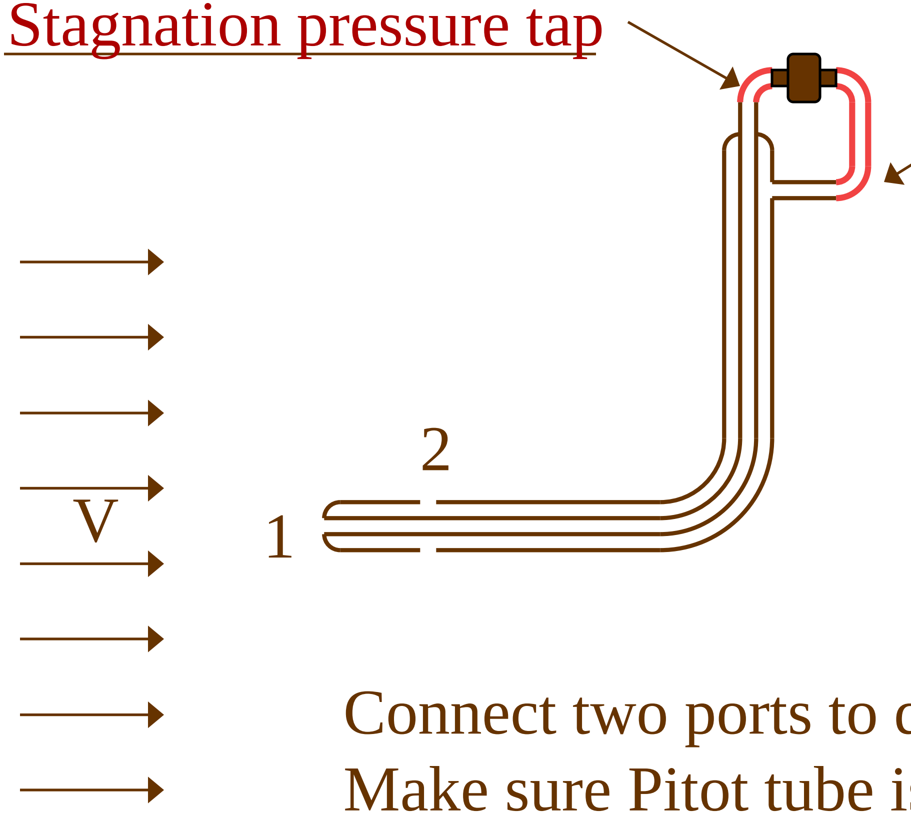
$$z_1 + \frac{V_1^2}{2g} = z_2 + \frac{V_2^2}{2g}$$

$$V_2 = \sqrt{2g\left(z_1 - z_2\right) + V_1^2}$$

Pitot Tubes

- Used to measure air speed on airplanes
- Can connect a differential pressure transducer to directly measure $V^2/2g$
- Can be used to measure the flow of water in pipelines <u>Point measurement!</u>

Pitot Tube



$$\frac{p_1}{\gamma} + \cancel{z_1} + \cancel{\frac{V_1^2}{2g}} = \frac{p_2}{\gamma} + \cancel{z_2} + \frac{V_2^2}{2g}$$

$$V_1 = \underline{0}$$

$$z_1 = z_2$$

$$V = \sqrt{\frac{2}{\rho}\left(p_1 - p_2\right)}$$

Connect two ports to differential pressure transducer.

Make sure Pitot tube is completely filled with the fluid that is being measured.

Solve for velocity as function of pressure difference

Relaxed Assumptions for Bernoulli Equation

- Frictionless (velocity not influenced by viscosity)

  Small energy loss (accelerating flow, short distances)

- Steady

  Or gradually varying

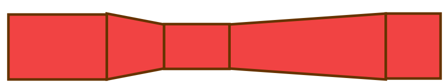
- Constant density (incompressible)

  Small changes in density

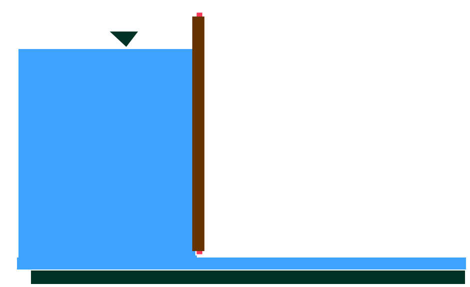
- Along a streamline

  Don't cross streamlines

Bernoulli Equation Applications

- Stagnation tube
- Pitot tube
- Free Jets
- Orifice
- Venturi 
- Sluice gate
- Sharp-crested weir

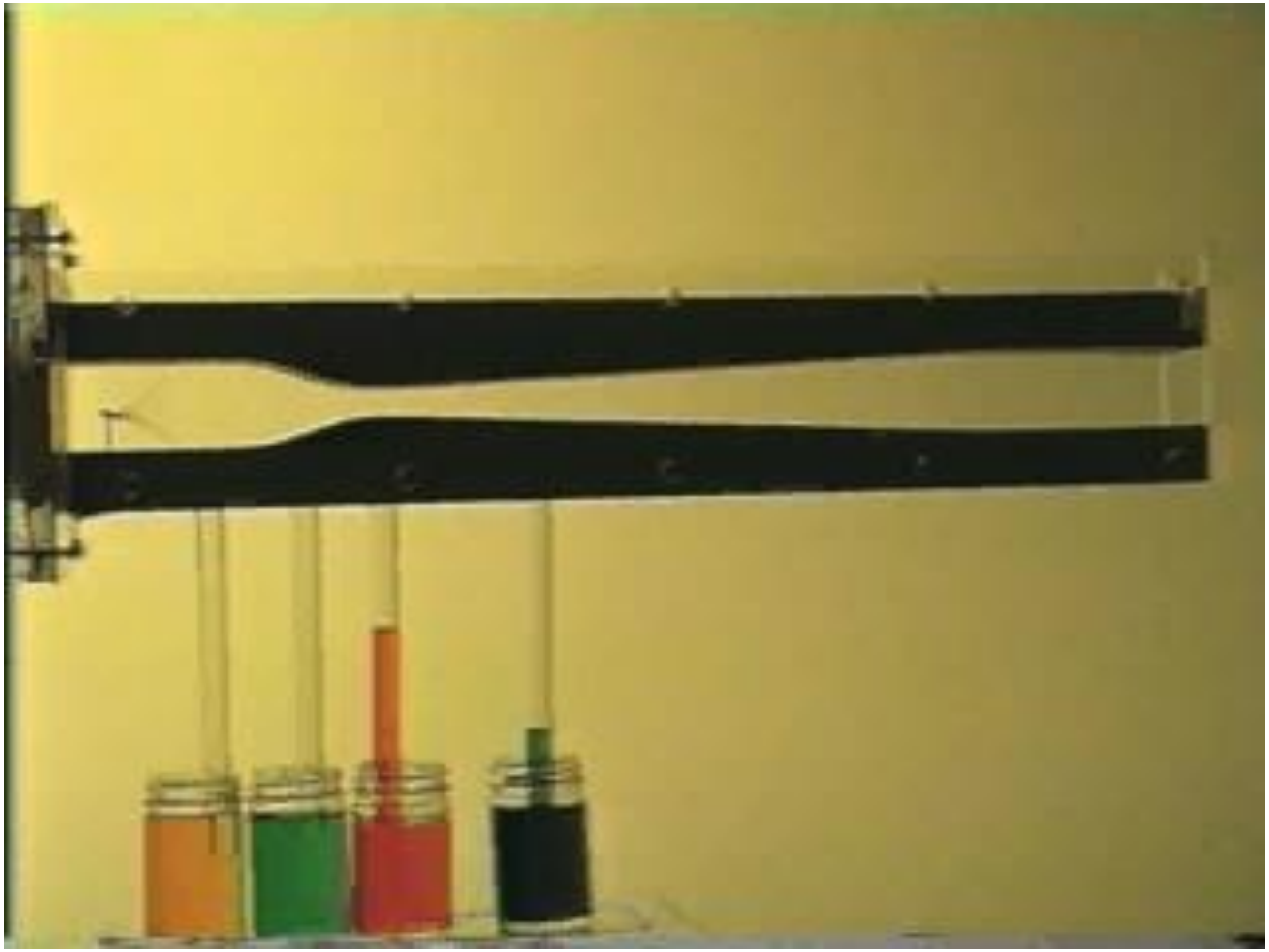
Applicable to contracting streamlines (accelerating flow).



Summary

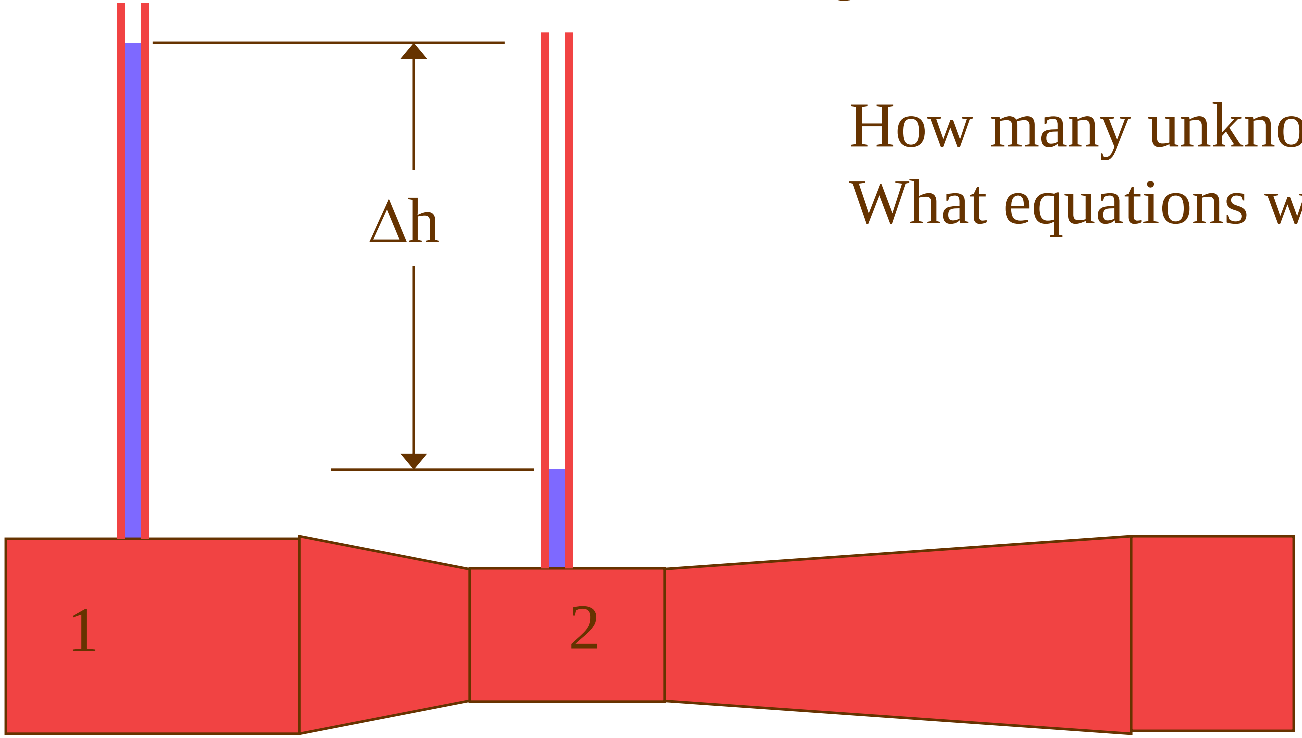
- By integrating F=ma along a streamline we found…
  - mechanical
    That energy can be converted between pressure, elevation, and velocity
  - That we can understand many simple flows by applying the Bernoulli equation
- However, the Bernoulli equation can not be applied to flows where viscosity is large, where mechanical energy is converted into thermal energy, or where there is shaft work.

Example: Venturi



Example: Venturi

How would you find the flow (Q) given the pressure drop between point 1 and 2 and the diameters of the two sections? You may assume the head loss is negligible. Draw the EGL and the HGL over the contracting section of the Venturi.



How many unknowns?

What equations will you use?

Example Venturi

$$\frac{p_1}{\gamma_1} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma_2} + z_2 + \frac{V_2^2}{2g}$$

$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\frac{p_1}{\gamma} - \frac{p_2}{\gamma} = \frac{V_2^2}{2g}\left[1 - \left(\frac{d_2}{d_1}\right)^4\right]$$

$$V_2 = \sqrt{\frac{2g(p_1 - p_2)}{\gamma\left[1 - (d_2/d_1)^4\right]}}$$

$$Q = C_v A_2 \sqrt{\frac{2g(p_1 - p_2)}{\gamma\left[1 - (d_2/d_1)^4\right]}}$$

$$Q = VA$$

$$V_1 A_1 = V_2 A_2$$

$$V_1 \frac{\pi d_1^2}{4} = V_2 \frac{\pi d_2^2}{4}$$

$$V_1 d_1^2 = V_2 d_2^2$$

$$V_1 = V_2 \frac{d_2^2}{d_1^2}$$

Chapter 5

Dimensional Analysis And Similitude

Introduction. DIMENSIONS AND UNITS

- A dimension is a measure of a physical quantity (without numerical values), while a unit is a way to assign a number to that dimension. For example, length is a dimension that is measured in units such as microns (μm), feet (ft), centimeters (cm), meters (m), kilometers (km), etc.
- There are seven primary dimensions (also called fundamental or basic dimensions)—mass, length, time, temperature, electric current, amount of light, and amount of matter.
- All nonprimary dimensions can be formed by some combination of the seven primary dimensions.
- For example, force has the same dimensions as mass times acceleration (by Newton's second law). Thus, in terms of primary dimensions,

$$\text{Dimensions of force:} \quad \{\text{Force}\} = \left\{\text{Mass}\,\frac{\text{Length}}{\text{Time}^2}\right\} = \{\text{mL/t}^2\}$$

Introduction. DIMENSIONS AND UNITS

Primary dimensions and their associated primary SI and English units

Dimension	Symbol*	SI Unit	English Unit
Mass	m	kg (kilogram)	lbm (pound-mass)
Length	L	m (meter)	ft (foot)
Time†	t	s (second)	s (second)
Temperature	T	K (kelvin)	R (rankine)
Electric current	I	A (ampere)	A (ampere)
Amount of light	C	cd (candela)	cd (candela)
Amount of matter	N	mol (mole)	mol (mole)

- Surface tension (σ_s), has dimensions of force per unit length. The dimensions of surface tension in terms of primary dimensions is

$$\text{Dimensions of surface tension:} \quad \{\sigma_s\} = \left\{\frac{\text{Force}}{\text{Length}}\right\} = \left\{\frac{m \cdot L/t^2}{L}\right\} = \{m/t^2\}$$

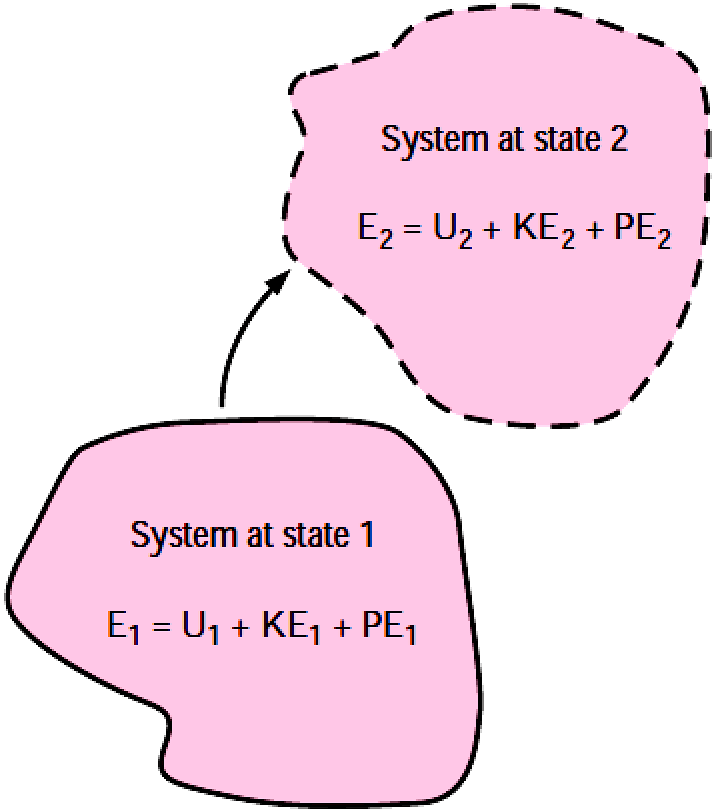
DIMENSIONAL HOMOGENEITY

- **Law of dimensional homogeneity:** Every additive term in an equation must have the same dimensions.
- Consider, for example, the change in total energy of a simple compressible closed system from one state and/or time (1) to another (2), as shown in the figure

- The change in total energy of the system (ΔE) is given by

$$\Delta E = \Delta U + \Delta KE + \Delta PE$$

- where E has three components: internal energy (U), kinetic energy (KE), and potential energy (PE).



DIMENSIONAL HOMOGENEITY

- These components can be written in terms of the system mass (m); measurable quantities and thermodynamic properties at each of the two states, such as speed (V), elevation (z), and specific internal energy (u); and the known gravitational acceleration constant (g),

$$\Delta U = m(u_2 - u_1) \qquad \Delta KE = \frac{1}{2}m(V_2^2 - V_1^2) \qquad \Delta PE = mg(z_2 - z_1)$$

- It is straightforward to verify that the left side of the change in Energy equation and all three additive terms on the right side have the same dimensions—energy.

$$\{\Delta E\} = \{\text{Energy}\} = \{\text{Force} \cdot \text{Length}\} \quad \rightarrow \quad \{\Delta E\} = \{mL^2/t^2\}$$

$$\{\Delta U\} = \left\{\text{Mass}\,\frac{\text{Energy}}{\text{Mass}}\right\} = \{\text{Energy}\} \quad \rightarrow \quad \{\Delta U\} = \{mL^2/t^2\}$$

DIMENSIONAL HOMOGENEITY

$$\{\Delta KE\} = \left\{\text{Mass}\,\frac{\text{Length}^2}{\text{Time}^2}\right\} \quad \rightarrow \quad \{\Delta KE\} = \{mL^2/t^2\}$$

$$\{\Delta PE\} = \left\{\text{Mass}\,\frac{\text{Length}}{\text{Time}^2}\,\text{Length}\right\} \quad \rightarrow \quad \{\Delta PE\} = \{mL^2/t^2\}$$

- In addition to dimensional homogeneity, calculations are valid only when the units are also homogeneous in each additive term.
- For example, units of energy in the above terms may be J, N·m , or kg·m^2/s^2, all of which are equivalent.
- Suppose, however, that kJ were used in place of J for one of the terms. This term would be off by a factor of 1000 compared to the other terms.
- It is wise to write out all units when performing mathematical calculations in order to avoid such errors.

Example 1. Dimensional Homogeneity of the Bernoulli Equation

- Probably the most well-known equation in fluid mechanics is the Bernoulli equation . One standard form of the Bernoulli equation for incompressible irrotational fluid flow is

$$P + \frac{1}{2}\rho V^2 + \rho g z = C$$

- (a) Verify that each additive term in the Bernoulli equation has the same dimensions. (b) What are the dimensions of the constant *C*?

SOLUTION We are to verify that the primary dimensions of each additive term in Eq. 1 are the same, and we are to determine the dimensions of constant *C*.

Analysis (*a*) Each term is written in terms of primary dimensions,

$$\{P\} = \{\text{Pressure}\} = \left\{\frac{\text{Force}}{\text{Area}}\right\} = \left\{\text{Mass}\,\frac{\text{Length}}{\text{Time}^2}\,\frac{1}{\text{Length}^2}\right\} = \left\{\frac{\text{m}}{\text{t}^2\text{L}}\right\}$$

$$\left\{\frac{1}{2}\rho V^2\right\} = \left\{\frac{\text{Mass}}{\text{Volume}}\left(\frac{\text{Length}}{\text{Time}}\right)^2\right\} = \left\{\frac{\text{Mass} \times \text{Length}^2}{\text{Length}^3 \times \text{Time}^2}\right\} = \left\{\frac{\text{m}}{\text{t}^2\text{L}}\right\}$$

$$\{\rho g z\} = \left\{\frac{\text{Mass}}{\text{Volume}}\,\frac{\text{Length}}{\text{Time}^2}\,\text{Length}\right\} = \left\{\frac{\text{Mass} \times \text{Length}^2}{\text{Length}^3 \times \text{Time}^2}\right\} = \left\{\frac{\text{m}}{\text{t}^2\text{L}}\right\}$$

Indeed, **all three additive terms have the same dimensions.**

(*b*) From the law of dimensional homogeneity, the constant must have the same dimensions as the other additive terms in the equation. Thus,

Primary dimensions of the Bernoulli constant: $$\{C\} = \left\{\frac{\text{m}}{\text{t}^2\text{L}}\right\}$$

Example 2 . Dimensional Homogeneity

- In Chap. 4 we discussed the differential equation for conservation of mass, the *continuity equation. In cylindrical* coordinates, and for steady flow,

$$\frac{1}{r}\frac{\partial(ru_r)}{\partial r}+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial u_z}{\partial z}=0$$

- Write the primary dimensions of each additive term in the equation, and verify that the equation is dimensionally homogeneous.

- **Solution.** We are to determine the primary dimensions of each additive term, and we are to verify that the equation is dimensionally homogeneous.
- **Analysis .**The primary dimensions of the velocity components are length/time. The primary dimensions of coordinates *r* and *z* are length, and the primary dimensions of coordinate θ are unity (it is a dimensionless angle). Thus each term in the equation can be written in terms of primary dimensions,

Example 2 . Dimensional Homogeneity

$$\left\{\frac{1}{r}\frac{\partial(ru_r)}{\partial r}\right\}=\left\{\frac{1}{\text{length}}\times\frac{\text{length}\frac{\text{length}}{\text{time}}}{\text{length}}\right\}=\left\{\frac{1}{\text{time}}\right\} \qquad \left\{\frac{1}{r}\frac{\partial(ru_r)}{\partial r}\right\}=\left\{\frac{1}{\text{t}}\right\}$$

$$\left\{\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}\right\}=\left\{\frac{1}{\text{length}}\times\frac{\frac{\text{length}}{\text{time}}}{1}\right\}=\left\{\frac{1}{\text{time}}\right\} \qquad \left\{\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}\right\}=\left\{\frac{1}{\text{t}}\right\}$$

$$\left\{\frac{\partial u_z}{\partial z}\right\}=\left\{\frac{\frac{\text{length}}{\text{time}}}{\text{length}}\right\}=\left\{\frac{1}{\text{time}}\right\} \qquad \left\{\frac{\partial u_z}{\partial z}\right\}=\left\{\frac{1}{\text{t}}\right\}$$

- Indeed, all three additive terms have the same dimensions, namely $\{t^{-1}\}$.

Nondimensionalization of Equations

- The law of dimensional homogeneity guarantees that every additive term in an equation has the same dimensions.
- It follows that if we divide each term in the equation by a collection of variables and constants whose product has those same dimensions, the equation is rendered **nondimensional**.

The nondimensionalized Bernoulli equation

$$\underbrace{\frac{P}{P_\infty}}_{\{1\}} + \underbrace{\frac{\rho V^2}{2P_\infty}}_{\{1\}} + \underbrace{\frac{\rho g z}{P_\infty}}_{\{1\}} = \underbrace{\frac{C}{P_\infty}}_{\{1\}}$$

A *nondimensionalized form of the* Bernoulli equation is formed by dividing each additive term by a pressure (here we use P_∞). *Each* resulting term is *dimensionless* (dimensions of {1}).

- If, in addition, the nondimensional terms in the equation are of order unity, the equation is called **normalized.**
- Each term in a nondimensional equation is dimensionless.

Nondimensionalization of Equations

- In the process of nondimensionalizing an equation of motion, **nondimensional parameters** often appear—most of which are named after a notable scientist or engineer (e.g., the Reynolds number and the Froude number).
- This process is sometimes called **inspectional analysis.**
- As a simple example, consider the equation of motion describing the elevation *z* of an object falling by gravity through a vacuum (no air drag).
- The initial location of the object is z_0 and its initial velocity is w_0 in the z-direction. From high school physics,

$$\frac{d^2z}{dt^2} = -g \qquad \text{……………….. (1)}$$

- **Dimensional variables** are defined as dimensional quantities that change or vary in the problem.

Nondimensionalization of Equations

- For the simple differential equation given in Eq. 1, there are two dimensional variables: z (dimension of length) and t (dimension of time).
- **Nondimensional (or dimensionless)** variables are defined as quantities that change or vary in the problem, but have no dimensions; an example is angle of rotation, measured in degrees or radians which are dimensionless units. Gravitational constant g, while dimensional, remains constant and is called a **dimensional constant.**
- Other dimensional constants are relevant to this particular problem are initial location z_0 and initial vertical speed w_0.
- While dimensional constants may change from problem to problem, they are fixed for a particular problem and are thus distinguished from dimensional variables.

Nondimensionalization of Equations

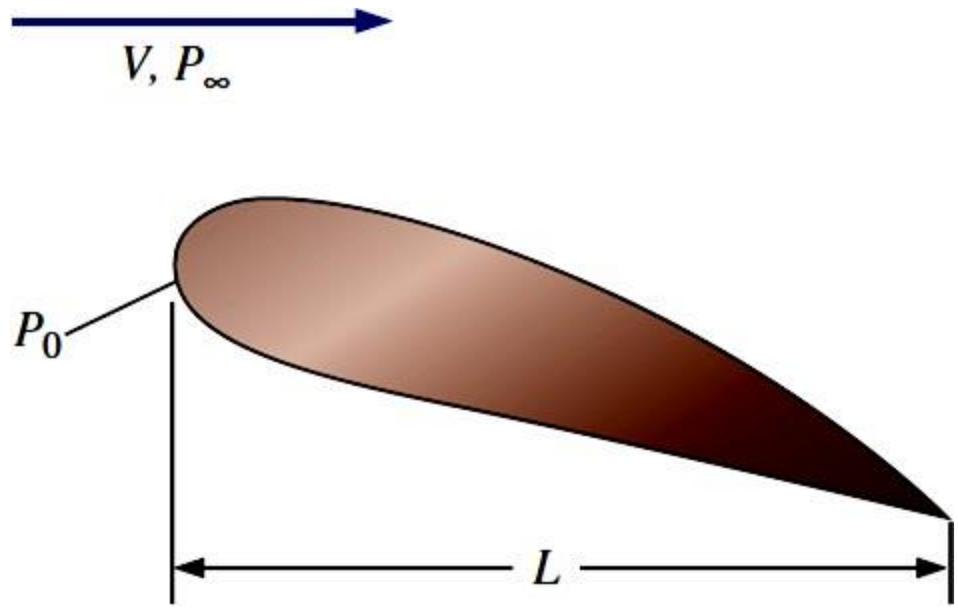
- We use the term **parameters** for the combined set of dimensional variables, nondimensional variables, and dimensional constants in the problem.
- Equation 1 is easily solved by integrating twice and applying the initial conditions. The result is an expression for elevation *z* at any time *t:*

$$z = z_0 + w_0 t - \frac{1}{2} g t^2 \qquad \ldots\ldots\ldots\ldots\ldots(2)$$

- The constant ½ and the exponent 2 in Eq. 2 are dimensionless results of the integration. Such constants are called **pure constants.** Other common examples of pure constants are Π and *e*.

Nondimensionalization of Equations

- To nondimensionalize Eq. 1, we need to select **scaling parameters,** based on the primary dimensions contained in the original equation.
- In fluid flow problems there are typically at least three scaling parameters, e.g., L, V, and $P_0 - P_\infty$, since there are at least three primary dimensions in the general problem (e.g., mass, length, and time).



In a typical fluid flow problem, the scaling parameters usually include a characteristic length L, a characteristic velocity V, and a reference pressure difference $P_0 - P_\infty$. Other parameters and fluid properties such as density, viscosity, and gravitational acceleration enter the problem as well.

Nondimensionalization of Equations

- In the case of the falling object being discussed here, there are only two primary dimensions, length and time, and thus we are limited to selecting only *two scaling parameters.*
- We have some options in the selection of the scaling parameters since we have three available dimensional constants g, z_0, *and* w_0. We choose z_0 and w_0. we can also do the analysis using g and z_0 and/or with g and w_0
- With these two chosen scaling parameters we nondimensionalize the dimensional variables *z and t.*
- The first step is to list the primary dimensions of all dimensional variables and dimensional constants in the problem,

Primary dimensions of all parameters:

$$\{z\} = \{L\} \qquad \{t\} = \{t\} \qquad \{z_0\} = \{L\} \qquad \{w_0\} = \{L/t\} \qquad \{g\} = \{L/t^2\}$$

Nondimensionalization of Equations

- The second step is to use our two scaling parameters to nondimensionalize *z* and *t* (by inspection) into nondimensional variables *z** and *t**,

Nondimensionalized variables:

$$z^* = \frac{z}{z_0} \qquad t^* = \frac{w_0 t}{z_0} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

- Substitution of Eq. 3 into Eq. 1 gives

$$\frac{d^2z}{dt^2} = \frac{d^2(z_0 z^*)}{d(z_0 t^*/w_0)^2} = \frac{w_0^2}{z_0}\frac{d^2z^*}{dt^{*2}} = -g \quad \rightarrow \quad \frac{w_0^2}{g z_0}\frac{d^2z^*}{dt^{*2}} = -1 \quad \ldots.(4)$$

- which is the desired nondimensional equation. The grouping of dimensional constants in Eq. 4 is the square of a well-known **nondimensional parameter** or **dimensionless group** called the **Froude number,**

Nondimensionalization of Equations

- Froude number: $$\mathrm{Fr} = \frac{w_0}{\sqrt{g z_0}}$$ ………… (5)

- Substitution of Eq. 5 into Eq. 4 yields
- *Nondimensionalized equation of motion:* $$\frac{d^2 z^*}{dt^{*2}} = -\frac{1}{\mathrm{Fr}^2}$$ …..(6)

- In dimensionless form, only one parameter remains, namely the Froude number.
- Equation 6 is easily solved by integrating twice and applying the initial conditions. The result is an expression for dimensionless elevation *z** as a function of dimensionless time *t**:
- *Nondimensional result:*
- $$z^* = 1 + t^* - \frac{1}{2\mathrm{Fr}^2} t^{*2}$$ ………….(7)

Nondimensionalization of Equations

- There are two key advantages of nondimensionalization
- First, it increases our insight about the relationships between key parameters. Equation 5 reveals, for example, that doubling w_0 has the same effect as decreasing z_0 by a factor of 4.
- Second, it reduces the number of parameters in the problem. For example, the original problem contains one dependent variable, z; one independent variable, t; and three additional dimensional constants, g, w_0, and z_0. The nondimensionalized problem contains one dependent parameter, z*; one independent parameter, t*; and only one additional parameter, namely the dimensionless Froude number, Fr. The number of additional parameters has been reduced from three to one!

Dimensional Analysis and Similarity

- Nondimensionalization of an equation by inspection is useful only when we know the equation to begin with.
- However, in many cases in real-life engineering, the equations are either not known or too difficult to solve; often times experimentation is the only method of obtaining reliable information.
- In most experiments, to save time and money, tests are performed on a geometrically scaled **model,** rather than on the full-scale **prototype**. In such cases, care must be taken to properly scale the results. We introduce here a powerful technique called **dimensional analysis.**

Dimensional Analysis and Similarity

- The three primary purposes of dimensional analysis are
  - ✓ To generate nondimensional parameters that help in the design of experiments (physical and/or numerical) and in the reporting of experimental results
  - ✓ To obtain scaling laws so that prototype performance can be predicted from model performance
  - ✓ To (sometimes) predict trends in the relationship between parameters
- There are three necessary conditions for complete similarity between a model and a prototype.
- The first condition is **geometric similarity**—the model must be the same shape as the prototype, but may be scaled by some constant scale factor.

Dimensional Analysis and Similarity

- The second condition is **kinematic similarity**, which means that the velocity at any point in the model flow must be proportional (by a constant scale factor) to the velocity at the corresponding point in the prototype flow.
- Specifically, for kinematic similarity the velocity at corresponding points must scale in magnitude and must point in the same relative direction.

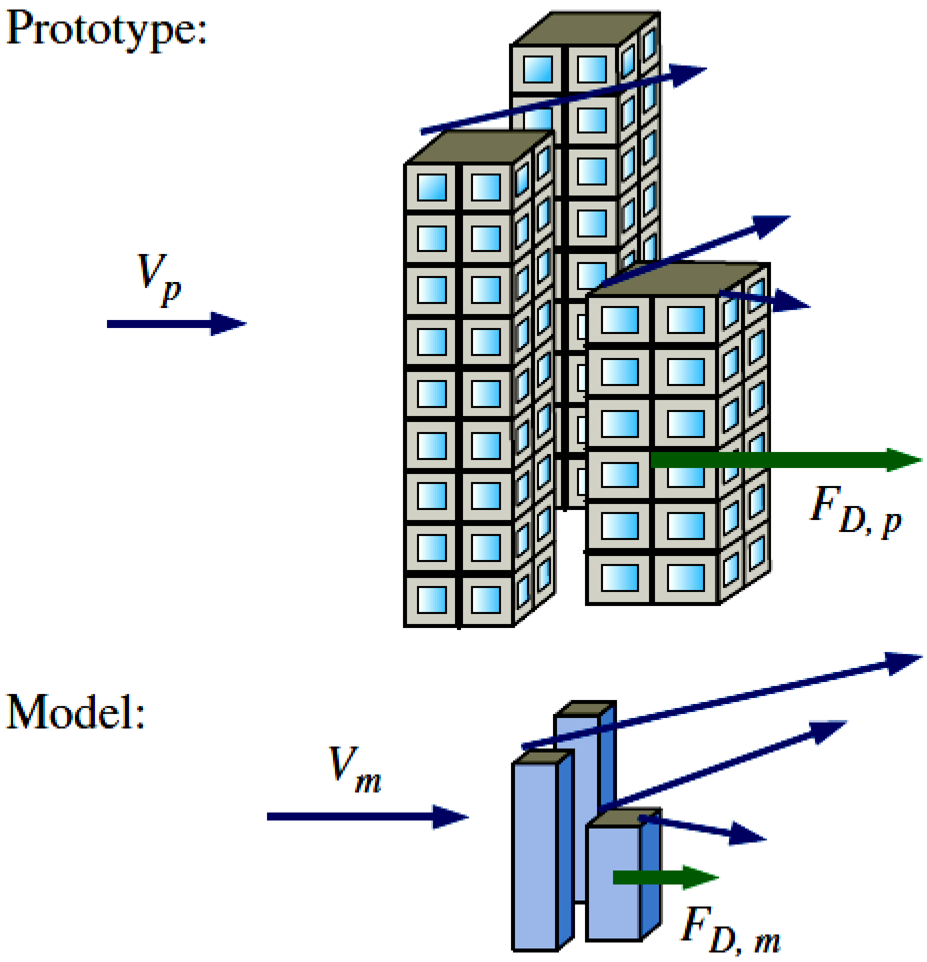


Fig. Kinematic similarity

Dimensional Analysis and Similarity

- *Kinematic similarity* is achieved when, at all locations, the speed in the model flow is proportional to that at corresponding locations in the prototype flow, and points in the same direction.
- *Geometric similarity is a prerequisite for kinematic similarity*
- Just as the geometric scale factor can be less than, equal to, or greater than one, so can the velocity scale factor.
- In Fig. above, for example, the geometric scale factor is less than one (model smaller than prototype), but the velocity scale is greater than one (velocities around the model are greater than those around the prototype).
- The third and most restrictive similarity condition is that of **dynamic similarity**. Dynamic similarity is achieved when all *forces* in the model flow scale by a constant factor to corresponding forces in the prototype flow (*force-scale equivalence).*

Dimensional Analysis and Similarity

- As with geometric and kinematic similarity, the scale factor for forces can be less than, equal to, or greater than one.
- In Fig. shown in slide 20 above for example, the force-scale factor is less than one since the force on the model building is less than that on the prototype.
- *Kinematic similarity is a necessary but insufficient condition for dynamic similarity.*
- It is thus possible for a model flow and a prototype flow to achieve both geometric and kinematic similarity, yet not dynamic similarity. All three similarity conditions must exist for complete similarity to be ensured.
- In a general flow field, complete similarity between a model and prototype is achieved only when there is geometric, kinematic, and dynamic similarity.

Dimensional Analysis and Similarity

- We let uppercase Greek letter Pi (Π) denote a nondimensional parameter. We have already discussed one Π, namely the Froude number, Fr.
- In a general dimensional analysis problem, there is one Π that we call the **dependent Π**, giving it the notation **Π_1**. The parameter $\mathbf{\Pi}_1$ is in general a function of several other **Π**'s, which we call **independent** Π's. The functional relationship is
- *Functional relationship between* **Π***'s:*

$$\Pi_1 = f(\Pi_2, \Pi_3, \ldots, \Pi_k)$$

- where k is the total number of Π's.
- Consider an experiment in which a scale model is tested to simulate a prototype flow.

Dimensional Analysis and Similarity

- To ensure complete similarity between the model and the prototype, each independent P of the model (subscript *m)* must be identical to the corresponding independent Π of the prototype (subscript p),

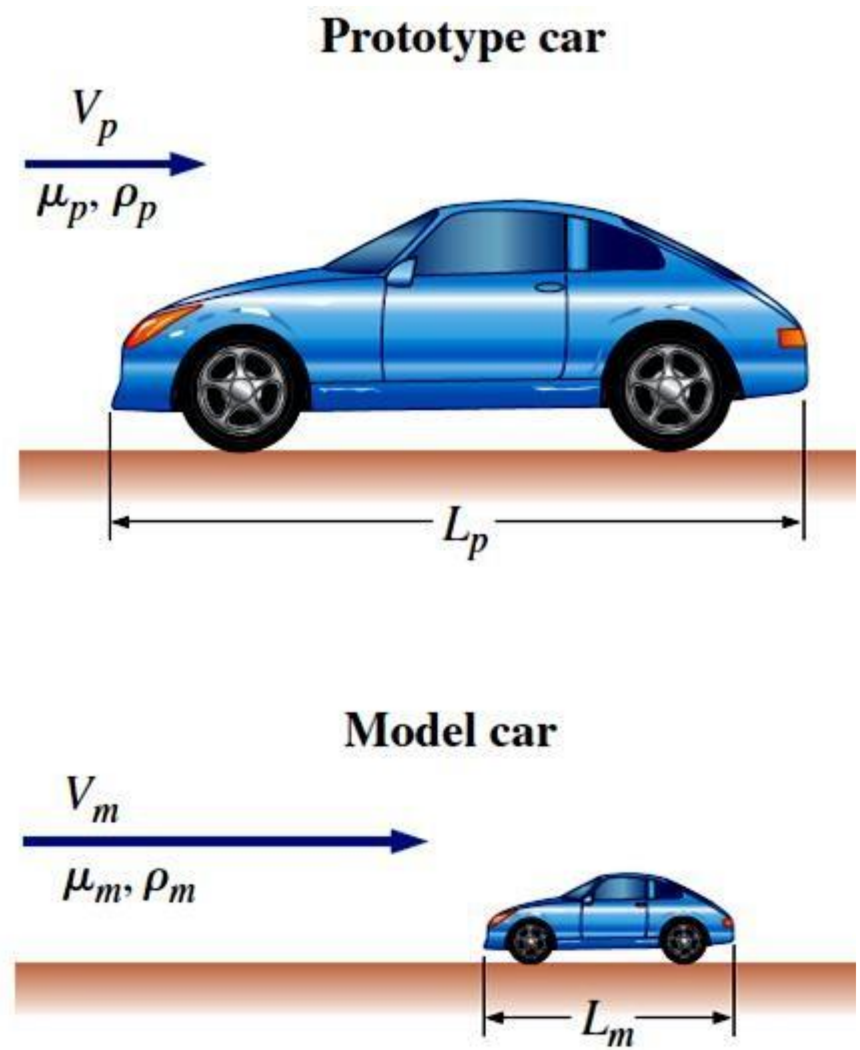
  i.e., $\Pi_{2,\,m} = \Pi_{2,\,p}$, $\Pi_{3,\,m} = \Pi_{3,\,p}$, ..., $\Pi_{k,\,m} = \Pi_{k,\,p}$.
- To ensure complete similarity, the model and prototype must be geometrically similar, and all independent Π groups must match between model and prototype.
- Under these conditions the *dependent* Π of the model ($\Pi_{1,\,m}$) is guaranteed to also equal the dependent Π of the prototype ($\Pi_{1,\,p}$).
- Mathematically, we write a conditional statement for achieving similarity,

$$\text{If} \quad \Pi_{2,m} = \Pi_{2,p} \quad \text{and} \quad \Pi_{3,m} = \Pi_{3,p} \ldots \quad \text{and} \quad \Pi_{k,m} = \Pi_{k,p},$$

$$\text{then} \quad \Pi_{1,m} = \Pi_{1,p}$$

Dimensional Analysis and Similarity

- Consider, for example, the design of a new sports car, the aerodynamics of which is to be tested in a wind tunnel. To save money, it is desirable to test a small, geometrically scaled model of the car rather than a full-scale prototype of the car.
- In the case of aerodynamic drag on an automobile, it turns out that if the flow is approximated as incompressible, there are only two Π's in the problem,



Dimensional Analysis and Similarity

$$\Pi_1 = f(\Pi_2)$$

- Where

$$\Pi_1 = \frac{F_D}{\rho V^2 L^2} \quad \text{and} \quad \Pi_2 = \frac{\rho V L}{\mu}$$

- The procedure used to generate these Π's will be discussed later in this chapter.
- In the above equation F_D is the magnitude of the aerodynamic drag on the car, ρ is the air density, V is the car's speed (or the speed of the air in the wind tunnel), L is the length of the car, and μ is the viscosity of the air. Π_1 is a nonstandard form of the drag coefficient, and Π_2 is the **Reynolds number, Re.**
- The Reynolds number is the most well known and useful dimensionless parameter in all of fluid mechanics

Dimensional Analysis and Similarity

- In the problem at hand there is only one independent Π, and the above Eq. ensures that if the independent Π's match (the Reynolds numbers match: $\Pi_{2,\,m} = \Pi_{2,\,p}$), then the dependent Π's also match ($\Pi_{1,\,m} = \Pi_{1,\,p}$).
- This enables engineers to measure the aerodynamic drag on the model car and then use this value to predict the aerodynamic drag on the prototype car.

Example 3: Similarity between Model and Prototype Cars

- The aerodynamic drag of a new sports car is to be predicted at a speed of 50.0 mi/h at an air temperature of 25°C. Automotive engineers build a one fifth scale model of the car to test in a wind tunnel. It is winter and the wind tunnel is located in an unheated building; the temperature of the wind tunnel air is only about 5°C. Determine how fast the engineers should run the wind tunnel in order to achieve similarity between the model and the prototype.

Solution:

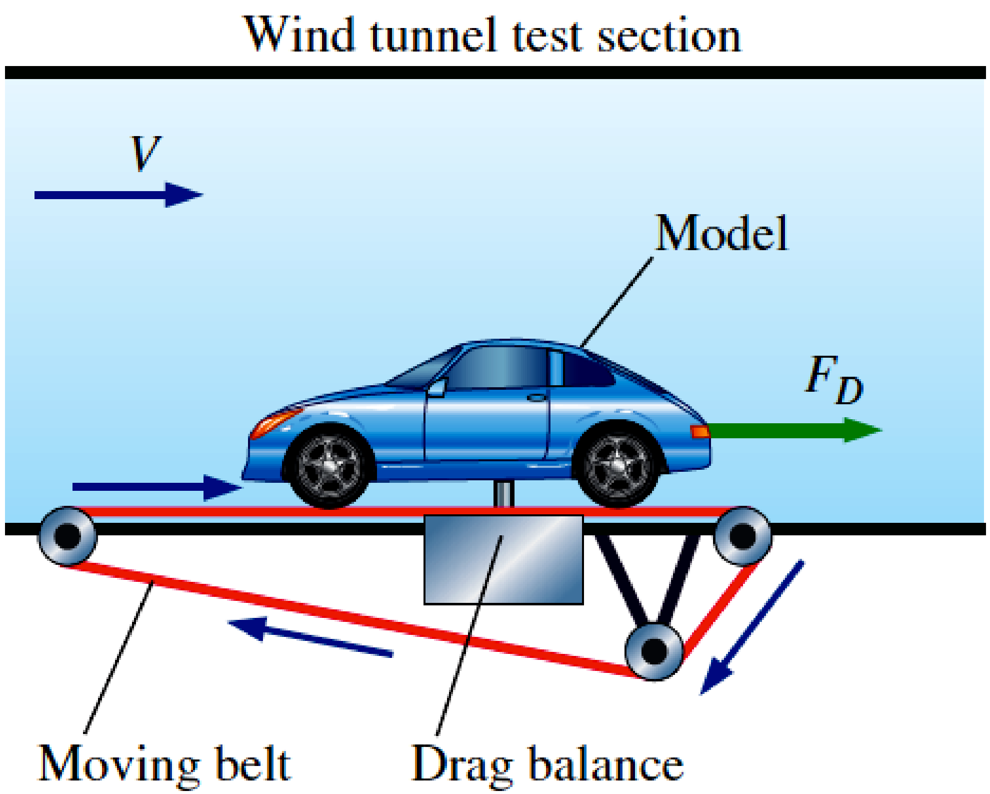
- We are to utilize the concept of similarity to determine the speed of the wind tunnel.

Assumptions:

- The model is geometrically similar to the prototype
- The wind tunnel walls are far enough away so as to not interfere with the aerodynamic drag on the model car.

Example 3: Similarity between Model and Prototype Cars

- The wind tunnel has a moving belt to simulate the ground under the car. (The moving belt is necessary in order to achieve kinematic similarity everywhere in the flow, in particular underneath the car.)



A *drag balance is a device used* in a wind tunnel to measure the aerodynamic drag of a body. When testing automobile models, a *moving belt is* often added to the floor of the wind tunnel to simulate the moving ground (from the car’s frame of reference).

Example 3: Similarity between Model and Prototype Cars

- ***Properties:*** For air at atmospheric pressure and at T = 25°C, ρ = 1.184 kg/m^3 and μ = 1.849 x 10^{-5} kg/m·s. Similarly, at T = 5°C, ρ = 1.269 kg/m^3 and μ = 1.754 x 10^{-5} kg/m·s.
- **Analysis:** Since there is only one independent Π in this problem, the similarity equation holds if $\Pi_{2,m} = \Pi_{2,p}$, where Π_2 is the Reynolds number. Thus, we write

$$\Pi_{2,m} = \mathrm{Re}_m = \frac{\rho_m V_m L_m}{\mu_m} = \Pi_{2,p} = \mathrm{Re}_p = \frac{\rho_p V_p L_p}{\mu_p}$$

- Thus

$$V_m = V_p\left(\frac{\mu_m}{\mu_p}\right)\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{L_p}{L_m}\right)$$
$$= (50.0\ \mathrm{mi/h})\left(\frac{1.754\times 10^{-5}\ \mathrm{kg/m\cdot s}}{1.849\times 10^{-5}\ \mathrm{kg/m\cdot s}}\right)\left(\frac{1.184\ \mathrm{kg/m^3}}{1.269\ \mathrm{kg/m^3}}\right)(5) = \mathbf{221\ mi/h}$$

Example 3: Similarity between Model and Prototype Cars

- The power of using dimensional analysis and similarity to supplement experimental analysis is further illustrated by the fact that the actual values of the dimensional parameters (density, velocity, etc.) are irrelevant. As long as the corresponding independent Π's are set equal to each other, similarity is achieved *even if different fluids are used.*
- *This explains why automobile* or aircraft performance can be simulated in a water tunnel, and the performance of a submarine can be simulated in a wind tunnel.
- Suppose, for example, that the engineers in Example above use a water tunnel instead of a wind tunnel to test their one-fifth scale model. Using the properties of water at room temperature (20°C is assumed), the water tunnel speed required to achieve similarity is easily calculated as

Example 3: Similarity between Model and Prototype Cars

$$V_m = V_p\left(\frac{\mu_m}{\mu_p}\right)\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{L_p}{L_m}\right)$$

$$= (50.0\ \text{mi/h})\left(\frac{1.002 \times 10^{-3}\ \text{kg/m}\cdot\text{s})}{1.849 \times 10^{-5}\ \text{kg/m}\cdot\text{s}}\right)\left(\frac{1.184\ \text{kg/m}^3}{998.0\ \text{kg/m}^3}\right)(5) = 16.1\ \text{mi/h}$$

- As can be seen, one advantage of a water tunnel is that the required water tunnel speed is much lower than that required for a wind tunnel using the same size model

The Method of Repeating Variables and the Buckingham Pi Theorem

- In this section we will learn how to generate the nondimensional parameters, i.e., the Π’s.
- There are several methods that have been developed for this purpose, but the most popular (and simplest) method is the **method of repeating variables,** popularized by Edgar Buckingham (1867–1940).
- We can think of this method as a step-by-step procedure or “recipe” for obtaining nondimensional parameters. There are six steps in this method as described below in detail

The Method of Repeating Variables and the Buckingham Pi Theorem

Step 1 List the parameters (dimensional variables, nondimensional variables, and dimensional constants) and count them. Let n be the total number of parameters in the problem, including the dependent variable. Make sure that any listed independent parameter is indeed independent of the others, i.e., it cannot be expressed in terms of them. (E.g., don't include radius r and area $A = \pi r^2$, since r and A are *not* independent.)

Step 2 List the primary dimensions for each of the n parameters.

Step 3 Guess the **reduction** j. As a first guess, set j equal to the number of primary dimensions represented in the problem. The expected number of Π's (k) is equal to n minus j, according to the **Buckingham Pi theorem,**

The Buckingham Pi theorem: $$k = n - j$$

If at this step or during any subsequent step, the analysis does not work out, verify that you have included enough parameters in step 1. Otherwise, go back and *reduce j by one* and try again.

The Method of Repeating Variables and the Buckingham Pi Theorem

Step 4 Choose *j* **repeating parameters** that will be used to construct each Π. Since the repeating parameters have the potential to appear in each Π, be sure to choose them *wisely*

Step 5 Generate the Π's one at a time by grouping the *j* repeating parameters with one of the remaining parameters, forcing the product to be dimensionless. In this way, construct all *k* Π's. By convention the first Π, designated as Π_1, is the *dependent* Π (the one on the left side of the list). Manipulate the Π's as necessary to achieve established dimensionless groups

Step 6 Check that all the Π's are indeed dimensionless.

The Method of Repeating Variables and the Buckingham Pi Theorem

Step 1: List the parameters in the problem and count their total number n.

Step 2: List the primary dimensions of each of the n parameters.

Step 3: Set the *reduction* j as the number of primary dimensions. Calculate k, the expected number of Π's,

$$k = n - j$$

Step 4: Choose j *repeating parameters*.

Step 5: Construct the k Π's, and manipulate as necessary.

Step 6: Write the final functional relationship and check your algebra.

- *Fig*. A concise summary of the six steps that comprise the *method of repeating variables*

The Method of Repeating Variables and the Buckingham Pi Theorem

- As a simple first example, consider a ball falling in a vacuum. Let us pretend that we do not know that Eq. 1 is appropriate for this problem, nor do we know much physics concerning falling objects.
- In fact, suppose that all we know is that the instantaneous elevation z of the ball must be a function of time t, initial vertical speed w_0, initial elevation z_0, and gravitational constant g.
- The beauty of dimensional analysis is that the only other thing we need to know is the primary dimensions of each of these quantities.
- As we go through each step of the method of repeating variables, we explain some of the subtleties of the technique in more detail using the falling ball as an example.

The Method of Repeating Variables

Step 1

- There are five parameters (dimensional variables, nondimensional variables, and dimensional constants) in this problem; $n = 5$. They are listed in functional form, with the dependent variable listed as a function of the independent variables and constants:

- *List of relevant parameters:*

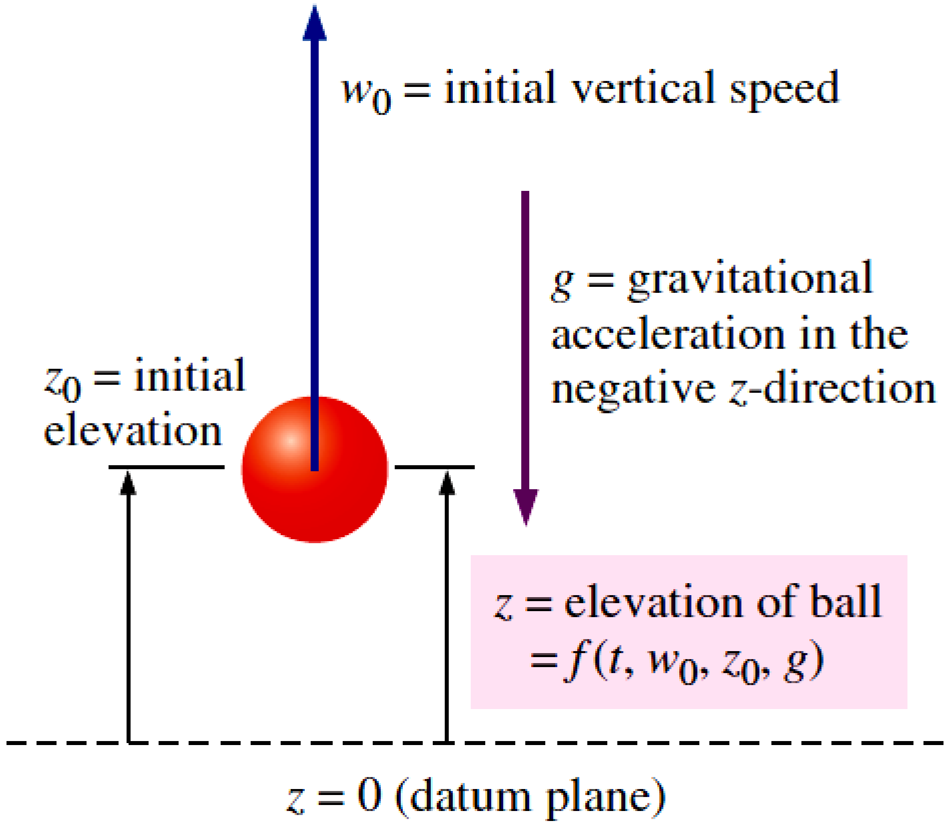


Fig. Setup for dimensional analysis of a ball falling in a vacuum. Elevation z is a function of time t, *initial vertical* speed w_0, *initial elevation* z_0, *and* gravitational constant g.

$$z = f(t, w_0, z_0, g) \qquad n = 5$$

The Method of Repeating Variables

Step 2

The primary dimensions of each parameter are listed here. We recommend writing each dimension with exponents since this helps with later algebra.

z	t	w_0	z_0	g
$\{L^1\}$	$\{t^1\}$	$\{L^1 t^{-1}\}$	$\{L^1\}$	$\{L^1 t^{-2}\}$

Step 3

As a first guess, j is set equal to 2, the number of primary dimensions represented in the problem (L and t).

Reduction: $j = 2$

If this value of j is correct, the number of Π's predicted by the Buckingham Pi theorem is

Number of expected Π*'s*: $k = n - j = 5 - 2 = 3$

The Method of Repeating Variables

Step 4

- We need to choose two repeating parameters since $j = 2$. Since this is often the hardest (or at least the most mysterious) part of the method of repeating variables, several guidelines about choosing repeating parameters are listed in Table 1.
- Following the guidelines of Table 1 on the next page, the wisest choice of two repeating parameters is w_0 *and* z_0.

Repeating parameters: w_0 *and* z_0

Step 5

- Now we combine these repeating parameters into products with each of the remaining parameters, one at a time, to create the Π's. The first Π is always the *dependent* Π and is formed with the dependent variable z.

Dependent Π *:* $\Pi_1 = z w_0^{a_1} z_0^{b_1}$ ....(1)

- where a_1 and b_1 are constant exponents that need to be determined.

The Method of Repeating Variables

- We apply the primary dimensions of step 2 into Eq. 1 and *force the Π to be* dimensionless by setting the exponent of each primary dimension to zero:
- *Dimensions of Π_1:*

$$\{\Pi_1\} = \{L^0 t^0\} = \{z w_0^{a_1} z_0^{b_1}\} = \{L^1 (L^1 t^{-1})^{a_1} L^{b_1}\}$$

- Since primary dimensions are by definition independent of each other, we equate the exponents of each primary dimension independently to solve for exponents a_1 and b_1

Time: $\{t^0\} = \{t^{-a_1}\}$ $\quad 0 = -a_1 \quad a_1 = 0$

Length: $\{L^0\} = \{L^1 L^{a_1} L^{b_1}\}$ $\quad 0 = 1 + a_1 + b_1 \quad b_1 = -1 - a_1 \quad b_1 = -1$

- Thus

$$\Pi_1 = \frac{z}{z_0}$$

The Method of Repeating Variables

- In similar fashion we create the first independent Π (Π_2) by combining the repeating parameters with independent variable *t*.

First independent Π: $\quad \Pi_2 = t w_0^{a_2} z_0^{b_2}$

Dimensions of Π_2: $\quad \{\Pi_2\} = \{L^0 t^0\} = \{t w_0^{a_2} z_0^{b_2}\} = \{t(L^1 t^{-1})^{a_2} L^{b_2}\}$

Equating exponents,

Time: $\quad \{t^0\} = \{t^1 t^{-a_2}\} \qquad 0 = 1 - a_2 \qquad a_2 = 1$

Length: $\quad \{L^0\} = \{L^{a_2} L^{b_2}\} \qquad 0 = a_2 + b_2 \qquad b_2 = -a_2 \qquad b_2 = -1$

Π_2 is thus

$$\Pi_2 = \frac{w_0 t}{z_0}$$

The Method of Repeating Variables

- Finally we create the second independent Π (Π_3) by combining the repeating parameters with *g and forcing the P to be dimensionless*

Second independent Π: $\quad \Pi_3 = g w_0^{a_3} z_0^{b_3}$

Dimensions of Π_3: $\quad \{\Pi_3\} = \{\mathrm{L}^0\mathrm{t}^0\} = \{g w_0^{a_3} z_0^{b_3}\} = \{\mathrm{L}^1\mathrm{t}^{-2}(\mathrm{L}^1\mathrm{t}^{-1})^{a_3}\mathrm{L}^{b_3}\}$

Equating exponents,

Time: $\quad \{\mathrm{t}^0\} = \{\mathrm{t}^{-2}\mathrm{t}^{-a_3}\} \quad 0 = -2 - a_3 \quad a_3 = -2$

Length: $\quad \{\mathrm{L}^0\} = \{\mathrm{L}^1\mathrm{L}^{a_3}\mathrm{L}^{b_3}\} \quad 0 = 1 + a_3 + b_3 \quad b_3 = -1 - a_3 \quad b_3 = 1$

Π_3 is thus

$$\Pi_3 = \frac{g z_0}{w_0^2}$$

The Method of Repeating Variables

- We can see that Π_1 and Π_2 are the same as the nondimensionalized variables *z* and t* defined by* Eq. 3 (See slide number 15)—no manipulation is necessary for these.
- However, we recognize that the third P must be raised to the power of -1/2 to be of the same form as an established dimensionless parameter, namely the Froude number of

$$\Pi_{3,\ \text{modified}} = \left(\frac{gz_0}{w_0^2}\right)^{-1/2} = \frac{w_0}{\sqrt{gz_0}} = \text{Fr}$$

- Such manipulation is often necessary to put the Π's into proper established form ("socially acceptable form" since it is a named, established nondimensional parameter that is commonly used in the literature.

The Method of Repeating Variables

Step 6

- We should double-check that the Π's are indeed dimensionless
- We are finally ready to write the functional relationship between the nondimensional parameters

Relationship between Π's:

$$\Pi_1 = f(\Pi_2, \Pi_3) \quad \rightarrow \quad \frac{z}{z_0} = f\left(\frac{w_0 t}{z_0}, \frac{w_0}{\sqrt{g z_0}}\right)$$

- The method of repeating variables properly predicts the functional relationship between dimensionless groups.
- However, the method of repeating variables cannot predict the exact mathematical form of the equation. This is a fundamental limitation of dimensional analysis and the method of repeating variables.

The Method of Repeating Variables

Table 1

Guidelines for choosing *repeating parameters* in step 4 of the method of repeating variables*

Guideline	Comments and Application to Present Problem
1. Never pick the *dependent* variable. Otherwise, it may appear in all the Π's, which is undesirable.	In the present problem we cannot choose z, but we must choose from among the remaining four parameters. Therefore, we must choose two of the following parameters: t, w_0, z_0, and g.
2. The chosen repeating parameters must not *by themselves* be able to form a dimensionless group. Otherwise, it would be impossible to generate the rest of the Π's.	In the present problem, any two of the independent parameters would be valid according to this guideline. For illustrative purposes, however, suppose we have to pick three instead of two repeating parameters. We could not, for example, choose t, w_0, and z_0, because these can form a Π all by themselves (tw_0/z_0).
3. The chosen repeating parameters must represent *all* the primary dimensions in the problem.	Suppose for example that there were *three* primary dimensions (m, L, and t) and *two* repeating parameters were to be chosen. You could not choose, say, a length and a time, since primary dimension mass would not be represented in the dimensions of the repeating parameters. An appropriate choice would be a density and a time, which together represent all three primary dimensions in the problem.
4. Never pick parameters that are already dimensionless. These are Π's already, all by themselves.	Suppose an angle θ were one of the independent parameters. We could not choose θ as a repeating parameter since angles have no dimensions (radian and degree are dimensionless units). In such a case, one of the Π's is already known, namely θ.
5. Never pick two parameters with the *same* dimensions or with dimensions that differ by only an exponent.	In the present problem, two of the parameters, z and z_0, have the same dimensions (length). We cannot choose both of these parameters. (Note that dependent variable z has already been eliminated by guideline 1.) Suppose one parameter has dimensions of length and another parameter has dimensions of volume. In dimensional analysis, volume contains only one primary dimension (length) and *is not dimensionally distinct from length*—we cannot choose both of these parameters.

The Method of Repeating Variables

6. Whenever possible, choose dimensional constants over dimensional variables so that only *one* Π contains the dimensional variable.	If we choose time t as a repeating parameter in the present problem, it would appear in all three Π's. While this would not be *wrong*, it would not be *wise* since we know that ultimately we want some nondimensional height as a function of some nondimensional time and other nondimensional parameter(s). From the original four independent parameters, this restricts us to w_0, z_0, and g.
7. Pick common parameters since they may appear in each of the Π's.	In fluid flow problems we generally pick a length, a velocity, and a mass or density (Fig. 7–25). It is unwise to pick less common parameters like viscosity μ or surface tension σ_s, since we would in general not want μ or σ_s to appear in each of the Π's. In the present problem, w_0 and z_0 are wiser choices than g.
8. Pick simple parameters over complex parameters whenever possible.	It is better to pick parameters with only one or two basic dimensions (e.g., a length, a time, a mass, or a velocity) instead of parameters that are composed of several basic dimensions (e.g., an energy or a pressure).

The Method of Repeating Variables

Table 2

Guidelines for manipulation of the Π's resulting from the method of repeating variables*

Guideline	Comments and Application to Present Problem
1. We may impose a constant (dimensionless) exponent on a Π or perform a functional operation on a Π.	We can raise a Π to any exponent n (changing it to Π^n) without changing the dimensionless stature of the Π. For example, in the present problem, we imposed an exponent of $-1/2$ on Π_3. Similarly we can perform the functional operation sin(Π), exp(Π), etc., without influencing the dimensions of the Π.
2. We may multiply a Π by a pure (dimensionless) constant.	Sometimes dimensionless factors of π, 1/2, 2, 4, etc., are included in a Π for convenience. This is perfectly okay since such factors do not influence the dimensions of the Π.
3. We may form a product (or quotient) of any Π with any other Π in the problem to replace one of the Π's.	We could replace Π_3 by $\Pi_3\Pi_1$, Π_3/Π_2, etc. Sometimes such manipulation is necessary to convert our Π into an established Π. In many cases, the established Π would have been produced if we would have chosen different repeating parameters.
4. We may use any of guidelines 1 to 3 in combination.	In general, we can replace any Π with some new Π such as $A\Pi_3{}^B \sin(\Pi_1{}^C)$, where A, B, and C are pure constants.
5. We may substitute a dimensional parameter in the Π with other parameter(s) of the same dimensions.	For example, the Π may contain the square of a length or the cube of a length, for which we may substitute a known area or volume, respectively, in order to make the Π agree with established conventions.

Table 3. Some common established nondimensional parameters

Some common established nondimensional parameters or Π's encountered in fluid mechanics and heat transfer*

Name	Definition	Ratio of Significance
Archimedes number	$\mathrm{Ar} = \dfrac{\rho_s g L^3}{\mu^2}(\rho_s - \rho)$	$\dfrac{\text{Gravitational force}}{\text{Viscous force}}$
Aspect ratio	$\mathrm{AR} = \dfrac{L}{W}$ or $\dfrac{L}{D}$	$\dfrac{\text{Length}}{\text{Width}}$ or $\dfrac{\text{Length}}{\text{Diameter}}$
Biot number	$\mathrm{Bi} = \dfrac{hL}{k}$	$\dfrac{\text{Surface thermal resistance}}{\text{Internal thermal resistance}}$
Bond number	$\mathrm{Bo} = \dfrac{g(\rho_f - \rho_v)L^2}{\sigma_s}$	$\dfrac{\text{Gravitational force}}{\text{Surface tension force}}$
Cavitation number	$\mathrm{Ca}\ (\text{sometimes } \sigma_c) = \dfrac{P - P_v}{\rho V^2}$ $\left(\text{sometimes } \dfrac{2(P - P_v)}{\rho V^2}\right)$	$\dfrac{\text{Pressure} - \text{Vapor pressure}}{\text{Inertial pressure}}$
Darcy friction factor	$f = \dfrac{8\tau_w}{\rho V^2}$	$\dfrac{\text{Wall friction force}}{\text{Inertial force}}$

Drag coefficient	$C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 A}$	$\dfrac{\text{Drag force}}{\text{Dynamic force}}$
Eckert number	$\mathrm{Ec} = \dfrac{V^2}{c_p T}$	$\dfrac{\text{Kinetic energy}}{\text{Enthalpy}}$
Euler number	$\mathrm{Eu} = \dfrac{\Delta P}{\rho V^2} \left(\text{sometimes } \dfrac{\Delta P}{\frac{1}{2}\rho V^2}\right)$	$\dfrac{\text{Pressure difference}}{\text{Dynamic pressure}}$
Fanning friction factor	$C_f = \dfrac{2\tau_w}{\rho V^2}$	$\dfrac{\text{Wall friction force}}{\text{Inertial force}}$
Fourier number	$\mathrm{Fo}\ (\text{sometimes } \tau) = \dfrac{\alpha t}{L^2}$	$\dfrac{\text{Physical time}}{\text{Thermal diffusion time}}$
Froude number	$\mathrm{Fr} = \dfrac{V}{\sqrt{gL}} \left(\text{sometimes } \dfrac{V^2}{gL}\right)$	$\dfrac{\text{Inertial force}}{\text{Gravitational force}}$
Grashof number	$\mathrm{Gr} = \dfrac{g\beta \lvert \Delta T \rvert L^3 \rho^2}{\mu^2}$	$\dfrac{\text{Buoyancy force}}{\text{Viscous force}}$
Jakob number	$\mathrm{Ja} = \dfrac{c_p(T - T_{sat})}{h_{fg}}$	$\dfrac{\text{Sensible energy}}{\text{Latent energy}}$
Knudsen number	$\mathrm{Kn} = \dfrac{\lambda}{L}$	$\dfrac{\text{Mean free path length}}{\text{Characteristic length}}$

Lewis number	$\mathrm{Le} = \dfrac{k}{\rho c_p D_{AB}} = \dfrac{\alpha}{D_{AB}}$	$\dfrac{\text{Thermal diffusion}}{\text{Species diffusion}}$
Lift coefficient	$C_L = \dfrac{F_L}{\frac{1}{2}\rho V^2 A}$	$\dfrac{\text{Lift force}}{\text{Dynamic force}}$
Mach number	Ma (sometimes M) $= \dfrac{V}{c}$	$\dfrac{\text{Flow speed}}{\text{Speed of sound}}$
Nusselt number	$\mathrm{Nu} = \dfrac{Lh}{k}$	$\dfrac{\text{Convection heat transfer}}{\text{Conduction heat transfer}}$
Peclet number	$\mathrm{Pe} = \dfrac{\rho L V c_p}{k} = \dfrac{LV}{\alpha}$	$\dfrac{\text{Bulk heat transfer}}{\text{Conduction heat transfer}}$
Power number	$N_p = \dfrac{\dot{W}}{\rho D^5 \omega^3}$	$\dfrac{\text{Power}}{\text{Rotational inertia}}$
Prandtl number	$\mathrm{Pr} = \dfrac{\nu}{\alpha} = \dfrac{\mu c_p}{k}$	$\dfrac{\text{Viscous diffusion}}{\text{Thermal diffusion}}$
Pressure coefficient	$C_p = \dfrac{P - P_\infty}{\frac{1}{2}\rho V^2}$	$\dfrac{\text{Static pressure difference}}{\text{Dynamic pressure}}$
Rayleigh number	$\mathrm{Ra} = \dfrac{g\beta \lvert \Delta T \rvert L^3 \rho^2 c_p}{k\mu}$	$\dfrac{\text{Buoyancy force}}{\text{Viscous force}}$

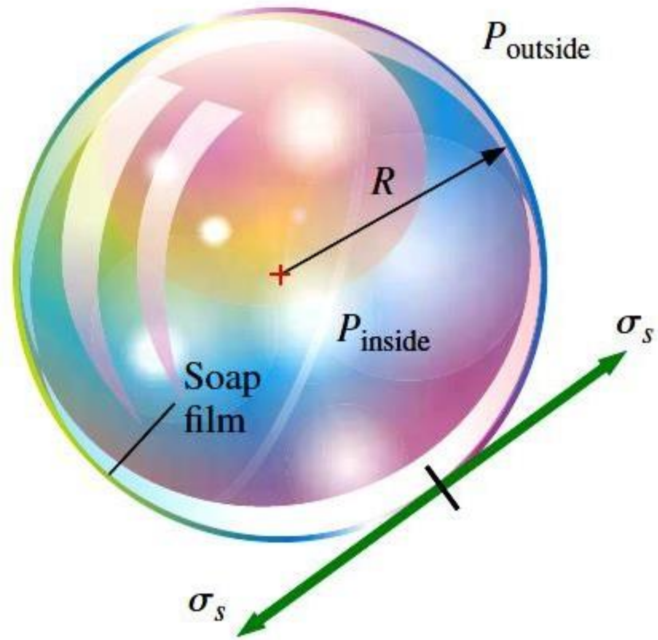
Reynolds number	$\mathrm{Re} = \dfrac{\rho V L}{\mu} = \dfrac{VL}{\nu}$	$\dfrac{\text{Inertial force}}{\text{Viscous force}}$
Richardson number	$\mathrm{Ri} = \dfrac{L^5 g \Delta\rho}{\rho \dot{V}^2}$	$\dfrac{\text{Buoyancy force}}{\text{Inertial force}}$
Schmidt number	$\mathrm{Sc} = \dfrac{\mu}{\rho D_{AB}} = \dfrac{\nu}{D_{AB}}$	$\dfrac{\text{Viscous diffusion}}{\text{Species diffusion}}$
Sherwood number	$\mathrm{Sh} = \dfrac{VL}{D_{AB}}$	$\dfrac{\text{Overall mass diffusion}}{\text{Species diffusion}}$
Specific heat ratio	$k \text{ (sometimes } \gamma) = \dfrac{c_p}{c_v}$	$\dfrac{\text{Enthalpy}}{\text{Internal energy}}$
Stanton number	$\mathrm{St} = \dfrac{h}{\rho c_p V}$	$\dfrac{\text{Heat transfer}}{\text{Thermal capacity}}$
Stokes number	$\mathrm{Stk} \text{ (sometimes St)} = \dfrac{\rho_p D_p^2 V}{18 \mu L}$	$\dfrac{\text{Particle relaxation time}}{\text{Characteristic flow time}}$
Strouhal number	$\mathrm{St} \text{ (sometimes S or Sr)} = \dfrac{fL}{V}$	$\dfrac{\text{Characteristic flow time}}{\text{Period of oscillation}}$
Weber number	$\mathrm{We} = \dfrac{\rho V^2 L}{\sigma_s}$	$\dfrac{\text{Inertial force}}{\text{Surface tension force}}$

EXAMPLE 4. Pressure in a Soap Bubble

- Some children are playing with soap bubbles, and you become curious as to the relationship between soap bubble radius and the pressure inside the soap bubble. You reason that the pressure inside the soap bubble must be greater than atmospheric pressure, and that the shell of the soap bubble is under tension, much like the skin of a balloon. You also know that the property surface tension must be important in this problem. Not knowing any other physics, you decide to approach the problem using dimensional analysis. Establish a relationship between pressure difference

$$\Delta P = P_{\text{inside}} - P_{\text{outside}}$$

- soap bubble radius R, *and the surface tension* σ_s *of the soap film.*



The pressure inside a soap bubble is greater than that surrounding the soap bubble due to surface tension in the soap film.

EXAMPLE 4. Pressure in a Soap Bubble

- **SOLUTION.** The pressure difference between the inside of a soap bubble and the outside air is to be analyzed by the method of repeating variables.
- ***Assumptions*** *1*. The soap bubble is neutrally buoyant in the air, and gravity is not relevant. 2 No other variables or constants are important in this problem.
- ***Analysis*** The step-by-step method of repeating variables is employed.

Step 1 There are three variables and constants in this problem; $n = 3$. They are listed in functional form, with the dependent variable listed as a function of the independent variables and constants:

List of relevant parameters: $\Delta P = f(R, \sigma_s) \qquad n = 3$

EXAMPLE 4. Pressure in a Soap Bubble

- **Step 2** The primary dimensions of each parameter are listed.

ΔP	R	σ_s
$\{m^1L^{-1}t^{-2}\}$	$\{L^1\}$	$\{m^1t^{-2}\}$

- **Step 3** As a first guess, *j is set equal to 3, the number of primary dimensions* represented in the problem (m, L, and t).

Reduction (first guess): $j = 3$

- If this value of *j is correct, the expected number of Π's is*

$k = n - j = 3 - 3 = 0.$

But how can we have zero P's? Something is obviously not right

- At times like this, we need to first go back and make sure that we are not neglecting some important variable or constant in the problem.
- Since we are confident that the pressure difference should depend only on soap bubble radius and surface tension, we reduce the value of *j by one,*

EXAMPLE 4. Pressure in a Soap Bubble

Reduction (second guess): $j = 2$

- If this value of j is correct, $k = n - j = 3 - 2 = 1$. Thus we expect one Π, which is more physically realistic than zero Π's.
- **Step 4** We need to choose two repeating parameters since $j = 2$. Following the guidelines of Table 1, our only choices are R and σ_s, since ΔP *is the* dependent variable.
- **Step 5** We combine these repeating parameters into a product with the dependent variable ΔP to create the dependent Π,

Dependent Π: $\Pi_1 = \Delta P R^{a_1} \sigma_s^{b_1}$ ....(1)

- We apply the primary dimensions of step 2 into Eq. 1 and force the Π to be dimensionless.

Dimensions of Π_1:

$$\{\Pi_1\} = \{m^0L^0t^0\} = \{\Delta P R^{a_1}\sigma_s^{b_1}\} = \{(m^1L^{-1}t^{-2})L^{a_1}(m^1t^{-2})^{b_1}\}$$

EXAMPLE 4. Pressure in a Soap Bubble

We equate the exponents of each primary dimension to solve for a_1 and b_1:

$$\begin{array}{llll}
\textit{Time}: & \{\mathrm{t}^0\} = \{\mathrm{t}^{-2}\mathrm{t}^{-2b_1}\} & 0 = -2 - 2b_1 & b_1 = -1 \\
\textit{Mass}: & \{\mathrm{m}^0\} = \{\mathrm{m}^1\mathrm{m}^{b_1}\} & 0 = 1 + b_1 & b_1 = -1 \\
\textit{Length}: & \{\mathrm{L}^0\} = \{\mathrm{L}^{-1}\mathrm{L}^{a_1}\} & 0 = -1 + a_1 & a_1 = 1
\end{array}$$

Fortunately, the first two results agree with each other, and Eq. 1 thus becomes

$$\Pi_1 = \frac{\Delta P R}{\sigma_s} \tag{2}$$

- From Table 3, the established nondimensional parameter most similar to Eq. 2 is the **Weber number,** defined as a pressure (ρV^2) *times a length* divided by surface tension. There is no need to further manipulate this Π.

EXAMPLE 4. Pressure in a Soap Bubble

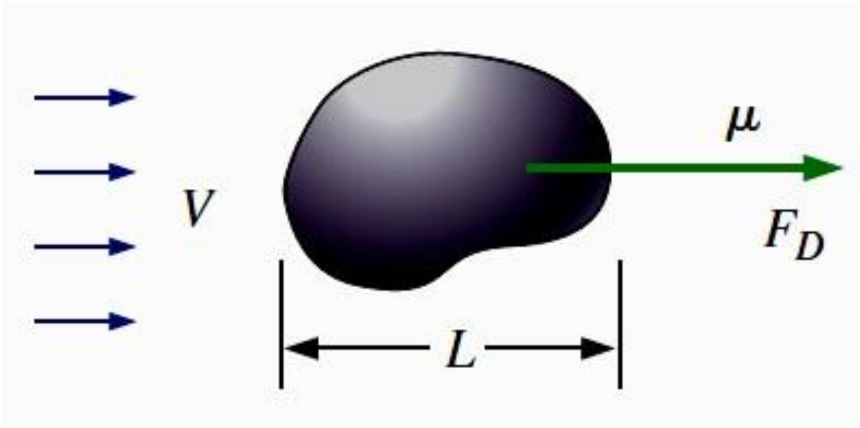
- **Step 6** We write the final functional relationship. In the case at hand, there is only one Π, which is a function of *nothing*. *This is possible only if* the Π is constant.
- *Relationship between Π's:*

$$\Pi_1 = \frac{\Delta P R}{\sigma_s} = f(\text{nothing}) = \text{constant} \quad \rightarrow \quad \Delta P = \text{constant}\,\frac{\sigma_s}{R} \qquad (3)$$

- This is an example of how we can sometimes predict *trends with* dimensional analysis, even without knowing much of the physics of the problem. For example, we know from our result that if the radius of the soap bubble doubles, the pressure difference decreases by a factor of 2. Similarly, if the value of surface tension doubles, ΔP increases by a factor of 2.
- Dimensional analysis cannot predict the value of the constant in Eq. 3; further analysis (or one experiment) reveals that the constant is equal to 4 (Chap. 1).

Example 5

- When small aerosol particles or microorganisms move through air or water, the Reynolds number is very small (Re << 1). Such flows are called **creeping flows. The aerodynamic** drag on an object in creeping flow is a function only of its speed V, some characteristic length scale L of the object, and fluid viscosity μ. Use dimensional analysis to generate a relationship for F_D as a function of the independent variables.



Solution We are to use dimensional analysis to find a functional relationship between F_D and variables V, L, and μ.

Assumptions 1 We assume Re << 1 so that the creeping flow approximation applies. 2 Gravitational effects are irrelevant. 3 No parameters other than those listed in the problem statement are relevant to the problem.

Analysis We follow the step-by-step method of repeating variables.

Step 1 There are four variables and constants in this problem; $n = 4$. They are listed in functional form, with the dependent variable listed as a function of the independent variables and constants:

List of relevant parameters: $$F_D = f(V, L, \mu) \qquad n = 4$$

Step 2 The primary dimensions of each parameter are listed.

F_D	V	L	μ
$\{m^1L^1t^{-2}\}$	$\{L^1t^{-1}\}$	$\{L^1\}$	$\{m^1L^{-1}t^{-1}\}$

Step 3 As a first guess, we set j equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $$j = 3$$

If this value of j is correct, the number of Πs expected is

Number of expected Πs: $$k = n - j = 4 - 3 = 1$$

Step 4 Now we need to choose three repeating parameters since $j = 3$. Since we cannot choose the dependent variable, our only choices are V, L, and μ.

Step 5 Now we combine these repeating parameters into a product with the dependent variable F_D to create the dependent Π,

Dependent Π: $$\Pi_1 = F_D V^{a_1} L^{b_1} \mu^{c_1} \qquad (1)$$

We apply the primary dimensions of Step 2 into Eq. 1 and force the Π to be dimensionless,

Dimensions of Π_1:

$$\{\Pi_1\} = \{\mathrm{m}^0\mathrm{L}^0\mathrm{t}^0\} = \{F_D V^{a_1} L^{b_1} \mu^{c_1}\} = \{(\mathrm{m}^1\mathrm{L}^1\mathrm{t}^{-2})(\mathrm{L}^1\mathrm{t}^{-1})^{a_1}(\mathrm{L}^1)^{b_1}(\mathrm{m}^1\mathrm{L}^{-1}\mathrm{t}^{-1})^{c_1}\}$$

Now we equate the exponents of each primary dimension to solve for exponents a_1 through c_1.

mass:	$\{\mathrm{m}^0\} = \{\mathrm{m}^1\mathrm{m}^{c_1}\}$	$0 = 1 + c_1$	$c_1 = -1$
time:	$\{\mathrm{t}^0\} = \{\mathrm{t}^{-2}\mathrm{t}^{-a_1}\mathrm{t}^{-c_1}\}$	$0 = -2 - a_1 - c_1$	$a_1 = -1$
length:	$\{\mathrm{L}^0\} = \{\mathrm{L}^1\mathrm{L}^{a_1}\mathrm{L}^{b_1}\mathrm{L}^{-c_1}\}$	$0 = 1 + a_1 + b_1 - c_1$	$b_1 = -1$

Equation 1 thus becomes

Π_1:

$$\Pi_1 = \frac{F_D}{\mu V L} \quad (2)$$

Step 6 We now write the functional relationship between the nondimensional parameters. In the case at hand, there is only one Π, which is a function of *nothing*. This is possible only if the Π is constant. Putting Eq. 2 into standard functional form,

Relationship between Πs:
$$\Pi_1 = \frac{F_D}{\mu V L} = f(\text{nothing}) = \text{constant} \qquad (3)$$

or

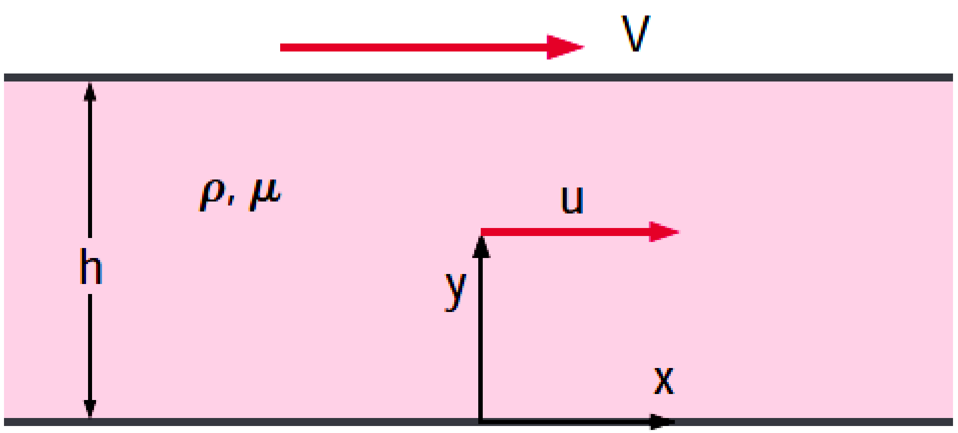
Result of dimensional analysis:
$$\boxed{F_D = \text{constant} \cdot \mu V L} \qquad (4)$$

Thus we have shown that for creeping flow around an object, the aerodynamic drag force is simply a constant multiplied by $\mu V L$, regardless of the shape of the object.

Discussion This result is very significant because all that is left to do is find the constant, which will be a function of the shape of the object (and its orientation with respect to the flow).

Example 6

- Consider fully developed **Couette flow**—flow between two infinite parallel plates separated by distance *h*, with the top plate moving and the bottom plate stationary as illustrated in the Fig. shown. The flow is steady, incompressible, and two-dimensional in the *xy*-plane. Use the method of repeating variables to generate a dimensionless relationship for the *x* component of fluid velocity *u* as a function of fluid viscosity μ, top plate speed *V*, distance h, fluid density *ρ, and* distance *y*.



Solution We are to use dimensional analysis to find the functional relationship between the given parameters.

Assumptions 1 The given parameters are the only relevant ones in the problem.

Analysis The step-by-step method of repeating variables is employed to obtain the nondimensional parameters (the Πs).

Step 1 There are six parameters in this problem; $n = 6$,

List of relevant parameters: $$u = f(\mu, V, h, \rho, y) \qquad n = 6 \qquad (1)$$

Step 2 The primary dimensions of each parameter are listed,

u	μ	V	h	ρ	y
$\{L^1t^{-1}\}$	$\{m^1L^{-1}t^{-1}\}$	$\{L^1t^{-1}\}$	$\{L^1\}$	$\{m^1L^{-3}\}$	$\{L^1\}$

Step 3 As a first guess, j is set equal to 3, the number of primary dimensions represented in the problem (m, L, and t).

Reduction: $$j = 3$$

If this value of j is correct, the expected number of Πs is

Number of expected Πs: $$k = n - j = 6 - 3 = 3$$

Step 4 We need to choose three repeating parameters since $j = 3$. Following the guidelines outlined in this chapter, we elect not to pick the viscosity. It is better to pick a fixed length (h) rather than a variable length (y); otherwise y would appear in each Pi, which would not be desirable. We choose

Repeating parameters: V, ρ, and h

Step 5 The dependent Π is generated:

$$\Pi_1 = uV^{a_1}\rho^{b_1}h^{c_1} \qquad \{\Pi_1\} = \left\{(L^1t^{-1})(L^1t^{-1})^{a_1}(m^1L^{-3})^{b_1}(L^1)^{c_1}\right\}$$

mass: $\{m^0\} = \{m^{b_1}\}$ $\quad 0 = b_1$ $\quad b_1 = 0$

time: $\{t^0\} = \{t^{-1}t^{-a_1}\}$ $\quad 0 = -1 - a_1$ $\quad a_1 = -1$

length: $\{L^0\} = \{L^1L^{a_1}L^{-3b_1}L^{c_1}\}$ $\quad 0 = 1 + a_1 - 3b_1 + c_1$ $\quad c_1 = 0$

The dependent Π is thus

Π_1: $$\Pi_1 = \frac{u}{V}$$

The second Pi (the first independent Π in this problem) is generated:

$$\Pi_2 = \mu V^{a_2}\rho^{b_2}h^{c_2} \qquad \{\Pi_2\} = \left\{(m^1L^{-1}t^{-1})(L^1t^{-1})^{a_2}(m^1L^{-3})^{b_2}(L^1)^{c_2}\right\}$$

mass: $\{m^0\} = \{m^1m^{b_2}\}$ $\quad 0 = 1 + b_2$ $\quad b_2 = -1$

time: $\{t^0\} = \{t^{-1}t^{-a_2}\}$ $0 = -1 - a_2$ $a_2 = -1$

length: $\{L^0\} = \{L^{-1}L^{a_2}L^{-3b_2}L^{c_2}\}$ $0 = -1 + a_2 - 3b_2 + c_2$ $c_2 = -1$

$0 = -1 - 1 + 3 + c_2$

which yields

Π_2: $$\Pi_2 = \frac{\mu}{\rho V h}$$

We recognize this Π as the inverse of the Reynolds number. So, after inverting,

Modified Π_2: $$\Pi_2 = \frac{\rho V h}{\mu} = \text{Reynolds number} = \text{Re}$$

The third Pi (the second independent Π in this problem) is generated:

$$\Pi_3 = yV^{a_3}\rho^{b_3}h^{c_3} \qquad \{\Pi_3\} = \left\{(L^1)(L^1t^{-1})^{a_3}(m^1L^{-3})^{b_3}(L^1)^{c_3}\right\}$$

mass: $\{m^0\} = \{m^{b_3}\}$ $0 = b_3$ $b_3 = 0$

time: $\{t^0\} = \{t^{-a_3}\}$ $0 = -a_3$ $a_3 = 0$

length: $\{L^0\} = \{L^1 L^{a_3} L^{-3b_3} L^{c_3}\}$ $0 = 1 + a_3 - 3b_3 + c_3$ $c_3 = -1$

$0 = 1 + c_3$

which yields

Π_3: $\Pi_3 = \frac{y}{h}$

Step 6 We write the final functional relationship as

Relationship between Π*s:* $$\boxed{\frac{u}{V} = f\left(\mathrm{Re}, \frac{y}{h}\right)} \quad (2)$$