QUESTION BANK - ODE & PDE

Partial Differential Equations (PDE) : Solution by Separation of Variables

1. The order of the PDE

   $$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$$

   (a) 2, (b) 1, (c) 3, (d) 0.

2. Check the nature of PDE's

   $$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$$

   (a) Linear & homogeneous (b) Non-linear & homogeneous, (c) Linear & non-homogeneous, (d) Non-linear & non- homogeneous.

3. Check the nature of PDE's

   $$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+2x$$

   (a) Linear & homogeneous (b) Non-linear & homogeneous, (c) Linear & non-homogeneous, (d) Non-linear & non- homogeneous.

4. Check the nature of PDE's

   $$\left(\frac{\partial u}{\partial t}\right)^2=\frac{\partial^2 u}{\partial x^2}+\sin x$$

   (a) Linear & homogeneous (b) Non-linear & homogeneous, (c) Linear & non-homogeneous, (d) Non-linear & non- homogeneous.

5. Check the nature of PDE's

   $$\frac{\partial u}{\partial t}=\sin x^2\,\frac{\partial^2 u}{\partial x^2}$$

   (a) Linear & homogeneous (b) Non-linear & homogeneous, (c) Linear & non-homogeneous, (d) Non-linear & non- homogeneous.

6. Check the nature of PDE's

   $$u\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$$

   (a) Linear & homogeneous (b) Non-linear & homogeneous, (c) Linear & non-homogeneous, (d) Non-linear & non- homogeneous.

7. Identify the following PDE

   $$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+5x$$

   (a) Parabolic, (b) Elliptical, (c) Hyperbolic, (d) None of the above.

8. Identify the following PDE

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

(a) Parabolic, (b) Elliptical, (c) Hyperbolic, (d) None of the above.

9. Identify the type of boundary condition

At x=0, $\frac{\partial u}{\partial x} = q$

(a) Dirichlet boundary condition, (b) Neumann boundary condition, (c) Robin mixed boundary condition.

10. Identify the type of boundary condition

At x=L, $-k\frac{\partial T}{\partial x} = h(T - T_\infty)$

(a) Dirichlet boundary condition, (b) Neumann boundary condition, (c) Robin mixed boundary condition.

11. $L = \frac{d^2}{dx^2} + \frac{d}{dx}$, L is which type of operator?

(a) Linear, (b) Non-linear, (c) None of the above.

12. For Eigen value problem:

$$\frac{d^2 y}{dx^2} + \lambda^2 y = 0$$

With B.C. at x=0, y=0;

x=1, y=0 Eigen functions are

(a) $\sin(n\pi x)$, (b) $\sin^2(n\pi x)$, (c) $\tan(n\pi x)$, (d) $\cos\left[(2n-1)\frac{\pi}{2}\right]x$.

13. $L = \frac{d^2}{dx^2}$ is a

(a) Self adjoint operator & linear operator, (b) Non self adjoint & linear operator, (c) Self adjoint & Non linear operator, (d) Non self adjoint & Non linear operator.

14. $\frac{d^2 y}{dx^2} + \lambda^2 y = 0$

At x=0, $\frac{dy}{dx} = 0$;

x=1, y=0;

What is the Eigen value of the above problem?

(a) nπ, (b) $(2n-1)\frac{\pi}{2}$, (c) $(n\pi)^2$, (d) $(2n+1)\frac{\pi}{2}$.

15. $\frac{d^2y}{dx^2}+\lambda^2 y=0$

At x=0, y=0;

x=1, $\frac{dy}{dx}+\beta y=0$;

What is the Eigen value of the above problem?

(a) nπ, (b) $(2n-1)\frac{\pi}{2}$,(c) roots of $\lambda_n+\beta\tan\lambda_n=0$, (d) roots of $\lambda_n-\beta\tan\lambda_n=0$.

Partial Differential Equations (PDE) for Engineers: Solution by Separation of Variables

1. Consider, Eigen value Problem,

   $$5\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+\lambda(\sin x)y=0$$

   Subject to at x=0, y=0;

   $$x=1,\quad \frac{dy}{dx}=0$$

   In this problem the weight function is
   (a) x, (b) λ, (c) 5, (d) sin x.

2. Consider the problem, Ly=0, Where

   $$L=5\frac{d^2}{dx^2}+7\frac{d}{dx}+9$$

   And boundary condition, at x=0, y=0
   x=1, y=0
   The adjoint operator of L^* is,
   (a) $5\frac{d^2}{dx^2}+7\frac{d}{dx}+9$ (b) $5\frac{d^2}{dx^2}-7\frac{d}{dx}+9$ (c) $5\frac{d^2}{dx^2}+7\frac{d}{dx}-9$ (d) $L=5\frac{d^2}{dx^2}+9\frac{d}{dx}-7$.

3. Consider the problem, Ly=0, where $L=5\frac{d^2}{dx^2}+9$

   With B.C, B at x=0, y=0; x=1, y=0

   (a) L=L*, B=B* (b) $L\neq L^*$, B=B* (c) L=L*, $B\neq B^*$
   Where * represent adjoint problem.

4. For Parabolic PDE:

   $$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$$

   At t=0, u=0;

   At x=0, u=0;

   x=1, u=1;
   Is it a
   (a) Well posed problem (b) Ill posed problem

5. For the Parabolic PDE:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

At t=0, u=1

At x=0, u=2

$$x = 1, \quad \frac{\partial u}{\partial x} = 3$$

This problem can be divided into primarily following sub-problems:

(a) 1 (b) 2 (c) 3 (d) 4.

6. For Parabolic PDE,

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

At t=0, u=1, u=0 on all six boundaries at x=0 & 1, y=0 & 1, z=0 & 1.

The final expression of solution contains following number of summations:

(a) 1 (b) 2, (c) 3, (d) 4

7. Consider

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

At t=0, u=u_0

At x=0, u=0; at y=0, $\frac{\partial u}{\partial y} = 0$ at z=0, u=0

x=1, u=0; y=1, u=0 z=1, $\frac{\partial u}{\partial z} + \beta u = 0$

In this problem, number of independent eigenvalues are

(a) 1, (b) 3, (c) 4, (d) 2.

8. In problem 7, The eigen function are

(a) $\sin(n\pi x)$, $\sin(n\pi y)$, $\sin(n\pi z)$

(b) $\sin(n\pi x)$, $\cos(n\pi y)$, $\cos(n\pi z)$

(c) $\sin(n\pi x)$, $\cos(n\pi y)$, $\sin(\lambda_n z)$

(d) $\sin(n\pi x)$, $\cos[(2n-1)\frac{\pi}{2} y]$, $\sin(\lambda_n z)$ Where λ_n are roots of $\lambda_n + \beta \tan \lambda_n = 0$

9. In problem 7, The eigen values are

(a) $n\pi$, $(2n-1)\frac{\pi}{2}$, λ_n; where λ_n are roots of $\lambda_n + \beta \tan \lambda_n = 0$

(b) $n\pi$ only,

(c) $(2n-1)\frac{\pi}{2}$

(d) $n\pi$ & $(2n-1)\frac{\pi}{2}$

10. Identify the type of boundary condition

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

At t=0, u=0;

At x=0, u=1;

$x = 1,\quad \frac{\partial u}{\partial x} = 0$

The appropriate sub-problems are

(a) $\frac{d^2 u_s}{dx^2} = 0$ at x=0, u_s=1; at x=1, $\frac{du_s}{dx} = 0$ & $\frac{\partial u_t}{\partial t} = \frac{\partial^2 u_t}{\partial x^2}$ at t=0, u_t =-u_s(x); at x=0, u_t=0; at x=1, $\frac{\partial u_t}{\partial x} = 0$

(b) $\frac{d^2 u_s}{dx^2} = 0$ at x=0, u_s=1; at x=1, $\frac{du_s}{dx} = 0$ & $\frac{\partial u_t}{\partial t} = \frac{\partial^2 u_t}{\partial x^2}$ at t=0, u_t =u_s(x); at x=0, u_t=0; at x=1, $\frac{\partial u_t}{\partial x} = 0$

(c) $\frac{d^2 u_s}{dx^2} = 0$ at x=0, u_s=1; at x=1, $\frac{du_s}{dx} = 0$ & $\frac{\partial u_t}{\partial t} = \frac{\partial^2 u_t}{\partial x^2}$ at t=0, u_t =-u_s(x); at x=0, u_t=0; at x=1, $\frac{\partial u_t}{\partial x} = 1$

(d) $\frac{d^2 u_s}{dx^2} = 0$ at x=0, u_s=1; at x=1, $\frac{du_s}{dx} = 0$ & $\frac{\partial u_t}{\partial t} = \frac{\partial^2 u_t}{\partial x^2}$ at t=0, u_t =u_s(x); at x=0, u_t=1; at x=1, $\frac{\partial u_t}{\partial x} = 1$

11. Consider,

$$\frac{d^2y}{dx^2} + \lambda^2 y = 0$$

Subject to

x=0, y=0;

x=1, $\frac{dy}{dx} = 0$;

In this problem eigenvalue & eigenfunction are

(a) $\lambda_n = n\pi$, $\sin(\lambda_n x)$

(b) $\lambda_n = (2n-1)\frac{\pi}{2}$, $\cos(\lambda_n x)$

(c) $\lambda_n = (2n-1)\frac{\pi}{2}$, $\sin(\lambda_n x)$

(d) $\lambda_n = n\pi$, $\tan(\lambda_n x)$

Consider PDE:

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

Subject to at x=0, $u=u_0$; y=0, u=0;

x=1, u=0; y=1, u=0;

The eigen functions are

(a) $\sin(n\pi x)$, (b) $\sin(n\pi y)$, (c) $\cos[(2n-1)\frac{\pi}{2}y]$.

2. Consider PDE:

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

Subject to at x=0, $u=u_0$; y=0, $\frac{\partial u}{\partial y}=0$

x=1, u=0; y=1, u=0;

The eigen functions are

(a) $\sin(n\pi x)$, (b) $\sin(n\pi y)$, (c) $\cos[(2n-1)\frac{\pi}{2}x]$, (d) $\cos[(2n-1)\frac{\pi}{2}y]$.

3. An elliptical PDE physically models a system:
   (a) At steady state (b) at unsteady state (c) at the start up of the plant

4. A hyperbolic PDE must contain
   (a) Dirichlet B.C (b) Neumann B.C (c) Robin-mixed B.C (d) Cauchy B.C

5. Consider hyperbolic PDE:

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$$

The BCs on x are homogeneous.

BCs on t cannot be:

(a) At t=0, $u=u_{01}$, $\frac{\partial u}{\partial t}=u_{02}$

(b) At t=0, u=0, $\frac{\partial u}{\partial t}=0$

(c) At t=0, u=0, $\frac{\partial u}{\partial t}=u_{02}$

(d) At t=0, $u=u_{01}$, $\frac{\partial u}{\partial t}=0$

6. Bessel functions are orthogonal to each w.r.t weight function
   (a) r (b) r^2, (c) sin r, (d) exp (r).

7. For one dimensional transient heat conduction in a solid cylinder, where wall temperature is kept at constant temperature, the boundary condition at centreline of cylinder at r=0 is an example of:

   (a) Dirichlet B.C (b) Neumann B.C (c) Physical B.C (d) None of the above.

8. What is BC at r=0 in problem 7:
   (a) T=infinite (b) $T=T_{ambient}$ (c) $T=T_{wall}$

9. For Bessel function $J_0(x)$, it is
   (a) An exponential function of x
   (b) It is an oscillatory function about x axis with diminishing magnitude
   (c) It is a linear function in x through origin

10. For Bessel function $Y_0(x)$ is
    (a) 0 at x=0
    (b) ∞ at x=0
    (c) $-\infty$ at x=0
    (d) 1 at x=0

11. m^{th} order Bessel function $J_m(\lambda x)$ are
    (a) Orthogonal functions
    (b) Non-Orthogonal functions

12. For 2 dimensional transient heat conduction problem in a cylinder without θ symmetry the BCs on θ are
    (a) $T\big|_{\pi} = T\big|_{-\pi}$ & $\frac{\partial T}{\partial \theta}\big|_{\theta=\pi} = \frac{\partial T}{\partial \theta}\big|_{\theta=-\pi}$
    (b) T=0 at θ=π & $\frac{\partial T}{\partial \theta}\big|_{\theta=\pi} = 0$ at θ=-π
    (c) T=1 at θ=π & $\frac{\partial T}{\partial \theta}\big|_{\theta=\pi} = 0$ at θ=-π
    (d) T=0 at θ=π & $\frac{\partial T}{\partial \theta}\big|_{\theta=\pi} = 1$ at θ=-π