Electromagnetic Fields

Coordinate system

Cartesian		*Cylindrical*		*Spherical*
x	$=$	$r\cos\phi$	$=$	$r\sin\theta\cos\phi$
y	$=$	$r\sin\phi$	$=$	$r\sin\theta\sin\phi$
z	$=$	z	$=$	$r\cos\theta$
$\mathbf{i}_x$	$=$	$\cos\phi\,\mathbf{i}_r - \sin\phi\,\mathbf{i}_\phi$	$=$	$\sin\theta\cos\phi\,\mathbf{i}_r + \cos\theta\cos\phi\,\mathbf{i}_\theta - \sin\phi\,\mathbf{i}_\phi$
$\mathbf{i}_y$	$=$	$\sin\phi\,\mathbf{i}_r + \cos\phi\,\mathbf{i}_\phi$	$=$	$\sin\theta\sin\phi\,\mathbf{i}_r + \cos\theta\sin\phi\,\mathbf{i}_\theta + \cos\phi\,\mathbf{i}_\phi$
$\mathbf{i}_z$	$=$	$\mathbf{i}_z$	$=$	$\cos\theta\,\mathbf{i}_r - \sin\theta\,\mathbf{i}_\theta$

Coordinate system contd…

Cylindrical		*Cartesian*		*Spherical*
r	$=$	$\sqrt{x^2+y^2}$	$=$	$r \sin\theta$
ϕ	$=$	$\tan^{-1} y/x$	$=$	ϕ
z	$=$	z	$=$	$r\cos\theta$
$\mathbf{i}_r$	$=$	$\cos\phi\,\mathbf{i}_x + \sin\phi\,\mathbf{i}_y$	$=$	$\sin\theta\,\mathbf{i}_r + \cos\theta\,\mathbf{i}_\theta$
$\mathbf{i}_\phi$	$=$	$-\sin\phi\,\mathbf{i}_x + \cos\phi\,\mathbf{i}_y$	$=$	$\mathbf{i}_\phi$
$\mathbf{i}_z$	$=$	$\mathbf{i}_z$	$=$	$\cos\theta\,\mathbf{i}_r - \sin\theta\,\mathbf{i}_\theta$

Cartesian Coordinate system

Cartesian Coordinates (x, y, z)

$$\nabla f = \frac{\partial f}{\partial x}\mathbf{i}_x + \frac{\partial f}{\partial y}\mathbf{i}_y + \frac{\partial f}{\partial z}\mathbf{i}_z$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \mathbf{i}_x\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \mathbf{i}_y\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \mathbf{i}_z\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Cylindrical Coordinate system

Cylindrical Coordinates $(\mathrm{r}, \phi, \mathrm{z})$

$$\nabla f = \frac{\partial f}{\partial \mathrm{r}}\mathbf{i}_\mathrm{r} + \frac{1}{\mathrm{r}}\,\frac{\partial f}{\partial \phi}\mathbf{i}_\phi + \frac{\partial f}{\partial \mathrm{z}}\mathbf{i}_\mathrm{z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{\mathrm{r}}\,\frac{\partial}{\partial \mathrm{r}}(\mathrm{r}A_\mathrm{r}) + \frac{1}{\mathrm{r}}\,\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_\mathrm{z}}{\partial \mathrm{z}}$$

$$\nabla \times \mathbf{A} = \mathbf{i}_\mathrm{r}\left(\frac{1}{\mathrm{r}}\,\frac{\partial A_\mathrm{z}}{\partial \phi} - \frac{\partial A_\phi}{\partial \mathrm{z}}\right) + \mathbf{i}_\phi\left(\frac{\partial A_\mathrm{r}}{\partial \mathrm{z}} - \frac{\partial A_\mathrm{z}}{\partial \mathrm{r}}\right) + \mathbf{i}_\mathrm{z}\frac{1}{\mathrm{r}}\left[\frac{\partial(\mathrm{r}A_\phi)}{\partial \mathrm{r}} - \frac{\partial A_\mathrm{r}}{\partial \phi}\right]$$

$$\nabla^2 f = \frac{1}{\mathrm{r}}\,\frac{\partial}{\partial \mathrm{r}}\left(\mathrm{r}\frac{\partial f}{\partial \mathrm{r}}\right) + \frac{1}{\mathrm{r}^2}\,\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial \mathrm{z}^2}$$

Spherical Coordinate system

Spherical Coordinates (r, θ, ϕ)

$$\nabla f = \frac{\partial f}{\partial r}\mathbf{i}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\mathbf{i}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\mathbf{i}_\phi$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta A_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla \times \mathbf{A} = \mathbf{i}_r\frac{1}{r\sin\theta}\left[\frac{\partial(\sin\theta A_\phi)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right]$$

$$+\mathbf{i}_\theta\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial(rA_\phi)}{\partial r}\right] + \mathbf{i}_\phi\frac{1}{r}\left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right]$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$$

Static electric fields

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Coulomb's Law :

Statement:

Coulomb's Law states that the force between two point charges Q1and Q2 is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

Point charge is a hypothetical charge located at a single point in space. It is an idealized model of a particle having an electric charge.

Mathematically,

$$F = \frac{kQ_1Q_2}{R^2} \qquad k = \frac{1}{4\pi\varepsilon_0}$$

Where k is the proportionality constant. And ε_0 , is called the permittivity of free space

In SI units, Q1 and Q2 are expressed in Coulombs(C) and R is in meters.

Force F is in Newton's (N)

Electric field Intensity

Electric Field intensity:

The electric field intensity or the electric field strength at a point is defined as the force per unit charge. That is

$$\vec{E} = \lim_{Q \to 0} \frac{\vec{F}}{Q} \quad \text{or,} \quad \vec{E} = \frac{\vec{F}}{Q} \quad \text{.......................................(4)}$$

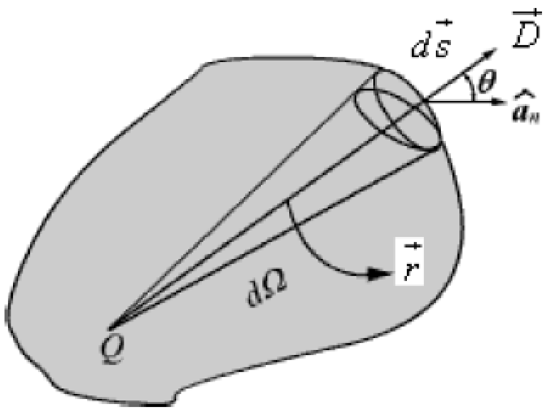
The electric field intensity E at a point r (observation point) due a point charge Q located at $\vec{r'}$ (source point) is given by:

$$\vec{E} = \frac{Q(\vec{r} - \vec{r'})}{4\pi\varepsilon_0 \left|\vec{r} - \vec{r'}\right|^3} \quad \text{..(5)}$$

Gauss’s Law

Gauss's Law:

Gauss's law is one of the fundamental laws of electromagnetism and it states that the total electric flux through a closed surface is equal to the total charge enclosed by the surface.



Let us consider a point charge Q located in an isotropic homogeneous medium of dielectric constant .

The flux density at a distance r on a surface enclosing the charge is given by

$$\vec{D} = \varepsilon\vec{E} = \frac{Q}{4\pi r^2}\hat{a}_r \quad \text{..(13)}$$

If we consider an elementary area ds, the amount of flux passing through the elementary area is given by

$$d\psi = \vec{D}.ds = \frac{Q}{4\pi r^2} ds\cos\theta \quad \text{.......................................(14)}$$

But $\frac{ds\cos\theta}{r^2} = d\Omega$, is the elementary solid angle subtended by the area $d\vec{s}$ at the location of Q.

Therefore we can write $d\psi = \frac{Q}{4\pi} d\Omega$

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For a closed surface enclosing the charge, we can write $\psi = \oint_s d\psi = \frac{Q}{4\pi} \oint_s d\Omega = Q$

Conductors and insulators

Conductors vs. Insulators: Comparison Chart

Conductors	Insulators
Conductors are materials that allow free flow of electrons from one atom to another.	Insulators won't allow free of electrons from one atom to another.
Conductors conduct electricity because of the free electrons present in them.	Insulators insulate electricity because of the tightly bound electrons present within atoms.
These materials can pass electricity through them.	Insulating materials cannot pass electric current through them.
Atoms are not able to hold onto their electrons tightly.	Atoms have tightly bound electrons thereby unable to transfer electrical energy well.
Materials that are good conductors generally have high conductivity.	Good insulating materials usually have low conductivity.
Mostly metals are good conductors such as copper, aluminum, silver, iron, etc.	Common insulators include rubber, glass, ceramic, plastic, asphalt, pure water, etc.

Boundary conditions

Boundary Conditions:

Boundary conditions is the condition that the field must satisfy at the interface separating the media

The boundary conditions at an interface separating:

- Dielectric and dielectric
- Conductor and dielectric
- Conductor and free space

To determine the boundary conditions, we need to use Maxwell's equation:

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0$$

And

$$\oint \mathbf{D} \cdot d\mathbf{S} = Q_{\text{enc}}$$

- Decomposing the electric field intensity E into orthogonal components

$$\mathbf{E} = \mathbf{E}_t + \mathbf{E}_n$$

where and are, respectively, the tangential and normal components of E to the interface of interest

1.Dielectric – dielectric boundary conditions:

E_1 and E_2 in media 1 and 2 can be decomposed as

$$\mathbf{E}_1 = \mathbf{E}_{1t} + \mathbf{E}_{1n}$$

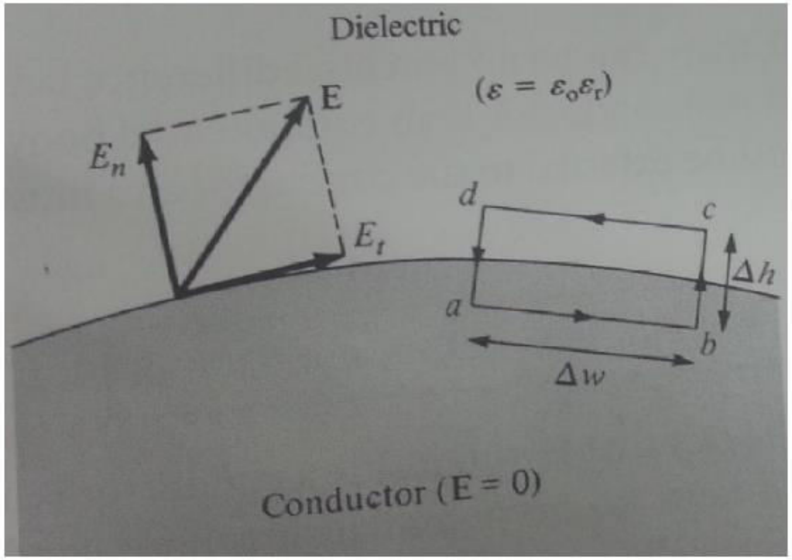
$$\mathbf{E}_2 = \mathbf{E}_{2t} + \mathbf{E}_{2n}$$

Applying Maxwell's equation to the closed path (abcda)

$$0 = E_{1t}\,\Delta w - E_{1n}\frac{\Delta h}{2} - E_{2n}\frac{\Delta h}{2} - E_{2t}\,\Delta w + E_{2n}\frac{\Delta h}{2} + E_{1n}\frac{\Delta h}{2} \qquad (1)$$

As $\Delta h -> 0$, equation (1) becomes

$$\boxed{E_{1t} = E_{2t}} \qquad (2)$$



is said to be continuous across the boundary

- Since D = = + , eq. (2) can be written as

$$\frac{D_{1t}}{\varepsilon_1} = E_{1t} = E_{2t} = \frac{D_{2t}}{\varepsilon_2}$$

Or

$$\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}$$

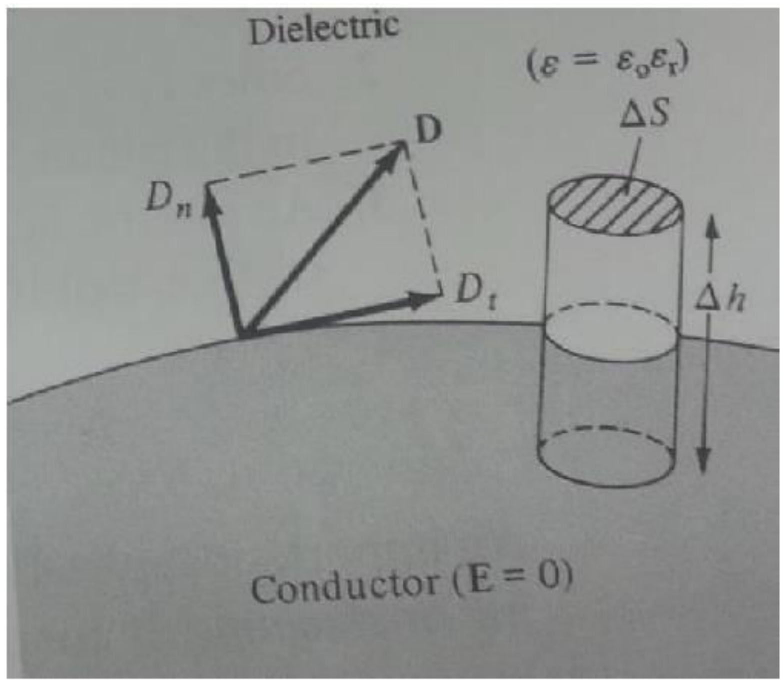
is said to be discontinuous across the interface

Applying the Gauss's law , we have

$$\oint \mathbf{D} \cdot d\mathbf{S} = Q_{\text{enc}}$$

Allowing Δh-> gives

$$\boxed{D_{1n} - D_{2n} = \rho_S}$$



If no free charges exist at the interface , so

$$\boxed{D_{1n} = D_{2n}} \qquad (1)$$

is continuous across the interface , since = ,eq. (1) can be written as

$$\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$$

The normal component of (E) is discontinuous at the boundary

Ohm's law in point form

Ohm's law in point form:

- Rather, it suffers constant collision with the atomic lattice and drifts from one atom to another.
- If the electron with mass 'm' is moving in an electric field $\overline{E}$ with an average drift velocity $\overline{u}$, according to Newton's law, the average charge in momentum of the free electron must match the applied force. Thus,

$$\frac{m\overline{u}}{T} = -e\overline{E}$$

$$or$$

$$\overline{u} = -\frac{eT}{m}\overline{E}$$

Where T is the average lime interval between collisions.

- If there are 'n' electrons per unit volume, the electronic charge density is given by,

$f_v = -ne$

- Thus, the conduction current density is,

$$\overline{J} = f_v\,\overline{u} = \frac{ne^2T}{m}.\overline{E} \begin{bmatrix} \because f_v = -ne \\ \overline{u} = -\dfrac{eT}{m}\overline{E} \end{bmatrix}$$

$$\boxed{\therefore \overline{J} = 6\overline{E}} \quad \text{----------(8)}$$

Where $6 = \frac{ne^2T}{m}$, is the conductivity of the conductor.

Maxwell's equations

Maxwell's Equations for Time-Varying Fields

Differential form	Integral form
$\nabla \times \overline{E} = -\dfrac{\partial \overline{B}}{\partial t}$	$\oint E.d\ell = -\iint \dfrac{\partial B}{\partial t}.ds$
$\nabla \times \overline{H} = J + \dfrac{\partial \overline{D}}{\partial t}$	$\oint H.d\ell = I + \iint \dfrac{\partial D}{\partial t}.ds$
$\nabla . \overline{D} = \rho$	$\oint D.ds = \iiint \rho\, dv$
$\nabla . \overline{B} = 0$	$\oint B.ds = o$
$\overline{D} = \varepsilon \overline{E}$	
$\overline{B} = \mu \overline{H}$	
$\overline{J} = \sigma \overline{E}$	

References

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- https://nptel.ac.in/courses/108/106/108106073/
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