Chapter 1

SOM-I

Engineering Stress

- The average engineering stress is defined as the force per unit area given as:

$$\sigma_{avg} = \frac{P}{A} \quad \left(psi \;\; or \;\; Pa \right)$$

- where P is the specific applied force and A is the original cross sectional area of the material before P is applied. The stress may be tension, compression or shear stress.

true stress

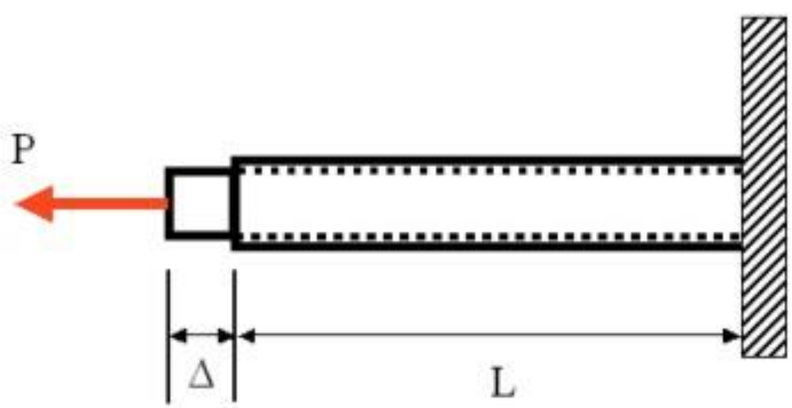
$$\sigma_{true} = \frac{P}{A}$$

- the term A is the instantaneous area of the deformed section under the action of the specified load P.

Engineering Strain

- This is measure of the deformation and measures changes in length and changes in angles. Extensional strain or normal strain is defined as the deformation per unit length:

$$\varepsilon_E = \frac{\Delta}{L} \quad \left(\mathit{in/in\ or\ mm/mm}\right)$$



Where Δ is the deformation and L is the original gage length of the specimen.

true strain, or Logarithmic Strain

- The true strain, or Logarithmic Strain is expressed as:

$$\varepsilon_{true} = \ln \frac{L'}{L}$$

- where L’ is the instantaneous gage length at any specified load, and L is the original gage length before the original load is applied, and since:

Relationship between Engineering Strain & True Strain

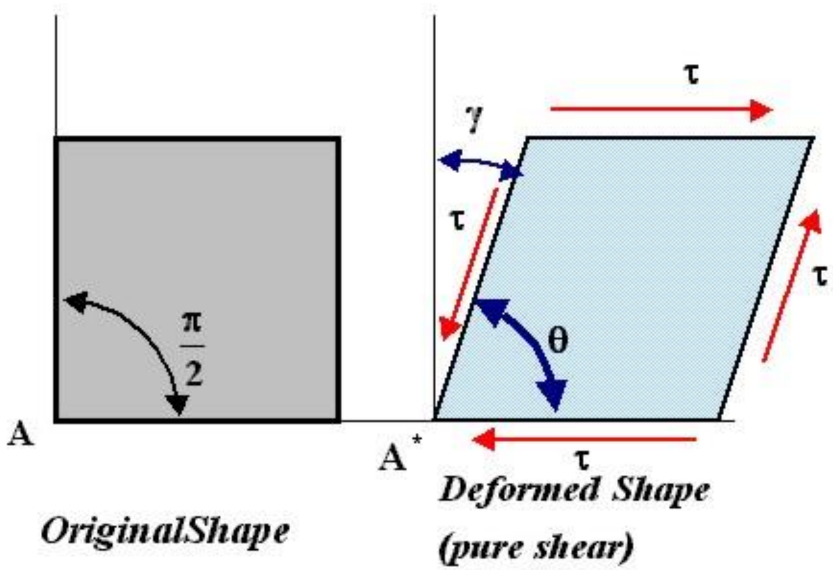
$$\mathbf{L}' = \mathbf{L} + \boldsymbol{\Delta}$$

then

$$\varepsilon_{true} = \ln\left(1 + \varepsilon_E\right)$$

shear strain

- The <u>shear strain</u> (g) is the angle measured in radians between two lines originally at right angles before the shear load is applied.



Note

- True stress and true strain are not very useful to the engineer or the designer, since most of the mechanical properties of the material are determined from the engineering stress vs. engineering strain curve.

Elastic Limit

- The elastic limit of a material is the maximum stress that a material can withstand without causing a permanent deformation.

Yield Strength

- The maximum stress that can be imposed one material without causing more than a specified permanent set or deformation. This set is determined by measuring the departure of the actual stress-strain diagram from an extension of the initial straight portion. The specified value is often taken as an offset of 0.002 in the strain (referred to as 0.2% strain).

Proportional Limit

- The maximum stress that can develop within a material without causing a deviation of the linear proportionality of the stress and the strain. The determination of this property requires a graph of stress vs. strain and is determined by the value of the stress at the point at which the curve deviates from a straight line. (There may be some nonlinear elastic behavior between the proportional limit and the yield strength.)

Yield Point

- The first stress (usually less than the ultimate strength) at which a material continues to deform appreciably without an increase in load. Most highly ductile materials such as steel have this property. Since steel is widely used, this property has common usage. The definition of yield point requires that the stress-strain curve has a point of zero slope other than that which may exist at the ultimate load. It is possible and quite often happens that there is more than one yield point. In this case the lower yield point is usually reported, as it is more readily reproducible. The upper yield point is the stress at which the stress-strain curve first becomes horizontal. The lower yield point is the lower and almost constant stress under which a material continues to deform. For those materials that exhibit a yield point, the value of Yield Strength that causes 0.1% or 0.2% offset (permanent deformation) may coincide with the value of the lower yield point.

Ultimate Strength

- The maximum stress that can be developed in a material is given as:

$$\sigma_{ultimate} = \frac{P}{A}$$

- Where P is a maximum load and A is the original cross sectional area of the material

- Ultimate strength is the highest, or the maximum value, obtained on the stress-strain curve. In general, when a ductile material reaches its ultimate strength under the action of an increasing load, it will undergo an appreciable change in geometry.

Poisson’s Ratio

- When a material is loaded in a uniaxial state of stress such as in a tension or compression test, deformation will take place in the direction of the loading. At the same time, a transverse deformation will also occur, even though there is no applied force in that direction.

- Within the proportional limit of the material the stress is directly proportional to the strain (Hooke's Law) and the axial strain:

$$\varepsilon_1 = \frac{\sigma_1}{E} \quad \textit{(tension or compression)}$$

- The absolute value of the ratio of the transverse strain to the axial strain is called Poisson’s Ratio n , and therefore:

$$\nu = \left| \frac{\varepsilon_2}{\varepsilon_1} \right| = -\frac{\varepsilon_2}{\varepsilon_1}$$

- For typical engineering materials, the value of Poisson’s ratio ranges from 0.2 to 0.4. This is true for values of applied load below the proportional limit of the material. However, for inelastic deformation the value of Poisson’s ratio approaches 0.5 and Hooke’s Law is no longer valid.

Modulus of Elasticity (or Young's Modulus) (E)

- The modulus of elasticity is the mechanical property that indicates its stiffness. It is the rate of change of the stress with respect to the strain. Typically, it is measured as the slope of the initial straight line of the stress-strain curve:

$$E = \frac{\Delta\sigma}{\Delta\varepsilon}$$

$\Delta\varepsilon$ *is an increment in strain*

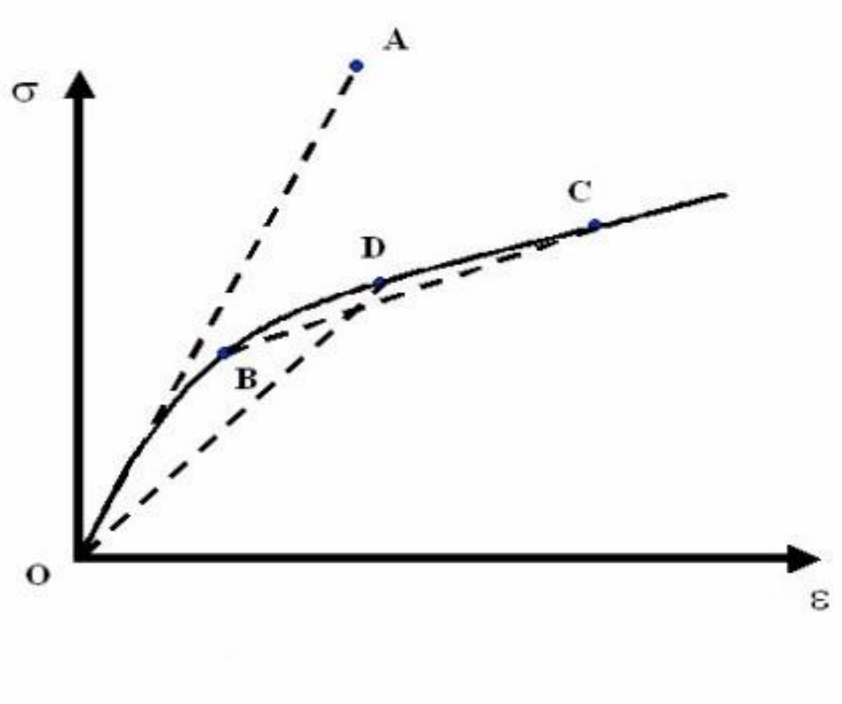
$\Delta\sigma$ *is an increment in stress*

- The measure of the stiffness applies to normal or shearing stress. Young’s Modulus of Elasticity (E) is associated with normal stress. For a most materials the values of modulus of elasticity in tension and compression are approximately the same.

- In terms of the stress-strain curve for ductile materials, the modulus of elasticity is determined by the slope of the stress-strain curves up to the proportional limit. However, some materials have stress-strain diagrams that are curved from the very beginning, and, therefore, it is necessary to determine the slope of the curve at zero stress.

- The Tangent Modulus is a measure of stiffness of a material at some given value of stress. The slope of the stress-strain curve at zero stress determines the value.
- The Chord Modulus is a measure of the average stiffness of a material between two values of stress on a stress-strain curve.
- The Secant Modulus is a special chord modulus at a specified stress and with the initial value of the stress is being zero, as in Figure 1.

- The slope of the line OA will give the initial tangent modulus of elasticity, and the slope of the chord BC will give the chord modulus of elasticity whereas the slope of the line OD will give the secant modulus of elasticity.



Shear Modulus (or Modulus of Rigidity) (G)

- This term is associated with shearing stresses:

$$G = \frac{\Delta\tau}{\Delta\gamma} \quad \left(psi\ or\ Pa\right)$$

$\Delta\varepsilon$ *is an increment in shearing strain*

$\Delta\sigma$ *is an increment in shearing stress*

- The relation between the modulus of rigidity to the modulus of elasticity is E:

$$\mathbf{G} = \frac{\mathbf{E}}{\mathbf{2(1+v)}}$$

Modulus of Resilience

- The modulus of rupture is the ultimate stress of a material as determined by the torsion formula or the flexure formula when a torsional or bending member is subjected to fracture loads. The modulus of rupture for bending can be determined from the equation:

$$\sigma = \frac{Mc}{I}$$

- and the modulus of rupture for torsion can be determined from the equation:

$$\tau = \frac{\mathbf{TR}}{\mathbf{J}}$$

- These values of stresses are not the actual stresses unless the relationship between stress and strain is linear for the entire range of loading of the material. The modulus of rupture is used as a comparison between the relative strengths of similarly shaped members when subjected to the same loading either in torsion or bending.

Modulus of Toughness

- The amount of energy absorbed per unit volume of a material when it is stressed to fracture is called the modulus of toughness. It is equel to the area under the entire stress-strain curve up to the point of fracture. The toughness of a material is related to its ultimate strength and its ductility. The ability to resist an impact load is related to a material's toughness.

Ductility and Brittleness

- A ductile material is the one that is able to withstand under a certain set of conditions a large amount of plastic deformation before rupture. Ductile behavior is associated with metals and it should not be confused with malleability. Malleability is the ability of a material to be hammered into thin sheets. Gold and lead are good examples of such materials.

- There is no absolute quantitative measure of ductility, however, for the relative comparison of various metals, the most common used measurements are:

1. The percentage of elongation
2. The percentage of reduction of area

- To determine the percentage of elongation and the percentage of reduction of area a tensile test must be performed on a specimen until it breaks. Usually a gage length of 2 inches is marked on the specimen before the test is carried out and the original cross sectional area of the uniform section is also measured. After the specimen is fractured, the two fractured ends are placed together and the distance between the marked points of the gage length is measured again. The new area is also calculated for the fractured section.

- The percentage of elongation will be obtained as:

$$\%\,elongation = \frac{(Final\ gage\ length - Original\ gage\ length)}{(Original\ gage\ length)} \times 100\%$$

- and the percentage of area reduction:

$$\%\,reduction\ in\ area = \frac{(Original\ area - Final\ area)}{(Original\ area)} \times 100\%$$

- The gage length is associated with the percentage of elongation and, therefore. It should be stated whenever the percentage of elongation is given; for example, 18 % elongation in 2 inches gage length, or 22 % in 6 inches gage length, and so onThe environmental conditions such as the temperature play a great role in the behavior of the material. For example, the material might have a ductile behavior at certain temperatures and a brittle property at other temperatures. In general, most materials behave in a more brittle fashion when subjected to lower temperature. In addition, the state of stress to which the material is subjected, that is, uniaxial, biaxial, or triaxial, is another factor on how the material will ultimately fracture, whether in a brittle or a ductile fashion.

- Materials that fracture at a low level of strain are called **brittle**. They will generally exhibit little eof no plastic strain before failure. Often materials that exhibit less than 5% elongation are called brittle. Cast iron is an example of a brittle material. Brittle materials fail on planes where the maximum normal stress occurs.

Creep

- Creep is defined as the continuous deformation of a material when subjected to a constant stress at a specified temperature. It is also possible that creep might continue to fracture. Several materials creep at room temperature and others require a considerable increase in temperature before a measurable deformation will take place under a constant load.

Relaxation

- Relaxation is the opposite phenomena of creep. It is the continuous decrease of stress when a material is subjected to constant strain at a specified temperature. The same properties that influence creep of a material will usually influence its relaxation.

Fatigue

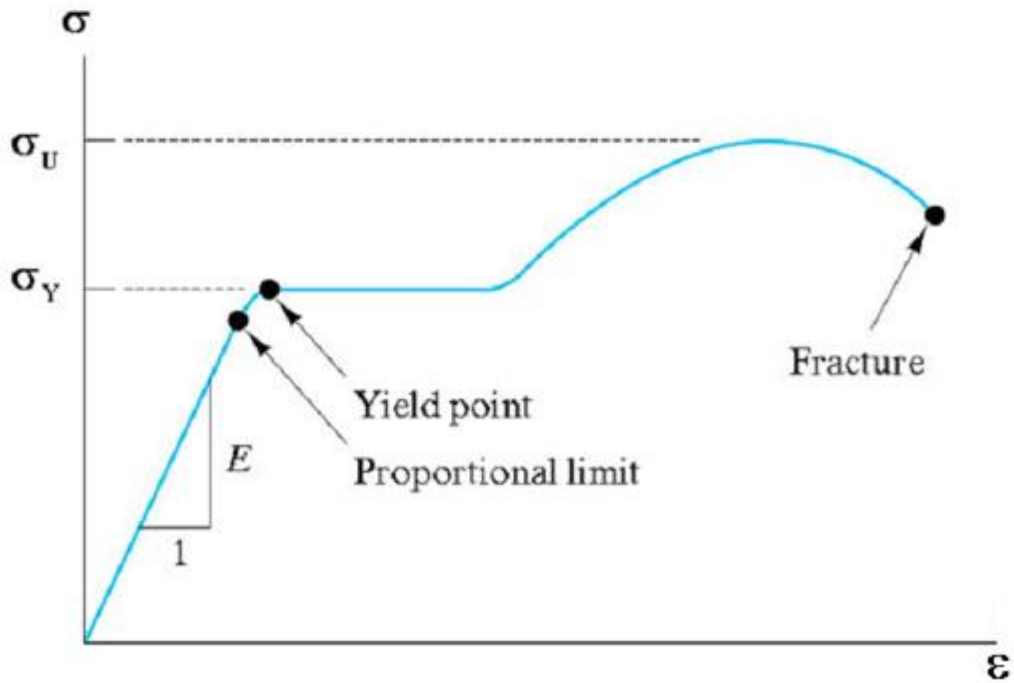
- Fatigue is associated with the behavior of the material when subjected to a cyclic or repeated loading. Generally, the stress value at which the material might fail due to the action of the cyclic loading is less than its ultimate stress. A material might show a considerable ductility under the action of the static load, however, when subjected to a repeated load it might fail in a brittle way. Several engineering components such as springs, connecting rods, axles, aircraft landing gears, and ships' hulls are subjected in actual practice to repeated cyclic stresses, and, therefore, fatigue tests must be carried out on such materials before they are put in service in order to determine their fatigue properties and strengths.

Stress- Strain Diagram

- The axial stress s is found by dividing the axial load P by the original cross-sectional area A. This is called the nominal stress, the conventional stress, or the engineering stress. Another measure of stress is the true stress obtained by dividing the axial load P by the current or deformed cross-sectional area A. In a tension test, the true stress will be larger than the nominal stress. The average strain e is found by dividing the measured elongation d by the gage length L.

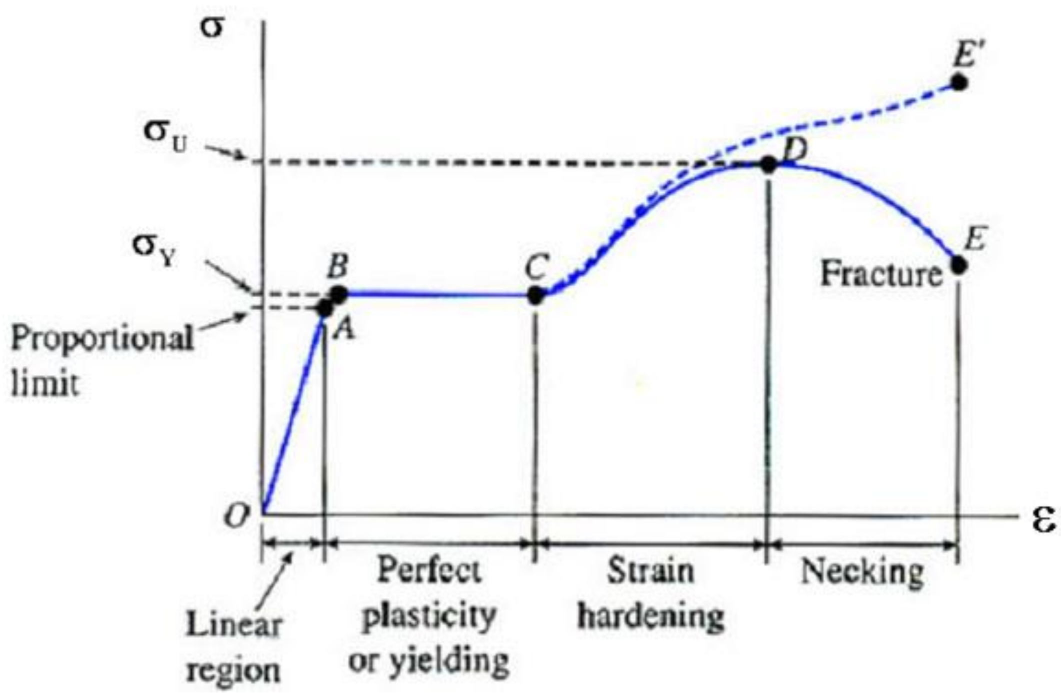
- When the stress and strain have been measured a plot of stress vs. strain can be drawn. This is called a stress-strain diagram. Many material properties can be found from the information on a stress-strain diagram for a material.

- Let us now discuss the stress-strain behavior of a common engineering material, structural steel. A typical stress-strain diagram (not to scale) for steel is shown below.

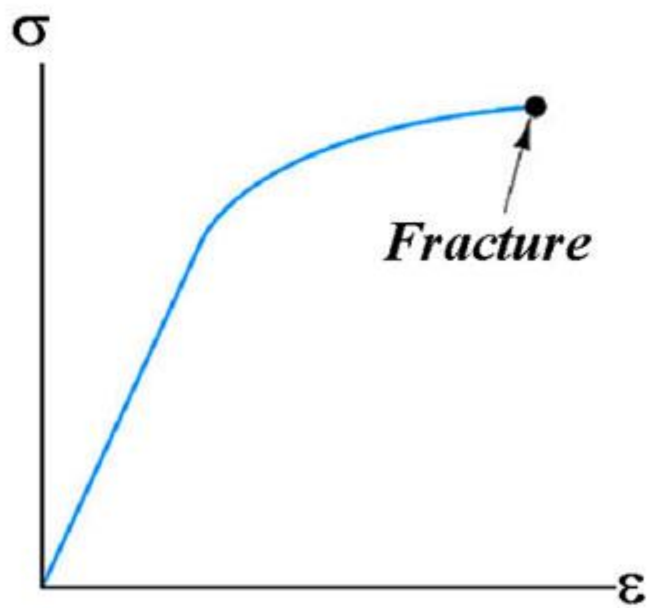

σ
σ_U
σ_Y
E
1
Yield point
Proportional limit
Fracture
ε

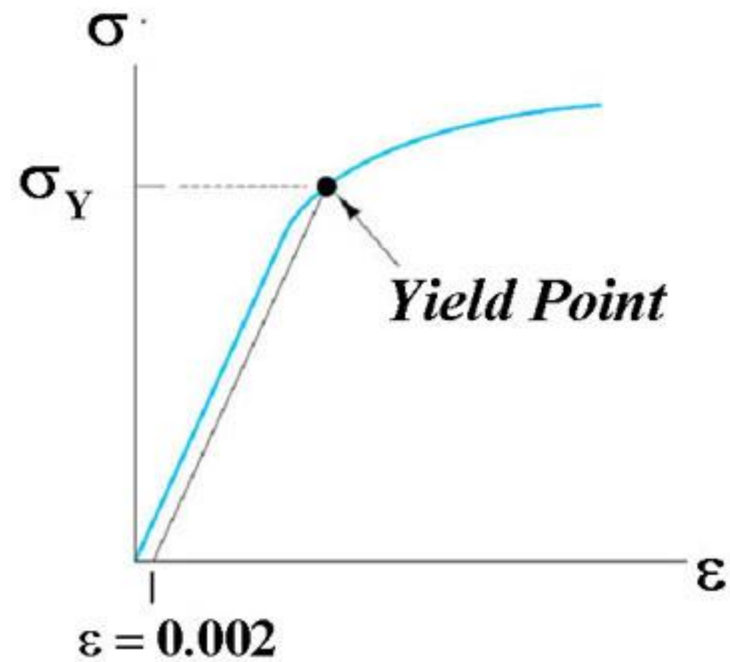
- We observe an initial linear portion of the curve. The slope of this straight line portion is the modulus of elasticity E for the material. The units for the modulus are the same as for stress. The linear behavior continues until the stress and strain are no longer proportional. This value of stress is known as the proportional limit. If the stress in increased past the proportional limit the strain begins to increase more rapidly with increases in stress. The curve will become horizontal at a value of stress called the yield point s_Y. At the yield point, considerable elongation occurs with no noticeable increase in the stress. This is known as yielding of the material. Next, the steel begins to strain harden. During this phase, both the stress and strain increase until the stress reaches a maximum value known as the ultimate stress s_U. At this point the nominal stress decreases with increasing strain until the specimen fractures or breaks. During this final phase a great deal of necking or reduction of the cross-sectional are occurs

- This behavior is shown in more detail in this figure. Point A is the proportional limit, B is the yield point, C is the end of the yielding or perfectly plastic zone, D is the ultimate stress, and E is the fracture stress. The dotted line shows the true stress.



- Some materials, such as aluminum, do not have a clearly observable yield point. When this is true, an arbitrary value of the yield stress is determined from the offset method. A straight line is drawn on the stress-strain diagram parallel to the initial linear part of the curve, but offset by some standard amount such as 0,002 or 0,2%. The intersection of the offset line with the stress-strain curve is defined as the yield point. This procedure is illustrated in the figures below.


σ
Fracture
ε


σ
σY
Yield Point
ε
ε = 0.002