Question Bank

Mathematics – II **2nd Semester (Civil)** **BTAM 201-18**

I.

Classify each of the following differential equations as ordinary or partial differential equations; state the order of each equation; and determine whether the equation under consideration is linear or nonlinear.

1. $\frac{dy}{dx} + x^2 y = xe^x.$

2. $\frac{d^3y}{dx^3} + 4\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 3y = \sin x.$

3. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$

4. $x^2\,dy + y^2\,dx = 0.$

5. $\frac{d^4y}{dx^4} + 3\left(\frac{d^2y}{dx^2}\right)^5 + 5y = 0.$

6. $\frac{\partial^4 u}{\partial x^2\,\partial y^2} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + u = 0.$

7. $\frac{d^2y}{dx^2} + y\sin x = 0.$

8. $\frac{d^2y}{dx^2} + x\sin y = 0.$

9. $\frac{d^6x}{dt^6} + \left(\frac{d^4x}{dt^4}\right)\left(\frac{d^3x}{dt^3}\right) + x = t.$

10. $\left(\frac{dr}{ds}\right)^3 = \sqrt{\frac{d^2r}{ds^2} + 1}.$

II.

Show that each of the functions defined in Column I is a solution of the corresponding differential equation in Column II on every interval $a < x < b$ of the x axis.

	I	**II**
(a)	$f(x) = x + 3e^{-x}$	$\frac{dy}{dx} + y = x + 1$
(b)	$f(x) = 2e^{3x} - 5e^{4x}$	$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = 0$
(c)	$f(x) = e^x + 2x^2 + 6x + 7$	$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 4x^2$
(d)	$f(x) = \frac{1}{1 + x^2}$	$(1 + x^2)\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 0$

III.

(b) Show that every function g defined by $g(x) = c_1e^{2x} + c_2xe^{2x} + c_3e^{-2x}$, where c_1, c_2, and c_3 are arbitrary constants, is a solution of the differential equation

$$\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 8y = 0.$$

(a) Show that every function f defined by $f(x) = c_1e^{4x} + c_2e^{-2x}$, where c_1 and c_2 are arbitrary constants, is a solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 8y = 0.$$

IV. Solve the following differential equations:

1. $\frac{dy}{dx} + \frac{3y}{x} = 6x^2.$

2. $x^4\frac{dy}{dx} + 2x^3y = 1.$

3. $\frac{dy}{dx} + 3y = 3x^2e^{-3x}.$

4. $\frac{dy}{dx} + 4xy = 8x.$

5. $\frac{dx}{dt} + \frac{x}{t^2} = \frac{1}{t^2}.$

6. $(u^2 + 1)\frac{dv}{du} + 4uv = 3u.$

7. $x\frac{dy}{dx} + \frac{2x + 1}{x + 1}y = x - 1.$

8. $(x^2 + x - 2)\frac{dy}{dx} + 3(x + 1)y = x - 1.$

9. $x\,dy + (xy + y - 1)\,dx = 0.$

10. $y\,dx + (xy^2 + x - y)\,dy = 0.$

11. $\frac{dr}{d\theta} + r\tan\theta = \cos\theta.$

12. $\cos\theta\,dr + (r\sin\theta - \cos^4\theta)\,d\theta = 0.$

13. $(\cos^2 x - y\cos x)\,dx - (1 + \sin x)\,dy = 0.$

14. $(y\sin 2x - \cos x)\,dx + (1 + \sin^2 x)\,dy = 0.$

15. $\frac{dy}{dx} - \frac{y}{x} = -\frac{y^2}{x}.$

16. $x\frac{dy}{dx} + y = -2x^6y^4.$

17. $dy + (4y - 8y^{-3})x\,dx = 0.$

18. $\frac{dx}{dt} + \frac{t + 1}{2t}x = \frac{t + 1}{xt}.$

V. Solve the following problems using method of variation of parameters:

1. $y'' + y = \cot x.$

2. $y'' + y = \tan^2 x.$

3. $y'' + y = \sec x.$

4. $y'' + y = \sec^3 x.$

Question No. V. continued…

5. $y'' + 4y = \sec^2 2x$.

6. $y'' + y = \tan x \sec x$.

7. $y'' + 4y' + 5y = e^{-2x} \sec x$.

8. $y'' - 2y' + 5y = e^x \tan 2x$.

9. $y'' + 6y' + 9y = \dfrac{e^{-3x}}{x^3}$.

10. $y'' - 2y' + y = xe^x \ln x$ $(x > 0)$.

11. $y'' + y = \sec x \csc x$.

12. $y'' + y = \tan^3 x$.

13. $y'' + 3y' + 2y = \dfrac{1}{1 + e^x}$.

14. $y'' + 3y' + 2y = \dfrac{1}{1 + e^{2x}}$.

15. $y'' + y = \dfrac{1}{1 + \sin x}$.

16. $y'' - 2y' + y = e^x \sin^{-1} x$.

17. $y'' + 3y' + 2y = \dfrac{e^{-x}}{x}$.

18. $y'' - 2y' + y = x \ln x$ $(x > 0)$.

VI.

Find the general solution of each of the differential equations in Exercises 1–22. In each case assume $x > 0$.

1. $x^2y'' - 3xy' + 3y = 0$.

2. $x^2y'' + xy' - 4y = 0$.

3. $4x^2y'' - 4xy' + 3y = 0$.

4. $x^2y'' - 3xy' + 4y = 0$.

5. $x^2y'' + xy' + 4y = 0$.

6. $x^2y'' - 3xy' + 13y = 0$.

7. $3x^2y'' - 4xy' + 2y = 0$.

8. $x^2y'' + xy' + 9y = 0$.

9. $9x^2y'' + 3xy' + y = 0$.

10. $x^2y'' - 5xy' + 10y = 0$.

11. $x^3y''' - 3x^2y'' + 6xy' - 6y = 0$.

12. $x^3y''' + 2x^2y'' - 10xy' - 8y = 0$.

13. $x^3y''' - x^2y'' - 6xy' + 18y = 0$.

14. $x^4y^{\text{iv}} - 4x^2y'' + 8xy' - 8y = 0$.

15. $x^2y'' - 4xy' + 6y = 4x - 6$.

16. $x^2y'' - 5xy' + 8y = 2x^3$.

17. $x^2y'' + 4xy' + 2y = 4 \ln x$.

VII.

Obtain a power series solution in powers of x of each of the initial-value problems in Exercises 1–8 by (a) the Taylor series method and (b) the method of undetermined coefficients.

1. $y' = x + y, \quad y(0) = 1.$

2. $y' = x^2 + 2y^2, \quad y(0) = 4.$

3. $y' = 1 + xy^2, \quad y(0) = 2.$

4. $y' = x^3 + y^3, \quad y(0) = 3.$

5. $y' = x + \sin y, \quad y(0) = 0.$

6. $y' = 1 + x \sin y, \quad y(0) = 0.$

7. $y' = e^x + x \cos y, \quad y(0) = 0.$

8. $y' = x^4 + y^4, \quad y(0) = 1.$

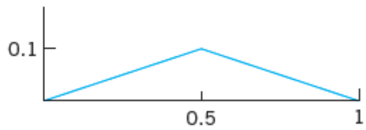
VIII. Deflection of the string:

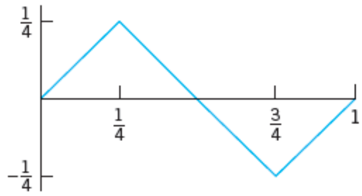
Find $u(x, t)$ for the string of length $L = 1$ and $c^2 = 1$ when the initial velocity is zero and the initial deflection with small k (say, 0.01) is as follows. Sketch or graph $u(x, t)$ as in Fig. 291 in the text.

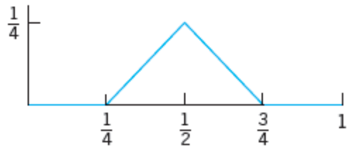
$k \sin 3\pi x$

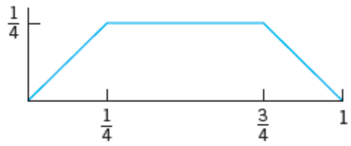
$k\,(\sin \pi x - \frac{1}{2} \sin 2\pi x)$

$kx(1 - x)$ **8.** $kx^2(1 - x)$









$2x - 4x^2$ if $0 < x < \frac{1}{2}$, $\quad 0$ if $\frac{1}{2} < x < 1$

IX. By the method of continuation combined with D'Alembert's formula solve each of the following four problems:

$$\begin{cases} u_{tt} - 9u_{xx} = 0, & x > 0, \\ u|_{t=0} = 0, & x > 0, \\ u_t|_{t=0} = \cos(x), & x > 0, \\ u|_{x=0} = 0, & t > 0. \end{cases}$$

$$\begin{cases} u_{tt} - 9u_{xx} = 0, & x > 0, \\ u|_{t=0} = 0, & x > 0, \\ u_t|_{t=0} = \sin(x), & x > 0, \\ u|_{x=0} = 0, & t > 0. \end{cases}$$

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X.

Find the temperature $u(x, t)$ in a laterally insulated copper bar 80 cm long if the initial temperature is $100 \sin (\pi x/80)$ C and the ends are kept at 0C . How long will it take for the maximum temperature in the bar to drop to 50C First guess, then calculate. *Physical data for copper:* density 8.92 g/cm^3, specific heat 0.092 cal/(g C), thermal conductivity $0.95 \text{ cal/(cm sec C)}$.

XI. Solve the problem in exercise X. when the initial temperature $100 \sin \frac{3\pi x}{80}^{\circ} C$ and the other data is as before.

XII.

Find the temperature in a laterally insulated bar of length L whose ends are kept at temperature 0, assuming that the initial temperature is

$$f(x) = \begin{cases} x & \text{if} \quad 0 < x < L/2, \\ L - x & \text{if} \quad L/2 < x < L. \end{cases}$$

(The uppermost part of Fig. 295 shows this function for the special $L = \pi$.)

XIII.

Find the temperature $u(x, t)$ in a bar of silver of length 10 cm and constant cross section of area 1 cm^2 (density 10.6 g/cm^3, thermal conductivity $1.04 \text{ cal/(cm sec C)}$, specific heat 0.056 cal/(g C) that is perfectly insulated laterally, with ends kept at temperature 0 C and initial temperature $f(x)$ C, where

$f(x) = \sin 0.1\pi x$

$f(x) = 4 - 0.8|x - 5|$

$f(x) = x(10 - x)$

XIV.

Heat flow in a plate. The faces of the thin square plate in Fig. 297 with side $a = 24$ are perfectly insulated. The upper side is kept at 25C and the other sides are kept at 0 C. Find the steady-state temperature $u(x, y)$ in the plate.

XV.

Find the steady-state temperature in the plate in Prob. 21 if the lower side is kept at $_0$ C, the upper side at $_1$ C, and the other sides are kept at 0 C. *int:* Split into two problems in which the boundary temperature is 0 on three sides for each problem.

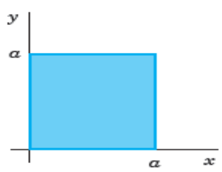
XVI.

CAS PROJECT. Isotherms. Find the steady-state solutions (temperatures) in the square plate in Fig. 297 with $a = 2$ satisfying the following boundary conditions. Graph isotherms.

(a) $u = 80 \sin \pi x$ on the upper side, 0 on the others.

(b) $u = 0$ on the vertical sides, assuming that the other sides are perfectly insulated.

(c) Boundary conditions of your choice (such that the solution is not identically zero).



References:

1. Advanced Engineering Mathematics by Erwin Ereyszig (Tenth Edition)
2. Partial Differential Equations by Victor Ivrii (University of Toronto)
3. Introduction to Ordinary Differential Equations by Shepley L. Ross (4^{th} Edition)