BABA BANDA SINGH BAHADUR ENGINEERING COLLEGE

Department of Applied Sciences

QUESTION BANK

Semester: First **Subject:** Mathematics Paper-I **Code:** BTAM101-18

Branch:E.C.E

Unit II: Matrix Algebra

Matrices, vectors: addition and scalar multiplication, matrix multiplication; Linear systems of equations, linear Independence, rank of a matrix, determinants, Cramer's Rule, inverse of a matrix, Gauss elimination and Gauss-Jordan elimination.

Name of the Faculty: Rajwinder Kaur

1. If $A=\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, then for what value of θ is A an identity matrix?
2. Construct a mxn matrix $A=\left[a_{ij}\right]$, where $a_{ij}=\dfrac{|2i-3j|}{2}$; $m=2, n=2$.
3. If $\begin{pmatrix} 2x+1 & 2y \\ 0 & y^2+1 \end{pmatrix}=\begin{pmatrix} x+3 & 10 \\ 0 & 26 \end{pmatrix}$, write the value of $y+x$.
4. Find non-zero values of x such that: $x\begin{pmatrix} 2x & 2 \\ 3 & x \end{pmatrix}+2\begin{pmatrix} 8 & 5x \\ 4 & 4x \end{pmatrix}=2\begin{pmatrix} x^2+8 & 24 \\ 10 & 6x \end{pmatrix}$.
5. Assume that *Y*, *W* and *P* are matrices of order 3 x *k*, *n* x 3 and *p* x *k* respectively. Find the restrictions on *n*, *p*, *k* so that *PY* + *WY* is defined.
6. If $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 5 \end{pmatrix}=\begin{pmatrix} 7 & 11 \\ k & 23 \end{pmatrix}$, then write the value of *k*.
7. A matrix *A* has *a* + *b* rows and a + 2 columns while the matrix *B* has *b* + 1 rows and *a* + 3 columns. Both matrices *AB* and *BA* exist. Find *a* and *b*. Can you say *AB* and *BA* are of same type? Are they equal?
8. If *A* and *B* are square matrices of same order and *k* is any scalar, prove that *A*-*k I* and *B*-*k I* commute if and only if *A* and *B* commute.

9. Give an example of two matrices A and B such that $AB = O$, where neither $A = O$ nor $B = O$.

10. If A is a matrix of order 2 x 3 and B is a matrix of order 3 x 5, what is the order of the matrix $(AB)^T$.

11. For what value of k, the matrix $\begin{pmatrix} 2-k & 3 \\ -5 & 1 \end{pmatrix}$ is not invertible?

12. Solve the following system of linear equations by matrix inversion method:

    (i) $x + y + z = 8, \quad x - y + 2z = 6, \quad 3x + 5y - 7z = 14$

    (ii) $x + 2y + 3z = 1, \quad 2x + 3y + 2z = 2, \quad 3x + 3y + 4z = 1$

13. Solve the following system of linear equations by Cramer's rule:

    (i) $2x + 3y = 5, \quad 11x - 5y = 6$

    (ii) $x + y + z = 6, \quad x - y + 2z = 5, \quad 3x + y + z = 8$

    (iii) $x + 2y + 3z = 1, \quad 2x + 3y + 2z = 2, \quad 3x + 3y + 4z = 1$

    (iv) $2x - y + 3z = 9, \quad y - z = -1, \quad x + y - z = 0$

14. Explain elementary transformations on a matrix.

15. If $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ then show that $A^n = A^{n-2} + A^2 - I$ for $n \geq 3$. Hence find A^{50}.

16. Compute the inverse of the following matrices by Gauss Jordan method (Elementary Row transformations):

    (i) $\begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$ *(ii)* $\begin{pmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}$

    (iii) $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$ *(iv)* $\begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$

17. Define rank of a matrix and give one example. What is the rank of a

    (a) Singular matrix of order n?

    (b) Non-singular matrix of order n?

18. Find the rank of the following matrices:

(i) $\begin{pmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & 3 \end{pmatrix}$ *(ii)* $\begin{pmatrix} 1 & 2 & -1 \\ 3 & 1 & 0 \\ 2 & -1 & 1 \end{pmatrix}$

19. Reduce the following matrices to normal form and hence find rank:

(i) $\begin{pmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{pmatrix}$ *(ii)* $\begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$ *(iii)* $\begin{pmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{pmatrix}$

(iv) $\begin{pmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{pmatrix}$ *(v)* $\begin{pmatrix} 2 & 2 & 2 \\ 1 & 2 & 1 \\ 3 & 4 & 3 \end{pmatrix}$

20. If $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{pmatrix}$; find two non-singular matrices P and Q such that PAQ is in the normal form. Also find A^{-1} (if it exists).

21. Solve the following system of linear equations by Gauss Elimination method:

(i) $x+y+z=6, \quad x-y+2z=5, \quad 3x+y+z=8$

(ii) $x+2y+3z=1, \quad 2x+3y+2z=2, \quad 3x+3y+4z=1$

22. Solve the following system of linear equations by Gauss Jordan method:

(i) $x+y+z=8, \quad x-y+2z=6, \quad 3x+5y-7z=14$

(ii) $x+2y+3z=1, \quad 2x+3y+2z=2, \quad 3x+3y+4z=1$

23. State the conditions in terms of rank of the coefficient matrix and rank of the augmented matrix for *(a)* Unique solution *(b)* No solution *(c)* Infinite many solution.

24. Investigate for consistency of the following equations and if possible find the solutions:

$$4x-2y+6z=8, \quad x+y-3z=-1, \quad 15x-3y+9z=21$$

25. Show that the equations $2x+6y+11=0, \quad 6x+20y-6z+3=0, \quad 6y-18z+1=0$ are not consistent.

26. For what values of λ and μ do the system of equations $x+y+z=6, x+2y+3z=10, x+2y+\lambda z=\mu$ have *(i)* No solution *(ii)* A unique solution *(iii)* An infinite number of solutions.

27. Find the real value of p for which the system of equations $x+2y+3z=px,\ 3x+y+2z=py,\ 2x+3y+z=pz$ have non-trivial solution.
28. For what value (s) of k, the equations $x+y+z=1, 2x+y+4z=k, 4x+y+10z=k^2$ have a solution? Solve them completely in each case.
29. Investigate the value of λ and μ so that the equations $2x+3y+5z=9, 7x+3y-2z=8, 2x+3y+\lambda z=\mu$ have *(i)* No solution *(ii)* A unique solution *(iii)* An infinite number of solutions.
30. Test the following system of equations for consistency and solve $x+2y+z=3,\ 2x+3y+2z=5,\ 3x-5y+5z=2,\ 3x+9y-z=4$
31. Show that the equations $3x+4y+5z=a,\ 4x+5y+6z=b,\ 5x+6y+7z=c$ do not have a solution unless $a+c=2b$.
32. For what value of k the system of equations $x+y+z=2,\ x+2y+z=-2,$ $x+y+(k-5)z=k$ has no solution?
33. For what value (s) of k, do the vectors $(k,1,1),(0,1,1),(k,0,k)$ are linearly independent.
34. Test whether the subset S of R_3 is L.I. or L.D. , given S = {(1,0,1), (1,1,0), (-1,0,-1)}
35. Define linear dependence of vectors and determine whether the vectors $(3,2,4),(1,0,2),(1,-1,-1)$ are linearly dependent or not?

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Unit IV: Linear Algebra

Eigen Values, Eigen Vectors, Symmetric, Skew-Symmetric and Orthogonal Matrices, Eigen bases; similar matrices, Diagonalisation.

1. Define Symmetric matrix. Also give an example.
2. Define Skew-symmetric matrix. Also give an example.
3. For what value of k, the matrix $\begin{pmatrix} 2k+3 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & -2k-3 \end{pmatrix}$ is skew- symmetric ?
4. Show that the matrix $B'AB$ is symmetric or skew symmetric according as A is symmetric or skew symmetric.
5. Show that the elements on the main diagonal of a skew symmetric matrix are all zero.
6. If a matrix A is symmetric as well as skew symmetric, then show that $A = O$.
7. If A and B are symmetric matrices of the same order, then show that AB is symmetric if and only if A and B commute.
8. Express the matrix $\begin{pmatrix} 14 & 17 & 18 \\ 19 & 6 & -7 \\ 1 & 2 & 5 \end{pmatrix}$ as the sum of a symmetric matrix and a skew-symmetric matrix.
9. Define orthogonal matrix. Also give an example.
10. Show that the product of two orthogonal matrices of same order is also an orthogonal matrix.
11. Show that transpose of an orthogonal matrix is also orthogonal.
12. If A be an orthogonal matrix, show that $|A| = \pm 1$.

13. Verify that the matrix $\begin{pmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \end{pmatrix}$ is orthogonal.

14. If $\langle l_i, m_i, n_i \rangle, \quad i = 1, 2, 3$ are the direction cosines of three mutually perpendicular lines referred to an orthogonal Cartesian coordinate system, prove that the matrix $\begin{pmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{pmatrix}$ is orthogonal.

15. If A is symmetric and P is orthogonal, show that $P^{-1}AP$ is symmetric.

16. Show that at least one latent root of every singular matrix is zero.

17. Show that, if zero is an eigen value of a matrix then it is singular.

18. Show that a square matrix and its transpose have the same set of eigen values.

19. If λ is an eigen value of square matrix A, then show that λ^m is an eigen value of $A^m \ \ \forall \ \ m \in N$.

20. If A is a non-singular matrix, prove that the eigen value of A^{-1} are the reciprocal of the eigen values of A.

21. Define similar matrices and prove that similar matrices have same characteristic roots.

22. If $A = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$, find the eigen values of A^4.

23. The characteristic roots of $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & k & -4 \\ 2 & -4 & 3 \end{pmatrix}$ are 0, 3, 15. Find the value of k.

24. Determine the eigen values and corresponding eigen values of the following matrices:

(i) $\begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$ *(ii)* $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ *(iii)* $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

(iv) $\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$ *(v)* $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ *(vi)* $\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -7 & 5 & 1 \end{pmatrix}$

25. Diagonalize the matrix $\begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix}$.

26. Diagonalize, if possible, the matrix $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 1 & -1 & 4 \end{pmatrix}$.

27. Diagonalize the following matrices:

(i) $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ *(ii)* $\begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$

28. Diagonalize $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ and hence find A^8.

29. If $A = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$, find the eigen values of A^{-1}.

30. Show that inverse of an orthogonal matrix is also orthogonal.

ANSWERS

3. $K = -\frac{3}{2}$ **22.** 16, 625 **23.** 7 **24.** ***(i)*** 1, 2, 3; $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$

(ii) 0, 3, 15; $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ ***(iii)*** 2, 2, 8; $\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

(iv) 2, 3, 5; $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ ***(v)*** -2, 3, 6; $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

(vi) 1, 1, 1; $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ **25.** $\begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$

26. Not diagonalizable

27. ***(i)*** $\begin{pmatrix} -2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{pmatrix}$ ***(ii)*** $\begin{pmatrix} 4 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}$ **28.** $\begin{pmatrix} 26215 & 78642 & 24574 \\ 13107 & 39322 & 11467 \\ 0 & 0 & 6561 \end{pmatrix}$

29. $\frac{-1}{2}, \frac{1}{5}$

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Rolle's theorem, Mean value theorems, Taylor's and Maclaurin theorems with remainders; Indeterminate forms and L'Hospital's rule; Maxima and minima. Evaluation of definite and improper integrals; Beta and Gamma functions and their properties; Applications of definite integrals to evaluate surface areas and volumes of revolutions.

1. State and prove Rolle's theorem.
2. State Lagrange's Mean Value theorem.
3. State and prove Cauchy's Mean Value theorem.
4. Verify Rolle's theorem for the following functions:

   (i) $f(x) = x(x-2)e^{\frac{3x}{4}}$ in $(0,2)$

   (ii) $f(x) = x^{2m-1}(a-x)^{2n}$ in $(0,a)$

   (iii) $f(x) = \dfrac{\sin x}{e^x}$ in $[0,\pi]$

   (iv) $f(x) = 2+(x-1)^{\frac{2}{3}}$ in $[0,2]$

   [Hint: Not differentiable at $x = 1$]
5. Find a root (solution) of the equation $x\ln x - 2 + x = 0$ lying in (1, 2).
6. Show that the polynomial equation $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_n = 0$ has at least one real root in (0, 1) if $\dfrac{a_0}{n+1} + \dfrac{a_1}{n} + ... + \dfrac{a_{n-2}}{2} + a_n = 0$ and $a_0, a_1, ...a_n$ are real numbers.

[Hint: Take $f(x)=\frac{a_0}{n+1}x^{n+1}+\frac{a_1}{n}x^n+..........+\frac{a_{n-2}}{2}x^2+a_nx$ in [0, 1]. Apply Rolle's theorem]

7. Deduce Lagrange's Mean Value theorem form Rolle's theorem.

   [Hint: Choose $g(x)=f(x)-f(a)-A(x-a),\ \ g(a)=0$, determine A such that $g(b)=0,\ g(x)$ satisfies all the conditions of Rolle's theorem]

8. Show that $\frac{h}{\tan^{-1}h}<\tan^{-1}h<h$ when $h\neq 0$ and $h>0$.

   [Hint: Take $f(x)=\tan^{-1}x$ in $0\le x\le h$]

9. Calculate approximately $\sqrt[5]{245}$ by using LMV theorem.

   [**Another form of LMV theorem:**

   We know that $\frac{f(b)-f(a)}{b-a}=f'(a)$, Take b = a + h, then we get

   $\frac{f(a+h)-f(a)}{a+h-a}=f'(a+\theta h)$ where $c=a+\theta h$ lies between a and b= a + h when $0<\theta<1$. Thus, $f(a+h)=f(a)+hf'(a+\theta h);\ \ 0<\theta<1$

   Hint: Use another form of LMV theorem *i.e.* $f(a+h)=f(a)+hf'(c)$, here choose $f(x)=x^{\frac{1}{5}}, a=243, b=245$ and $c=243$ approximately]

10. Calculate approximately $\sqrt[6]{65}$ by using LMV theorem.

11. Let $f(x)$ be continuous on $[a-1,a+1]$ and differentiable in $(a-1,a+1)$. Show that there exists a $\theta, 0<\theta<1$ such that

$$f(a-1)-2f(a)+f(a+1)=f'(a+\theta)-f'(a-\theta).$$

   [Hint: Define $\phi(t)=f(a+t)+f(a-t)$, Apply LMV theorem on $[0,1]$]

12. Use LMV theorem to prove that if $0<u<v$, $\frac{v-u}{1+v^2}<\tan^{-1}v-\tan^{-1}u<\frac{v+u}{1+v^2}$. Also, deduce that $\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\frac{4}{3}<\frac{\pi}{4}+\frac{1}{6}$. [Take $f(x)=\tan^{-1}x,\ \ u<x<v$]

13. Verify Cauchy's Mean Value theorem for the functions:

    (i) $f(x)=x^4, g(x)=x^2$ in the interval [a, b]

    (ii) $f(x)=\ln x, g(x)=\frac{1}{x}$ in the interval [1, e]

(iii) $f(x) = e^x, g(x) = e^{-x}$ in the interval (a, b)

(iv) $f(x) = \sqrt{x}, g(x) = \dfrac{1}{\sqrt{x}}$ in the interval (a, b)

(v) $f(x) = \dfrac{1}{x^2}, g(x) = \dfrac{1}{x}$ in the interval (a, b)

(vi) $f(x) = x^3 - 3x^2 + 2x, g(x) = x^3 - 5x^2 + 6x$ in the interval $(0, 0.5)$