Circuit Theory

- What you will use this for
  - Power management
  - Signals between subsystems
  - Possible analog data types
- How the knowledge will help you
  - Understanding power and energy requirements
  - Behavior of digital electric signals
  - Analog signal conditioning and limitations
  - Understanding associated technologies

Circuit theory Topics

- Circuit Topology
- Voltage, Current and Power
- Kirchoff’s Laws
- Circuit components
- DC circuits
- AC circuits

Circuit Topology

- A *circuit* consists of a mesh of loops
- Represented as branches and nodes in an undirected graph.
- Circuit components reside in the branches
- Connectivity resides in the nodes
  - Nodes represent wires
  - Wires represent equipotentials

Voltage, Current and Power (1)

- The concept of charge
  - The Coulomb [C] – the SI unit of charge
  - An electron carries -1.6e-19 [C]
  - Conservation of charge
- The concept of potential
  - Attraction/repulsion of charges
  - The electric field
  - The energy of moving a charge in a field

Voltage, Current and Power (2)

- **Voltage** is a *difference* in electric potential
  - always taken between two points.
  - Absolute voltage is a nonsensical fiction.
  - The concept of *ground* is also a (useful) fiction.
- It is a line integral of the force exerted by an electric field on a unit charge.
- Customarily represented by v or V.
- The SI unit is the Volt [V].

Voltage, Current and Power (3)

- **Current** is a *movement* of charge.
- It is the time derivative of charge passing through a circuit branch.
- Customarily represented by i or I.
- The SI unit is the Ampere [A].

Voltage, Current and Power (4)

- **Power** is the product of voltage by current.
- It is the time derivative of energy delivered to or extracted from a circuit branch.
- Customarily represented by *P* or *W*.
- The SI unit is the Watt [W].

Kirchoff’s Laws

- These laws add up to nothing! Yet they completely characterize circuit behavior.
- Kirchoff’s Voltage Law (KVL) - The sum of voltages taken around any loop is zero.
  - The start and end points are identical; consequently there is no potential difference between them.
- Kirchoff’s Current Law (KCL) – The sum of currents entering any node is zero.
  - A consequence of the law of conservation of charge.

Circuit components

- Active vs. Passive components
  - Active ones may generate electrical power.
  - Passive ones may store but not generate power.
- Lumped vs. Distributed Constants
  - Distributed constant components account for propagation times through the circuit branches.
  - Lumped constant components ignore these propagation times. Appropriate for circuits small relative to signal wavelengths.
- Linear, time invariant (LTI) components are those with constant component values.

Active circuit components

- Conservation of energy: active components must get their power from *somewhere*!
- From non-electrical sources
  - Batteries (chemical)
  - Dynamos (mechanical)
  - Transducers in general (light, sound, etc.)
- From other electrical sources
  - Power supplies
  - Power transformers
  - Amplifiers

Passive lumped constants

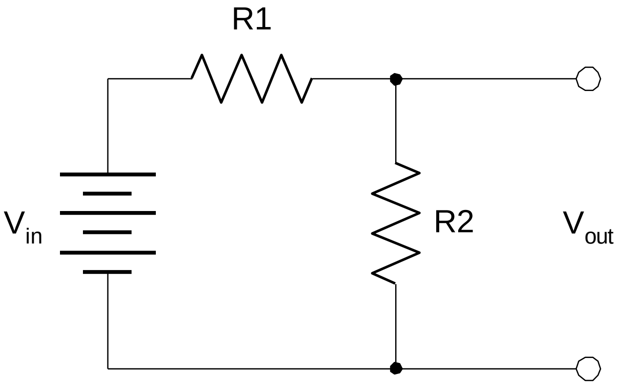
- Classical LTI
  - Resistors are AC/DC components.
  - Inductors are AC components (DC short circuit).
  - Capacitors are AC components (DC open circuit).
- Other components
  - Rectifier diodes.
  - Three or more terminal devices, e.g. transistors.
  - Transformers.

DC circuits

- The basic LTI component is the Resistor
  - Customarily represented by R.
  - The SI unit is the Ohm [Ω].
- Ohm's Law: $V = I R$

Ohm's and Kirchoff's laws *completely* prescribe the behavior of any DC circuit comprising LTI components.

Example: voltage divider



Assume no current is drawn at the output terminals in measuring V_{out}. Ohm's Law requires that $V_{R1} = I_{R1} R_1$ and $V_{R2} = I_{R2} R_2$, which is also V_{out}. KCL says the current leaving resistor R1 must equal the current entering R2, or $I_{R1} = I_{R2}$, so we can write $V_{out} = I_{R1} R_2$. KVL says the voltage around the loop including the battery and both resistors is 0, therefore $V_{in} = V_{R1} + V_{out}$, or $V_{in} = I_{R1} R_1 + I_{R1} R_2$. Thus, $I_{R1} = V_{in} / (R_1 + R_2)$, and

$$V_{out} = V_{in} R_2 / (R_1 + R_2).$$

AC circuits -- Components

- Basic LTI components
  - Resistor, R, [Ω] (Ohms)
  - Inductor, L, [H] (Henrys)
  - Capacitor, C, [F] (Farads)
- Frequency
  - Repetition rate, f, [Hz] (Hertz)
  - Angular, $\omega = 2\pi f$, [1/s] (radians/sec)

AC Components: Inductors

- Current in an inductor generates a magnetic field,

$$B = K_1 I$$

- Changes in the field induce an inductive voltage.

$$V = K_2 (dB/dt)$$

- The instantaneous voltage is

$$V = L(dI/dt),$$

where $L = K_1K_2$.

This is the time domain behavior of an inductor.

AC Components: Capacitors

- Charge in a capacitor produces an electric field E, and thus a proportional voltage,

$$Q = C\,V,$$

Where C is the *capacitance*.

- The charge on the capacitor changes according to

$$I = (dQ/dt).$$

- The instantaneous current is therefore

$$I = C(dV/dt).$$

This is the time domain behavior of a capacitor.

AC Circuits – Laplace Transform

- Transforms *differential* equations in time to *algebraic* equations in frequency (*s* domain).

$$F(t) \rightarrow f(s), \quad \frac{dF}{dt} \rightarrow s\ f(s), \quad \int F dt \rightarrow \frac{f(s)}{s},$$

where the frequency variable $s = \sigma + j\omega$.
For sinusoidal waves, $\sigma = 0$, and $s = j\omega$.

- Resistor behavior in *s* domain: $v = iR$.
- Inductor behavior in *s* domain: $v = i\,(j\omega L)$.
- Capacitor behavior in *s* domain: $i = v\,(j\omega C)$.

AC circuits -- Impedance

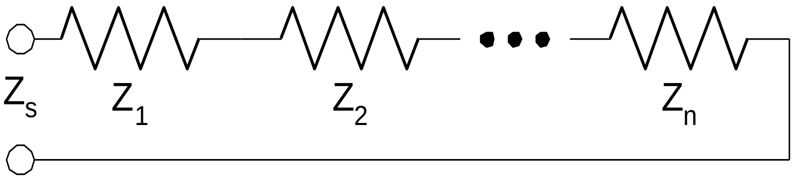
- Impedance and Ohm's Law for AC:
  - Impedance is $Z = R + jX$,

    where $j = \sqrt{-1}$, and X is the *reactance* in [Ω].
  - Ohm's AC Law in *s* domain: $v = i\,Z$
- Resistance *R* dissipates power as heat.
- Reactance *X* stores and returns power.
  - Inductors have positive reactance $X_l = \omega L$
  - Capacitors have negative reactance $X_c = -1/\omega C$

Impedance shortcuts

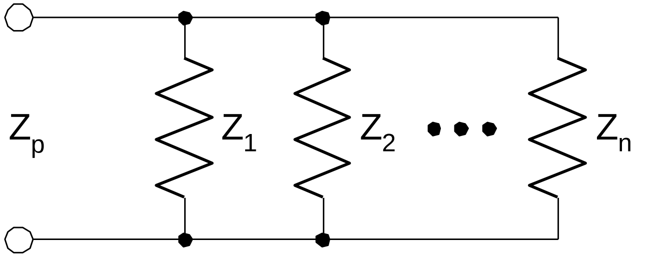
- The impedance of components connected in series is the complex sum of their impedances.

$$Z_s = Z_1 + Z_2 + \cdots Z_n$$

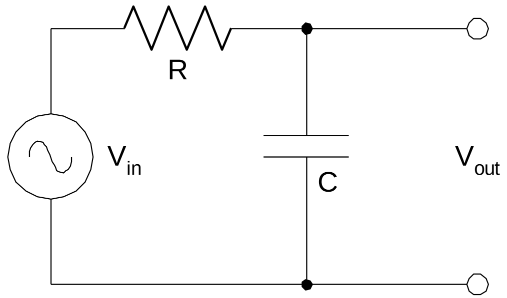


- The impedance of components connected in parallel is the reciprocal of the complex sum of their reciprocal impedances.

$$\frac{1}{Z_p} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots \frac{1}{Z_n}$$



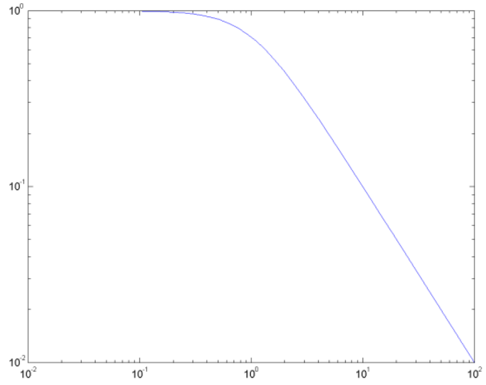
Example: low pass filter



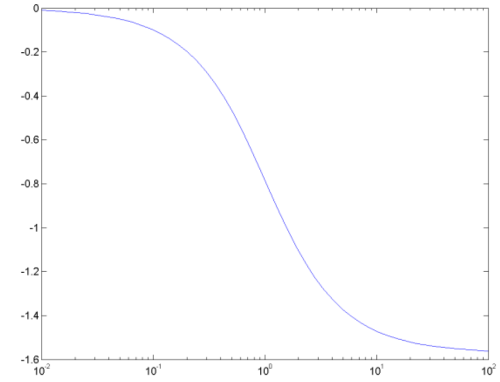
Generalizing from the DC example, $V_{out} = V_{in} \frac{Z_C}{Z_R + Z_C}$.

Recall that $Z_R = R$, and $Z_C = \frac{-j}{\omega C}$.

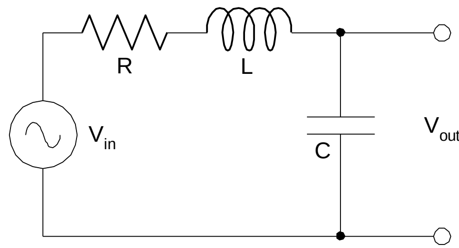
Define the filter gain $A = \frac{V_{out}}{V_{in}} = \frac{-j/\omega C}{R - j/\omega C} = \frac{1}{1 + j\omega RC}$.



Magnitude and phase plots of A, where RC=1. The magnitude plot is log/log, while the phase plot is linear radians vs. log freq.



Homework problem



Derive the filter gain of the pictured circuit.
Plot the magnitude and phase of the filter for
L = 6.3e-6 [H], R = 16 [Ω], and C = 1.0e-7 [F].
For extra credit, also plot for R = 7 [Ω] and 50 [Ω].

Superposition Theorem

- Total current through or voltage across a resistor or branch
  - Determine by adding effects due to each source acting independently
- Replace a voltage source with a short
- Replace a current source with an open
- Find results of branches using each source independently

Superposition Theorem

- Power
  - Not a linear quantity
  - Found by squaring voltage or current
- Theorem does not apply to power
  - To find power using superposition
  - Determine voltage or current
  - Calculate power

Maximum power transferTheorem

- Load should receive maximum amount of power from source
- Maximum power transfer theorem states
  - Load will receive maximum power from a circuit when resistance of the load is exactly the same as Thévenin (or Norton) equivalent resistance of the circuit

Continued…

- To calculate maximum power delivered by source to load
  - Use $P = V^2/R$
- Voltage across load is one half of Thévenin equivalent voltage

References..

- NPTEL Lectures
- Sudhakarsham Mohan
- A.C Chakravarti