Theory of machines

Chapter 1
Basic concepts of machines

Machine

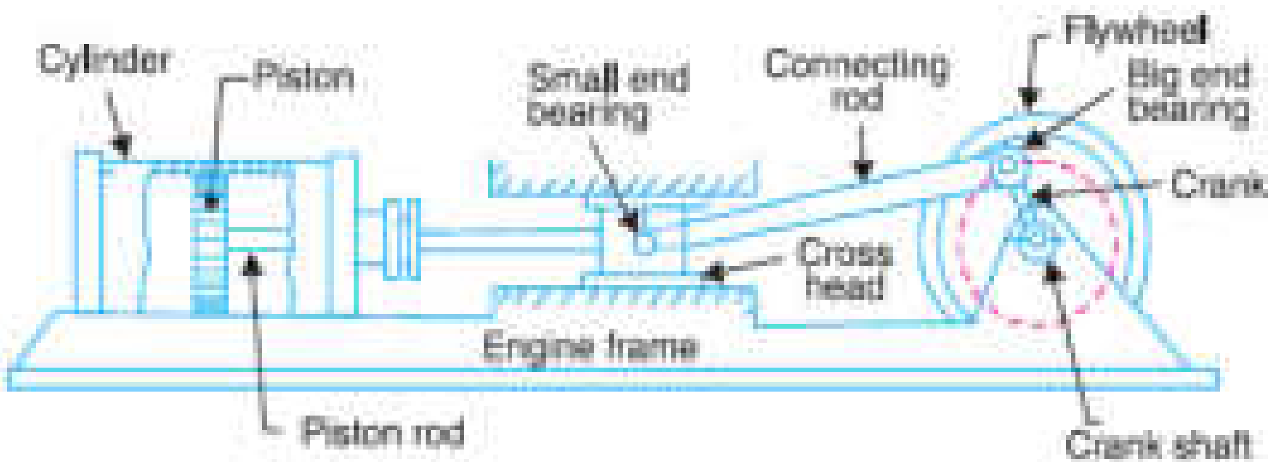
- receives energy and transforms it into useful form
- consists of a number of parts and bodies

Kinematic link

- part of a machine, which moves relative to some other part
- may consist of several parts, which are rigidly fastened together

Basic concepts of machines

Kinematic link examples



Basic concepts of machines

Types of Links

1. ***Rigid link***

   does not undergo any deformation while transmitting motion. Strictly speaking, rigid links do not exist.

2. ***Flexible link.***

   is partly deformed in a manner not to affect the transmission of motion. For example, belts, ropes, chains and wires (transmit tensile forces only).

3. ***Fluid link.***

   formed by having a fluid in a receptacle and the

   motion is transmitted through the fluid by pressure or compression only, as in the case of hydraulic presses, jacks and brakes.

Basic concepts of machines

Structure

is an assemblage of a number of resistant bodies (known as members) having no relative motion between them and meant for carrying loads having straining action. A railway bridge, a roof truss, machine frames etc., are the examples of a structure.

Basic concepts of machines

Difference Between a Machine and a Structure

1. The parts of a machine move relative to one another, whereas the members of a structure
   do not move relative to one another.
2. A machine transforms the available energy into some useful work, whereas in a structure
   no energy is transformed into useful work.
3. The links of a machine may transmit both power and motion, while the members of a
   structure transmit forces only.

Basic concepts of machines

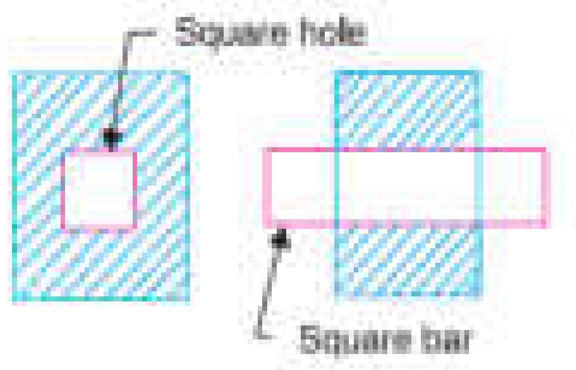
Kinematic pair.

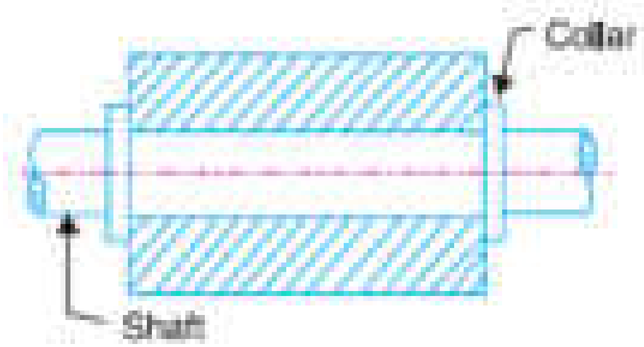
- two links or elements of a machine, in contact with each other.
- relative motion between links is completely or successfully constrained (*i.e. in a definite* direction),

Basic concepts of machines

Types of Constrained Motions

1. ***Completely constrained motion.*** motion between a pair is limited to a definite direction irrespective of the direction of force applied





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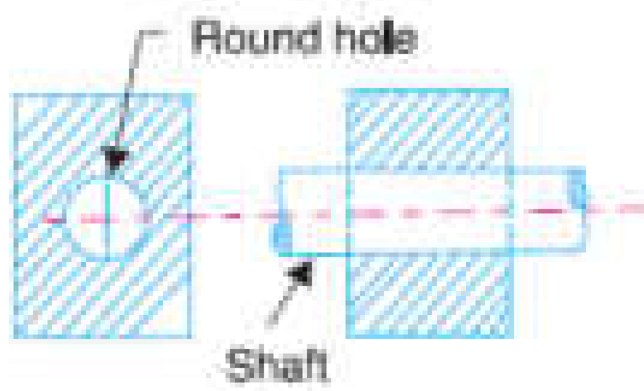
2. ***Incompletely constrained motion.*** motion between a pair can take place in more than one direction.



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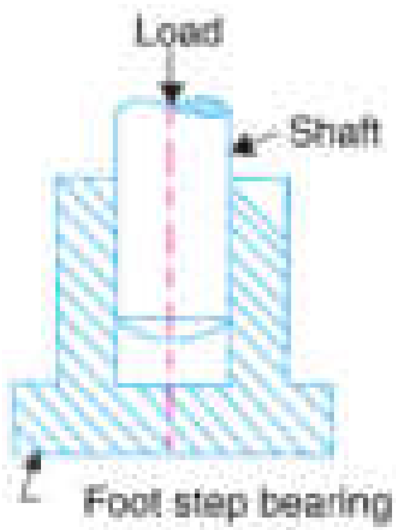
3. ***Successfully constrained motion.***

motion between the elements, forming a pair , is such that the constrained motion is not completed by itself, but by some other means,



Basic concepts of machines

- **Classification of Kinematic Pairs**

1. According to the type of relative motion between the elements.

(a) Sliding pair.
(b) Turning pair.
(c) Rolling pair.
(d) Screw pair.
(e) Spherical pair.

Basic concepts of machines

- Classification of Kinematic Pairs

2. According to the type of contact between the elements
(*a*) *Lower pair.* two elements of a pair have a surface contact when relative motion takes place and the surface of one element slides over the surface of the other.sliding pairs, turning pairs and screw pairs form lower pairs.
(*b*) *Higher pair.* two elements of a pair have a line or point contact when relative motion takes place and the motion between the two elements is partly turning and partly. A pair of friction discs, toothed gearing, belt and rope drives, ball and roller bearings and cam and follower are the examples of higher pairs.

Basic concepts of machines

- Classification of Kinematic Pairs

3. **According to the type of closure. :**

(a)Self closed pair.

two elements of a pair are connected together mechanically in a way that only required kind of relative motion occurs. The lower pairs are self closed pair.

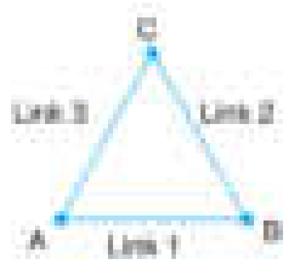
(b) Force - closed pair.

two elements of a pair are not connected mechanically but are kept in contact by the action of external forces. The cam and follower is an example of force closed pair, as it is kept in contact by the forces exerted by spring and gravity.

Basic concepts of machines

- Kinematic Chain

When the kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion (i.e. completely or successfully constrained motion)



Basic concepts of machines

- Types of Joints in a Chain

Binary joint.

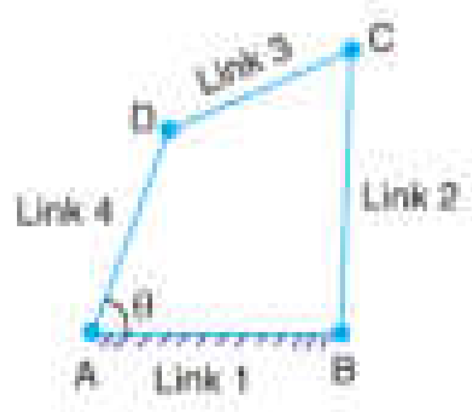
Ternary joint

Quaternary joint

Basic concepts of machines

- Mechanism(http://www.mechanisms101.com/fourbar01.shtml)

  When one of the links of a kinematic chain is fixed, the chain is known as ***mechanism***



Basic concepts of machines

- **Types of Kinematic Chains**

1. Four bar chain or quadric cyclic chain,
2. Single slider crank chain, and
3. Double slider crank chain.

Basic concepts of machines

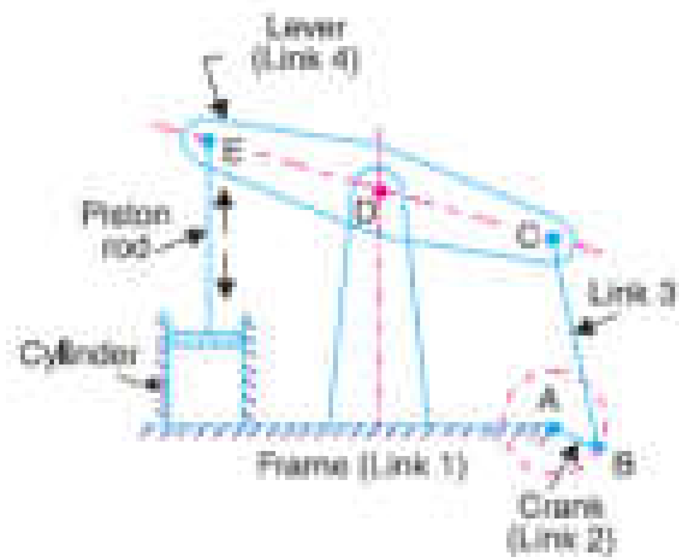
- **Inversion of Mechanism**

  method of obtaining different mechanisms by fixing different links in a kinematic chain, is known as ***inversion of the mechanism.***

Basic concepts of machines

- **Inversions of Four Bar Chain**

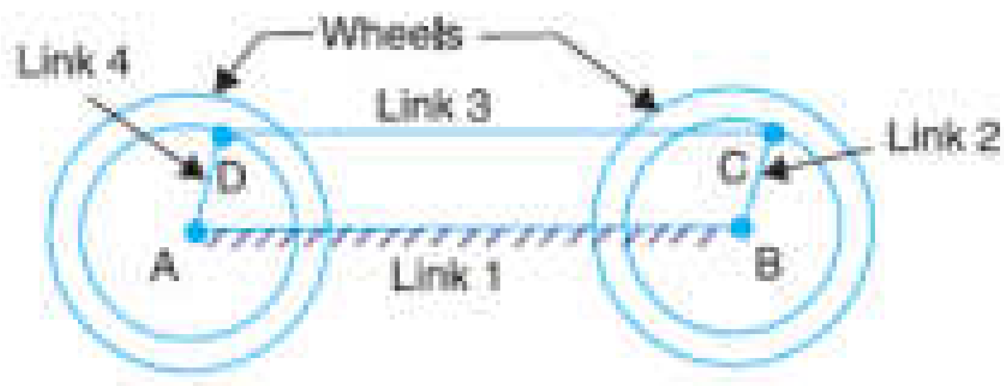
1. ***Beam engine (crank and lever mechanism).***



Basic concepts of machines

- Inversions of Four Bar Chain

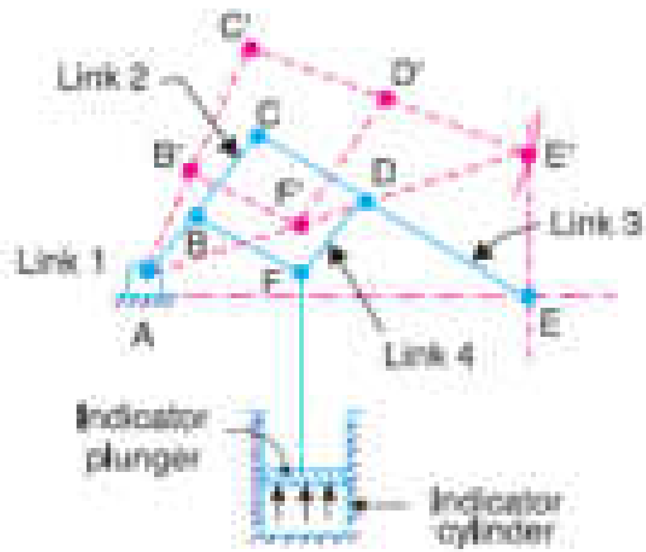
2. Coupling rod of a locomotive (Double crank mechanism).



Basic concepts of machines

- Inversions of Four Bar Chain

3. ***Watt's indicator mechanism (Double lever mechanism).***



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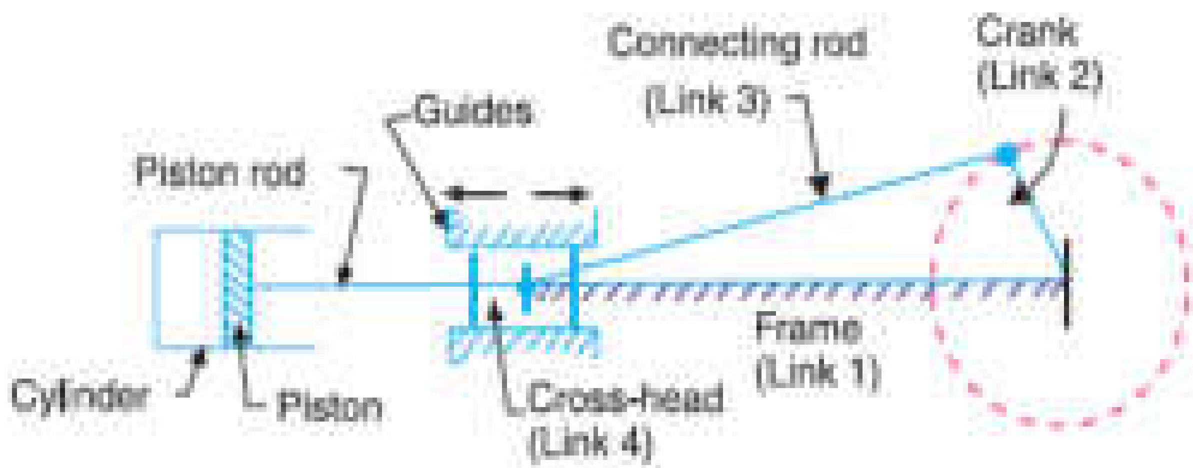
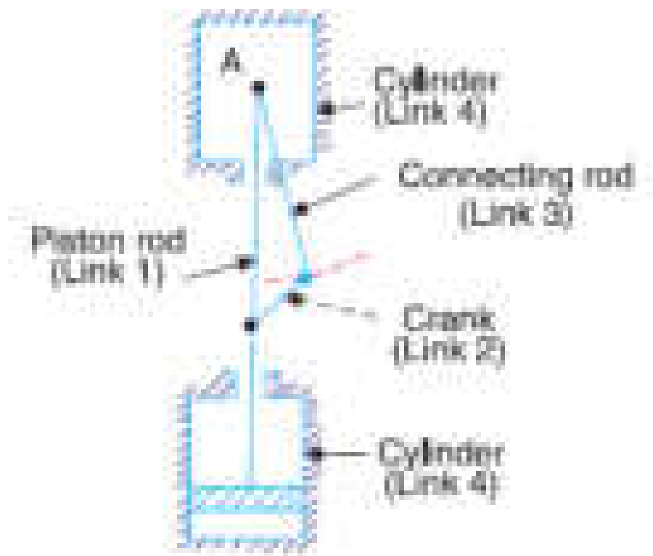
- **Single Slider Crank Chain**



Basic concepts of machines

- Inversions of Single Slider Crank Chain

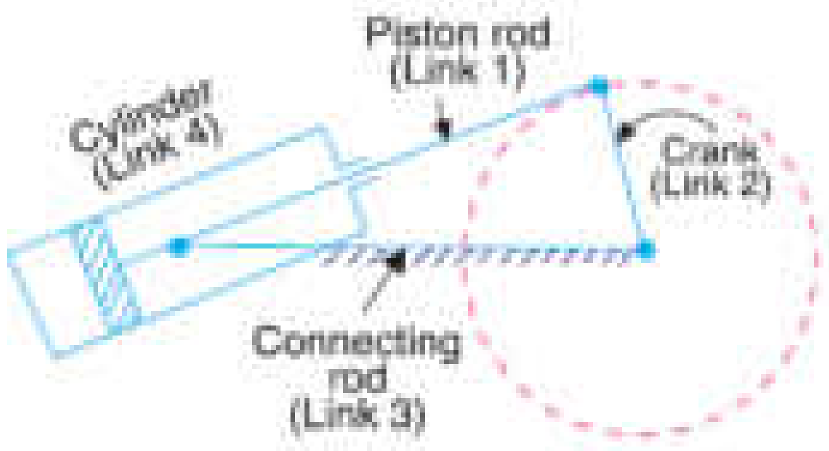
1. *Pendulum pump or Bull engine*



Basic concepts of machines

- Inversions of Single Slider Crank Chain

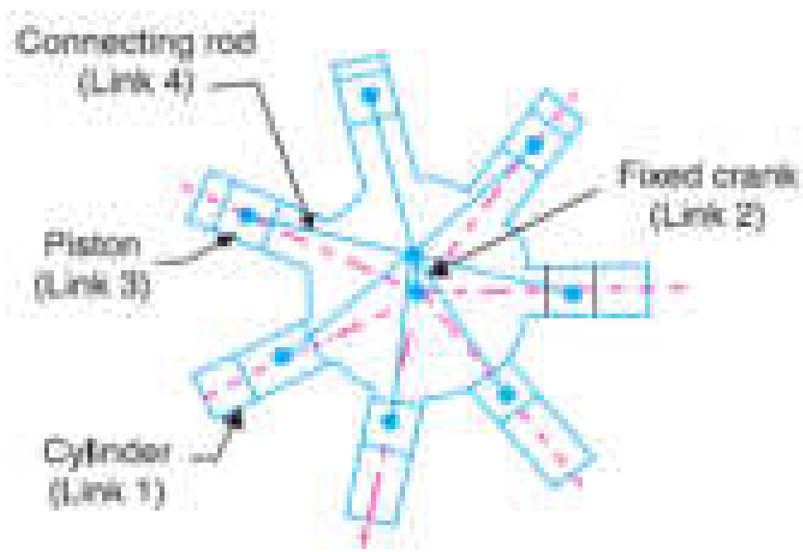
2. Oscillating cylinder engine.



Basic concepts of machines

- Inversions of Single Slider Crank Chain

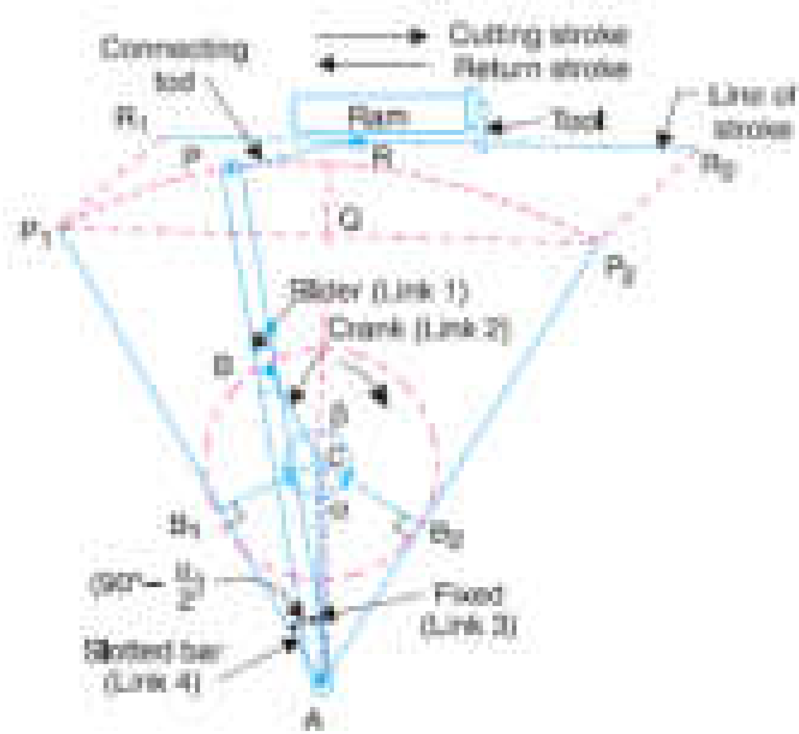
3. Rotary internal combustion engine or Gnome engine.



Basic concepts of machines

- Inversions of Single Slider Crank Chain

4. Crank and slotted lever quick return motion mechanism.



INERTIA FORCE

The inertia force is an imaginary force, which when acts upon a rigid body, brings it in an equilibrium position.

Inertia force = – Accelerating force

$$= -\,m.a$$

m = *Mass of the body, and*

a = *Linear acceleration of the centre* of gravity of the body.

D-Alembert's Principle

- The resultant force acting on a body together with the reversed effective force (or inertia force), are in equilibrium.

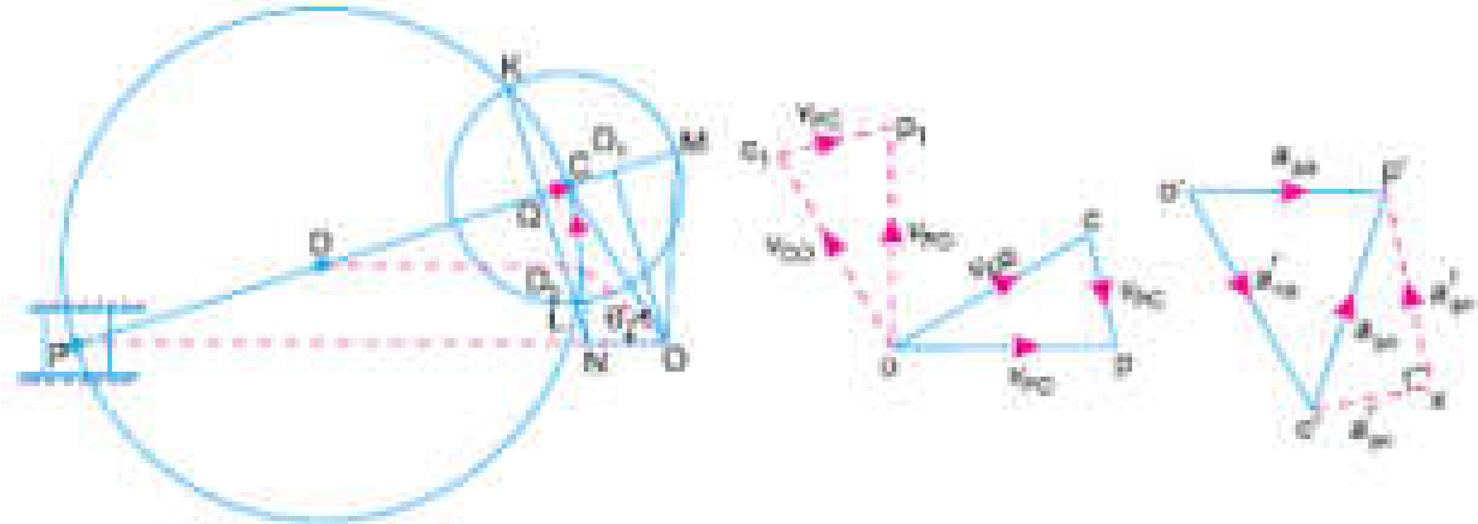
$$F + Fi = 0$$

F = *resultant external force*

Fi = *inertia force*

- is used to reduce a dynamic problem into an equivalent static problem

Klien's Construction

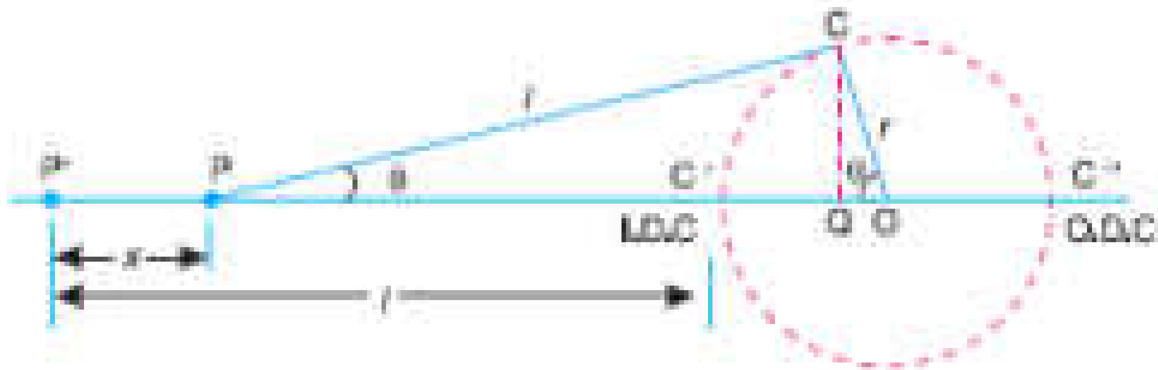


(a) Klien's acceleration diagram. (b) Velocity diagram. (c) Acceleration diagram.

$$\frac{oc_1}{OC} = \frac{op_1}{OM} = \frac{c_1p_1}{CM} = \omega \text{ (a constant)}$$

$$\frac{o'c'}{OC} = \frac{c'x}{CQ} = \frac{xp'}{QN} = \frac{o'p'}{NO} = \omega^2 \text{ (a constant)}$$

Analytical Method for Velocity and Acceleration of the Piston



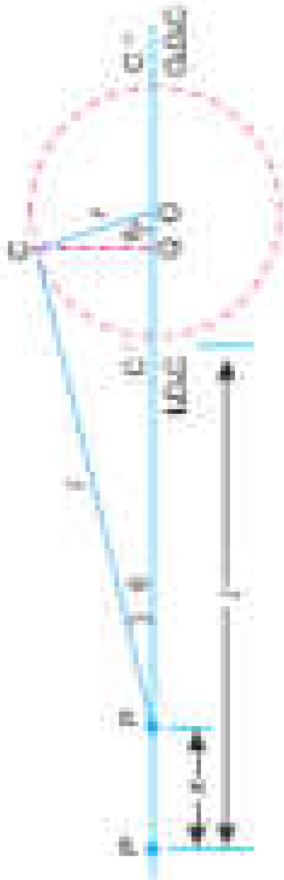
l = Length of connecting rod between the centres,

r = Radius of crank or crank pin circle,

φ = Inclination of connecting rod to the line of stroke PO, and

n = Ratio of length of connecting rod to the radius of crank = l/r.

Velocity of the piston



$$s = P'P = OP' - OP = (P'C' + C'O) - (PQ + QO)$$

$$= (l + r) - (l \cos \phi + r \cos \theta) \qquad \ldots \left(\because PQ = l \cos \phi, \text{ and } QO = r \cos \theta \right)$$

$$= r(1 - \cos \theta) + l(1 - \cos \phi) = r\left[(1 - \cos \theta) + \frac{l}{r}(1 - \cos \phi)\right]$$

$$= r[(1 - \cos \theta) + n(1 - \cos \phi)] \qquad \ldots(i)$$

From triangles CPQ and CQO,

$$CQ = l \sin \phi = r \sin \theta \quad \text{or} \quad l/r = \sin \theta / \sin \phi$$

$$\therefore \quad n = \sin \theta / \sin \phi \quad \text{or} \quad \sin \phi = \sin \theta / n \qquad \ldots(ii)$$

We know that, $\cos \phi = (1 - \sin^2 \phi)^{\frac{1}{2}} = \left(1 - \frac{\sin^2 \theta}{n^2}\right)^{\frac{1}{2}}$

Expanding the above expression by binomial theorem, we get

$$\cos \phi = 1 - \frac{1}{2} \times \frac{\sin^2 \theta}{n^2} + \ldots \qquad \ldots(\text{Neglecting higher terms})$$

$$1 - \cos \phi = \frac{\sin^2 \theta}{2n^2} \qquad \ldots(iii)$$

Velocity of the piston

Substituting the value of $(1 - \cos\phi)$ in equation (i), we have

$$x = r\left[(1-\cos\theta) + n \times \frac{\sin^2\theta}{2n^2}\right] = r\left[(1-\cos\theta) + \frac{\sin^2\theta}{2n}\right] \quad \text{...(iv)}$$

Differentiating equation (iv) with respect to θ,

$$\frac{dx}{d\theta} = r\left[\sin\theta + \frac{1}{2n} \times 2\sin\theta\cos\theta\right] = r\left(\sin\theta + \frac{\sin 2\theta}{2n}\right) \quad \text{...(v)}$$

$(\because 2\sin\theta\cos\theta = \sin 2\theta)$

∴ Velocity of P with respect to O or velocity of the piston P,

$$v_{PO} = v_P = \frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt} = \frac{dx}{d\theta} \times \omega$$

...(∵ Ratio of change of angular velocity $= d\theta/dt = \omega$)

Substituting the value of $dx/d\theta$ from equation (v), we have

$$v_{PO} = v_P = \omega r\left(\sin\theta + \frac{\sin 2\theta}{2n}\right) \quad \text{...(vi)}$$

We know that by Klien's construction,

$$v_P = \omega \times OM$$

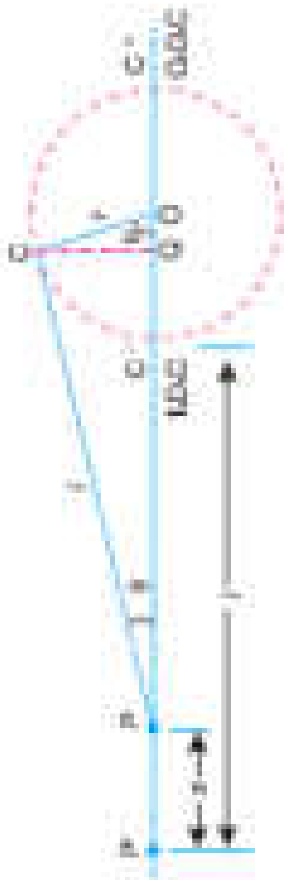
Comparing this equation with equation (vi), we find that

$$OM = r\left(\sin\theta + \frac{\sin 2\theta}{2n}\right)$$

Acceleration of the piston

Since the acceleration is the rate of change of velocity, therefore acceleration of the piston P,

$$a_P = \frac{dv_P}{dt} = \frac{dv_P}{d\theta} \times \frac{d\theta}{dt} = \frac{dv_P}{d\theta} \times \omega$$



Differentiating equation (iv) with respect to θ,

$$\frac{dv_P}{d\theta} = \omega r \left[\cos\theta + \frac{\cos 2\theta \times 2}{2n}\right] = \omega r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

Substituting the value of $\frac{dv_P}{d\theta}$ in the above equation, we have

$$a_P = \omega r \left[\cos\theta + \frac{\cos 2\theta}{n}\right] \times \omega = \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

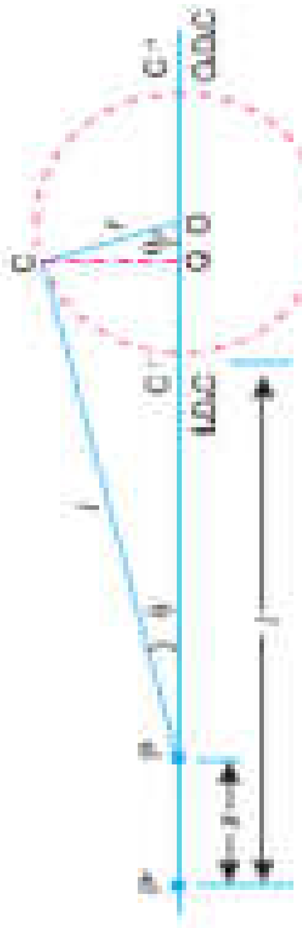
1. When crank is at the inner dead centre (I.D.C.), then $\theta = 0°$.

$$\therefore \quad a_P = \omega^2 r \left[\cos 0° + \frac{\cos 0°}{n}\right] = \omega^2 r \left(1 + \frac{1}{n}\right)$$

2. When the crank is at the outer dead centre (O.D.C.), then $\theta = 180°$.

$$\therefore \quad a_P = \omega^2 r \left[\cos 180° + \frac{\cos 2 \times 180°}{n}\right] = \omega^2 r \left(-1 + \frac{1}{n}\right)$$

Angular Velocity and Acceleration of the Connecting Rod



$$CQ = l \sin\phi = r \sin\theta$$

$$\therefore \quad \sin\phi = \frac{r}{l} \times \sin\theta = \frac{\sin\theta}{n}$$

Differentiating both sides with respect to time t,

$$\cos\phi \times \frac{d\phi}{dt} = \frac{\cos\theta}{n} \times \frac{d\theta}{dt} = \frac{\cos\theta}{n} \times \omega$$

$$\omega_{PC} = \frac{d\phi}{dt} = \frac{\cos\theta}{n} \times \frac{\omega}{\cos\phi} = \frac{\omega}{n} \times \frac{\cos\theta}{\cos\phi}$$

$$\omega_{PC} = \frac{\omega}{n} \times \frac{\cos\theta}{\left(1 - \frac{\sin^2\theta}{n^2}\right)^{\frac{1}{2}}} = \frac{\omega}{n} \times \frac{\cos\theta}{\frac{1}{n}(n^2 - \sin^2\theta)^{1/2}}$$

$$= \frac{\omega \cos\theta}{(n^2 - \sin^2\theta)^{1/2}}$$

Angular Velocity and acceleration of the Connecting Rod

Angular acceleration of the connecting rod PC.

$$\alpha_{PC} = \text{Angular acceleration of } P \text{ with respect to } C = \frac{d(\omega_{PC})}{dt}$$

We know that

$$\frac{d(\omega_{PC})}{dt} = \frac{d(\omega_{PC})}{d\theta} \times \frac{d\theta}{dt} = \frac{d(\omega_{PC})}{d\theta} \times \omega \quad \text{...(ii)}$$

$$\ldots [\because d\theta / dt = \omega]$$

Now differentiating equation (i), we get

$$\frac{d(\omega_{PC})}{d\theta} = \frac{d}{d\theta}\left[\frac{\omega \cos\theta}{(n^2 - \sin^2\theta)^{1/2}}\right]$$

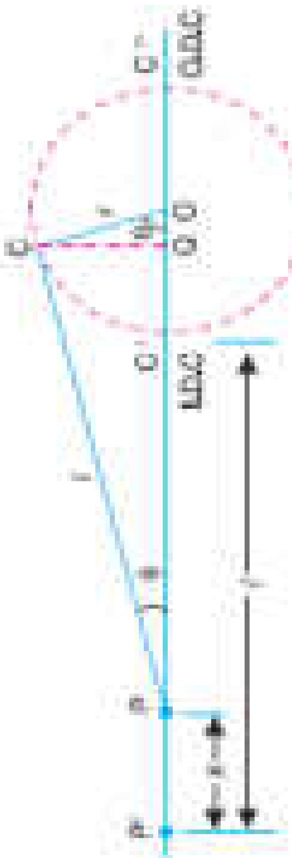
$$= \omega\left[\frac{(n^2 - \sin^2\theta)^{1/2}(-\sin\theta) - [(\cos\theta) \times \frac{1}{2}(n^2 - \sin^2\theta)^{-1/2} \times -2\sin\theta\cos\theta]}{n^2 - \sin^2\theta}\right]$$

$$= \omega\left[\frac{(n^2 - \sin^2\theta)^{1/2}(-\sin\theta) + (n^2 - \sin^2\theta)^{-1/2}\sin\theta\cos^2\theta}{n^2 - \sin^2\theta}\right]$$

$$= -\omega\sin\theta\left[\frac{(n^2 - \sin^2\theta)^{1/2} - (n^2 - \sin^2\theta)^{-1/2}\cos^2\theta}{n^2 - \sin^2\theta}\right]$$

$$= -\omega\sin\theta\left[\frac{(n^2 - \sin^2\theta) - \cos^2\theta}{(n^2 - \sin^2\theta)^{3/2}}\right] \quad \text{...[Dividing and multiplying by } (n^2 - \sin^2\theta)^{1/2}]$$

Angular Velocity and acceleration of the Connecting Rod



$$= \frac{-\omega \sin\theta}{(n^2 - \sin^2\theta)^{3/2}}\left[n^2 - (\sin^2\theta + \cos^2\theta)\right] = \frac{-\omega \sin\theta\,(n^2-1)}{(n^2 - \sin^2\theta)^{3/2}}$$

...($\because \sin^2\theta + \cos^2\theta = 1$)

$$\therefore \alpha_{PC} = \frac{d(\omega_{PC})}{d\theta} \times \omega = \frac{-\omega^2 \sin\theta\,(n^2-1)}{(n^2 - \sin^2\theta)^{3/2}}$$

...[From equation (ii)] ...(iii)

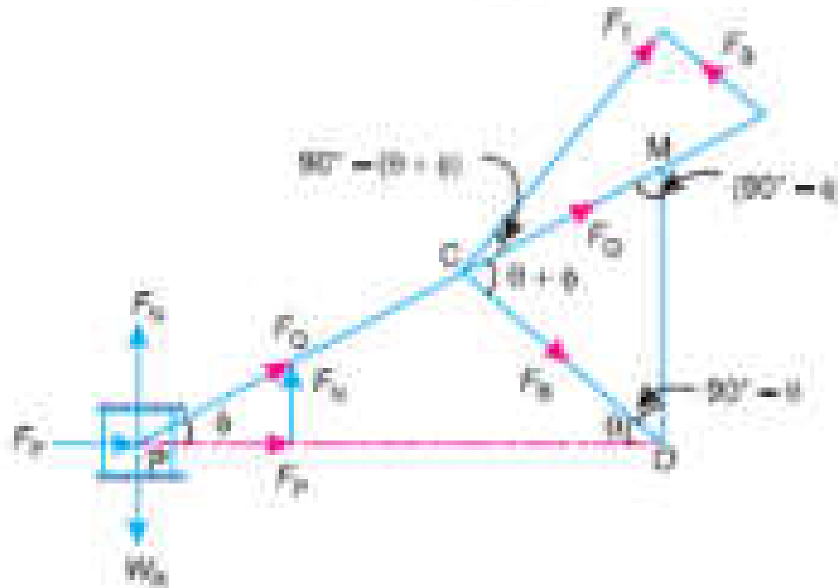
Notes: 1. Since $\sin^2\theta$ is small as compared to n^2, therefore it may be neglected. Thus, equations (i) and (iii) are reduced to

$$\omega_{PC} = \frac{\omega\cos\theta}{n}, \text{ and } \alpha_{PC} = \frac{-\omega^2 \sin\theta\,(n^2-1)}{n^3}$$

2. Also in equation (iii), unity is small as compared to n^2, hence the term unity may be neglected.

$$\therefore \quad \alpha_{PC} = \frac{-\omega^2 \sin\theta}{n}$$

Forces on the Reciprocating Parts of an Engine



m_R = Mass of the reciprocating parts, *e.g.* piston, crosshead pin or gadgeon pin etc., in kg, and

W_R = Weight of the reciprocating parts in newtons = $m_R.g$

Piston effort

- The net force acting on the piston or crosshead pin, along the line of stroke & is denoted by *Fp*

Piston effort.

$$F_P = \text{Net load on the piston} \mp \text{Inertia force}$$

$$= F_L \mp F_I \quad \text{...(Neglecting frictional resistance)}$$

$$= F_L \mp F_I - R_F \quad \text{...(Considering frictional resistance)}$$

R_F = Frictional resistance

$$F_L = p_1 A_1 - p_2 A_2 = p_1 A_1 - p_2 (A_1 - a)$$

p_1, A_1 = Pressure and cross-sectional area on the back end side of the piston,

p_2, A_2 = Pressure and cross-sectional area on the crank end side of the piston,

a = Cross-sectional area of the piston rod.

Force acting along the connecting rod

It is denoted by F_Q

$$F_Q = \frac{F_P}{\cos\phi}$$

$$\cos\phi = \sqrt{1 - \frac{\sin^2\theta}{n^2}}$$

$$F_Q = \frac{F_P}{\sqrt{1 - \frac{\sin^2\theta}{n^2}}}$$

Thrust on the sides of the cylinder walls

- It is denoted by F_N

$$F_N = F_Q \sin\phi = \frac{F_P}{\cos\phi} \times \sin\phi = F_P \tan\phi$$

Crank-pin effort and thrust on crank shaft bearings

- The component of F_Q perpendicular to the crank is known as crank-pin effort and it is denoted by F_T .The component of F_Q along the crank produces a thrust on the crank shaft bearings and it is denoted by $F_{B.}$

$$F_T = F_Q \sin(\theta + \phi) = \frac{F_P}{\cos\phi} \times \sin(\theta + \phi)$$

$$F_B = F_Q \cos(\theta + \phi) = \frac{F_P}{\cos\phi} \times \cos(\theta + \phi)$$

Crank effort

- The product of the crankpin effort (F_T) and the crank pin radius (r) is known as crank effort.

$$T = F_T \times r = \frac{F_p \sin(\theta + \phi)}{\cos\phi} \times r$$

$$= \frac{F_p (\sin\theta \cos\phi + \cos\theta \sin\phi)}{\cos\phi} \times r$$

$$= F_p \left(\sin\theta + \cos\theta \times \frac{\sin\phi}{\cos\phi} \right) \times r$$

$$= F_p (\sin\theta + \cos\theta \tan\phi) \times r$$

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{\sin\theta}{n} \times \frac{n}{\sqrt{n^2 - \sin^2\theta}} = \frac{\sin\theta}{\sqrt{n^2 - \sin^2\theta}}$$

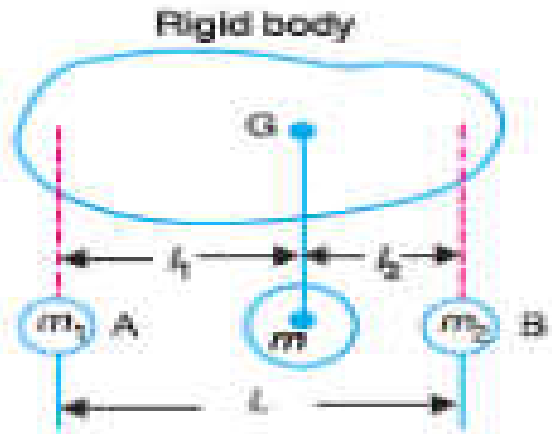
$$T = F_p \left(\sin\theta + \frac{\cos\theta \sin\theta}{\sqrt{n^2 - \sin^2\theta}} \right) \times r$$

$$= F_p \times r \left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}} \right)$$

Equivalent Dynamical System

- it is usually convenient to replace the rigid body by two masses placed at a fixed distance apart, in such a way that,
  - the sum of their masses is equal to the total mass of the body ;
  - the centre of gravity of the two masses coincides with that of the body ; and
  - the sum of mass moment of inertia of the masses about their centre of gravity is equal to the mass moment of inertia of the body.

Equivalent Dynamical System



$$m_1 + m_2 = m$$

$$m_1 . l_1 = m_2 . l_2$$

$$m_1 (l_1)^2 + m_2 (l_2)^2 = m (k_G)^2$$

Equivalent Dynamical System

$$m_1 = \frac{l_2 . m}{l_1 + l_2}$$

$$m_2 = \frac{l_1 . m}{l_1 + l_2}$$

$$l_1 . l_2 = (k_G)^2$$

Clutches

Basic purpose

- Used to disengage engine from input shaft
- Clutch is a mechanism which enables the rotary motion of one shaft to be transmitted, when desired, to a second shaft the axis of which is coincident with that of the first.

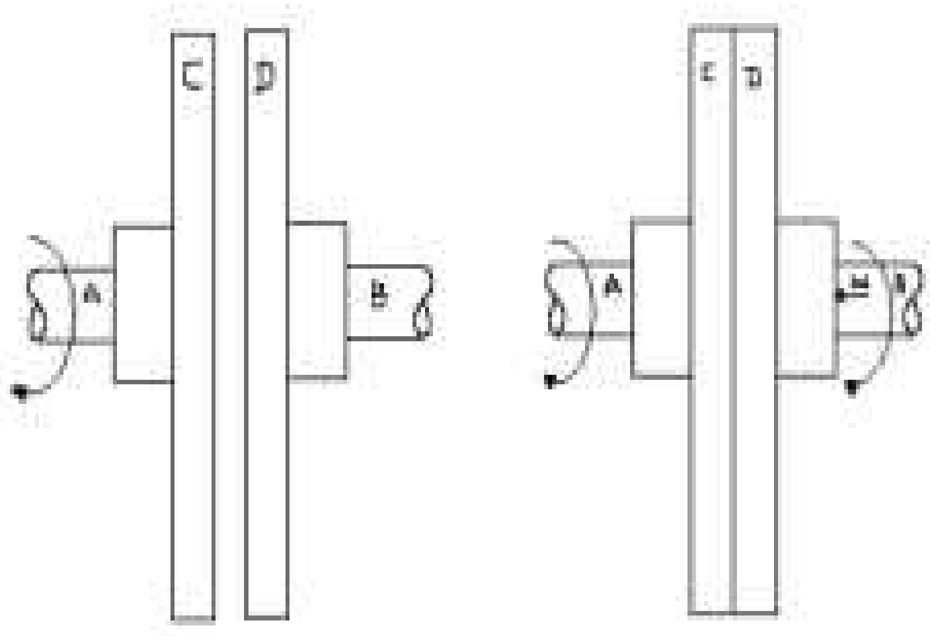
Requirements of clutch

- Torque transmission
- Gradual engagement
- Heat dissipation
- Dynamic balancing
- Vibration damping
- Size
- Inertia
- Clutch free pedal play

Types of clutches

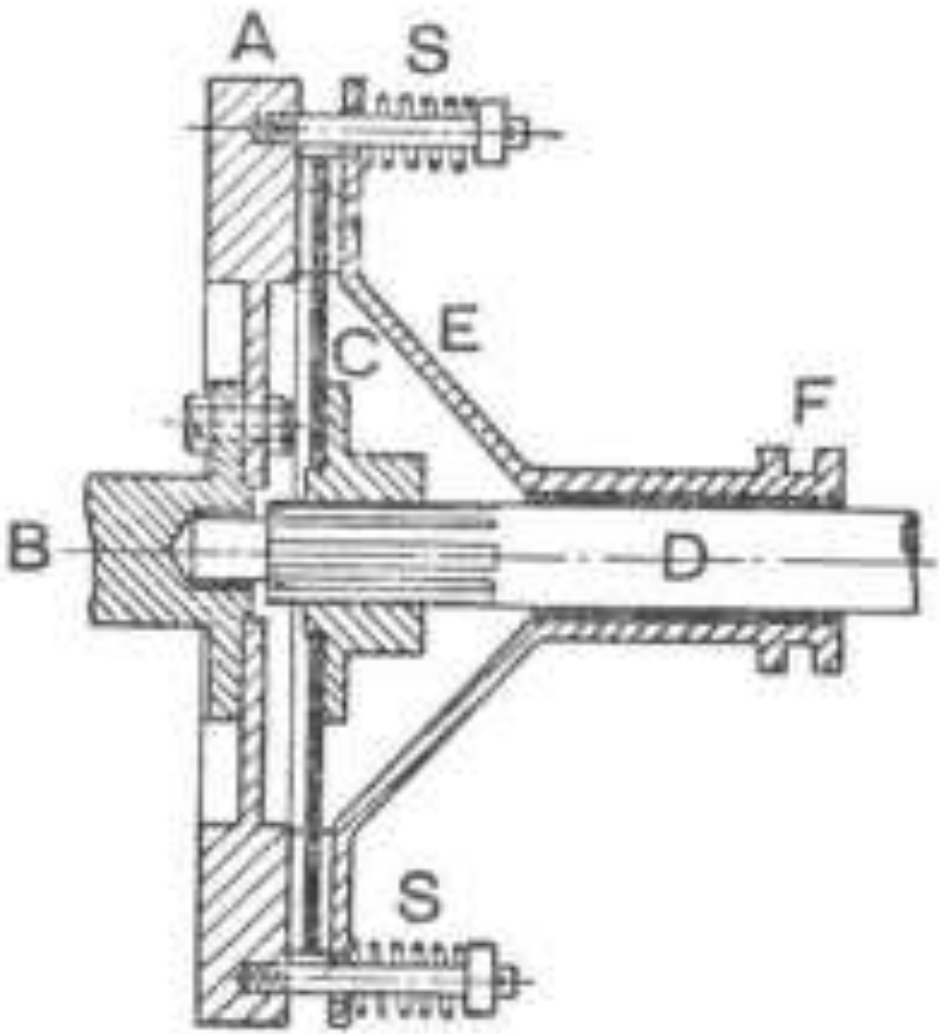
- Friction clutches
  - → Cone clutch.
  - → Single plate clutch.
  - → Multiplate clutch.
  - → Centrifugal clutch.
- Fluid flywheel.

Principle of friction clutch



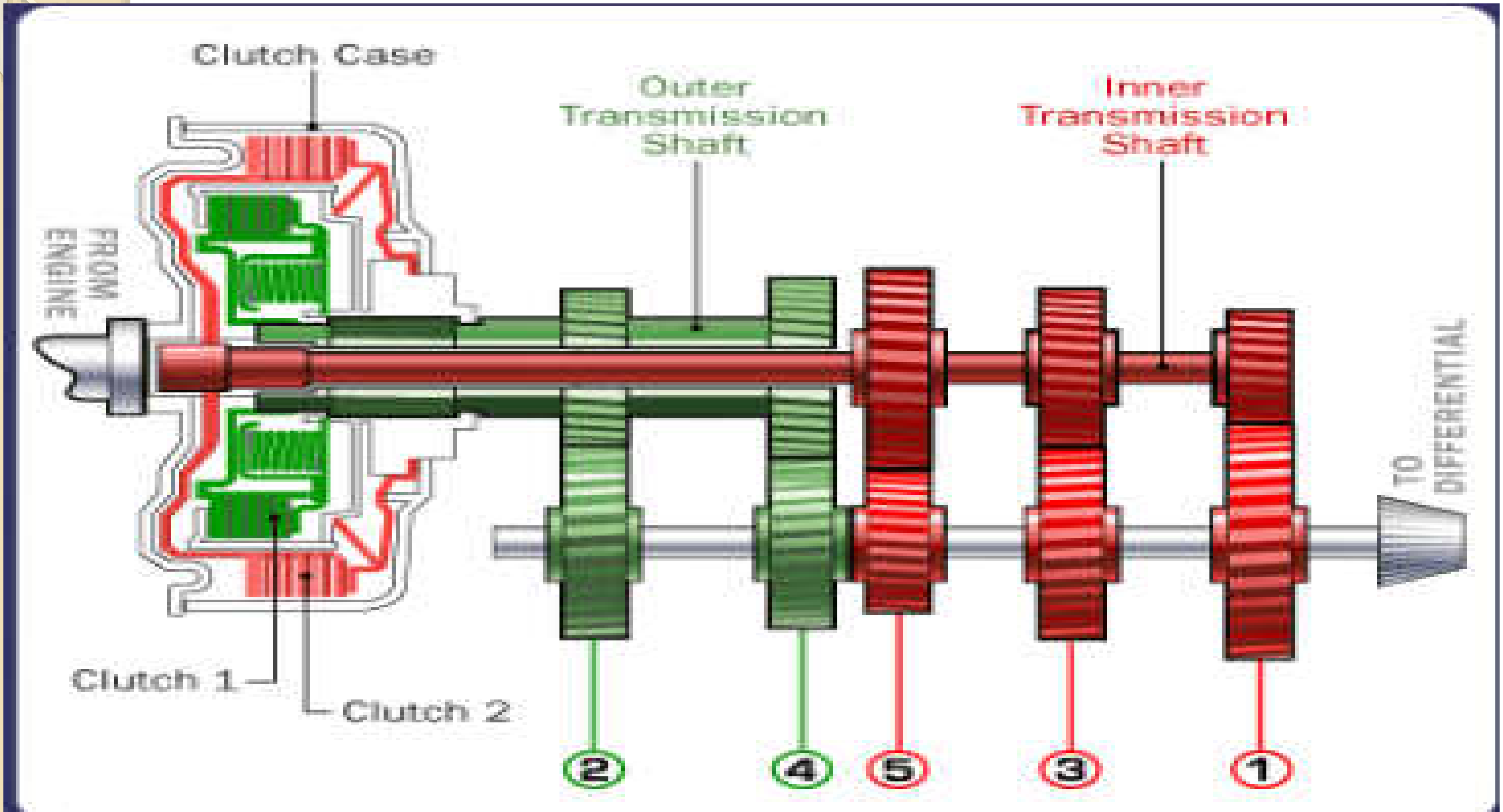
Cone clutch

Single plate clutch

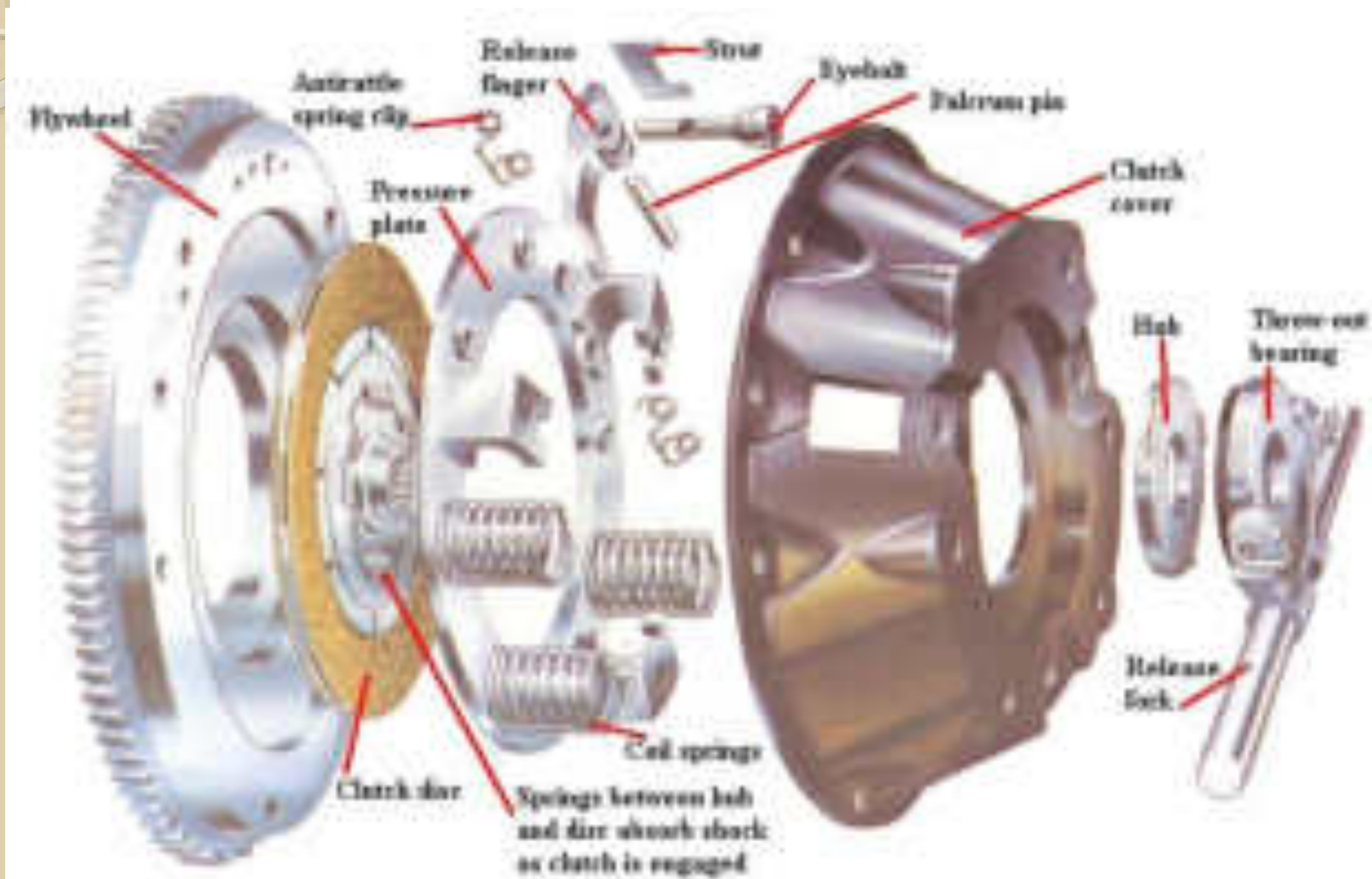


A-Fly wheel
B-Engine shaft
C-clutch plate
D- clutch shaft
E- pressure plate
F-bearing
S- clutch spring

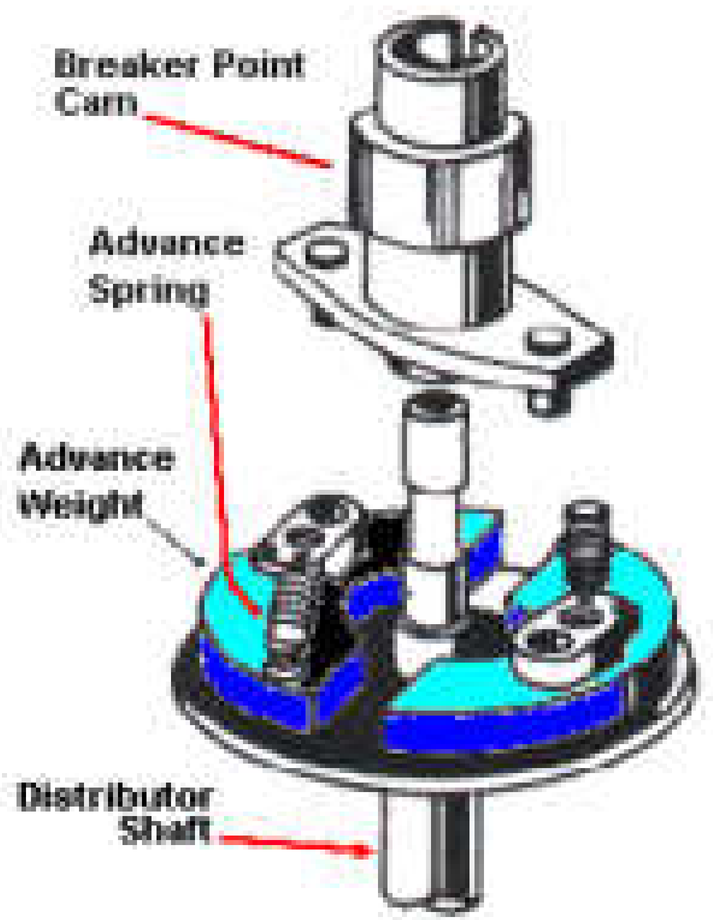
Multiplate clutch



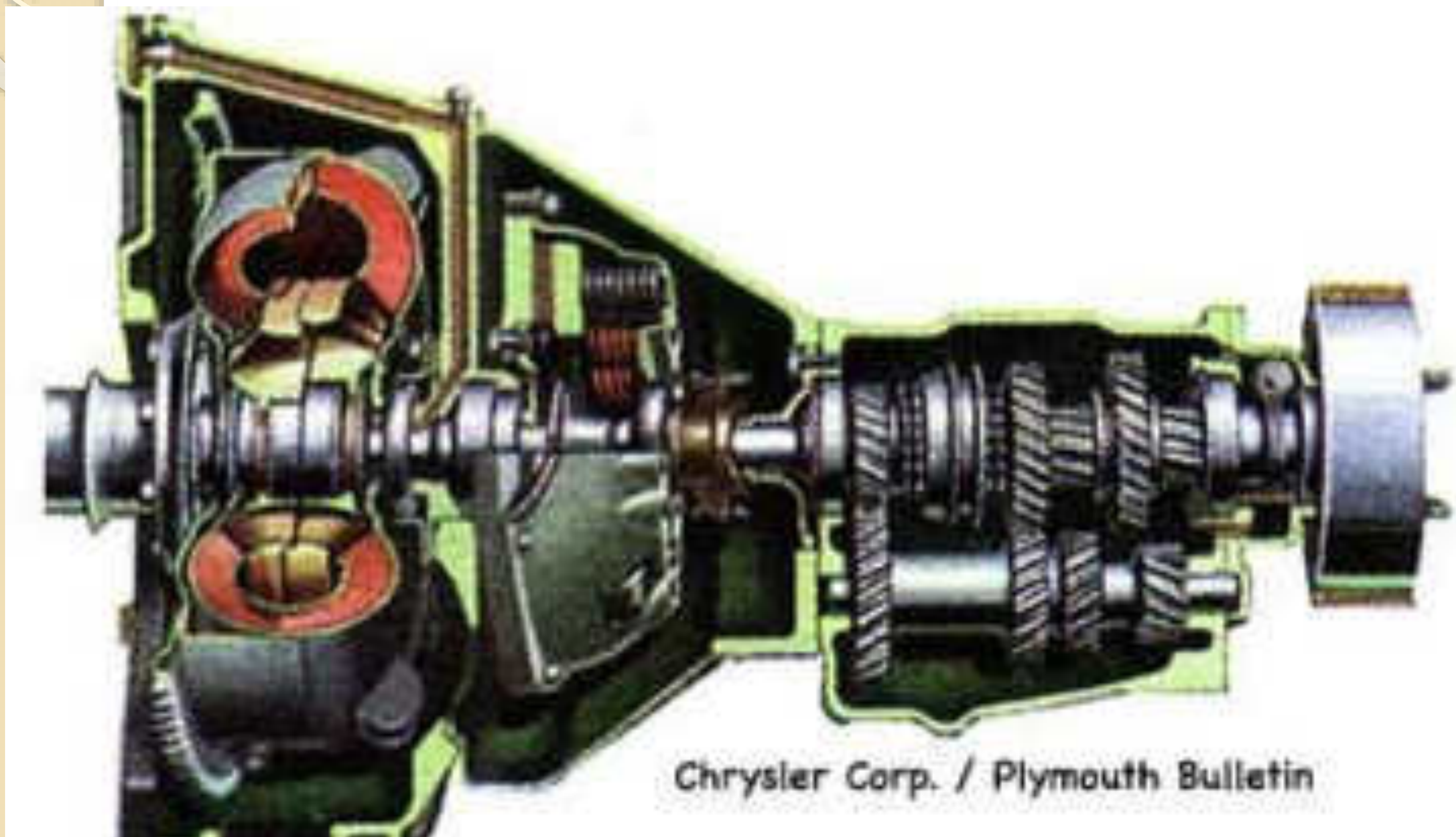
Semi-centrifugal clutch



Centrifugal clutch



Fluid flywheel



Clutch components

- Clutch plate.
- Clutch facing.
- Pressure plate.
- Springs.
- Bearing.

Clutch plate

Clutch facing

Pressure plate



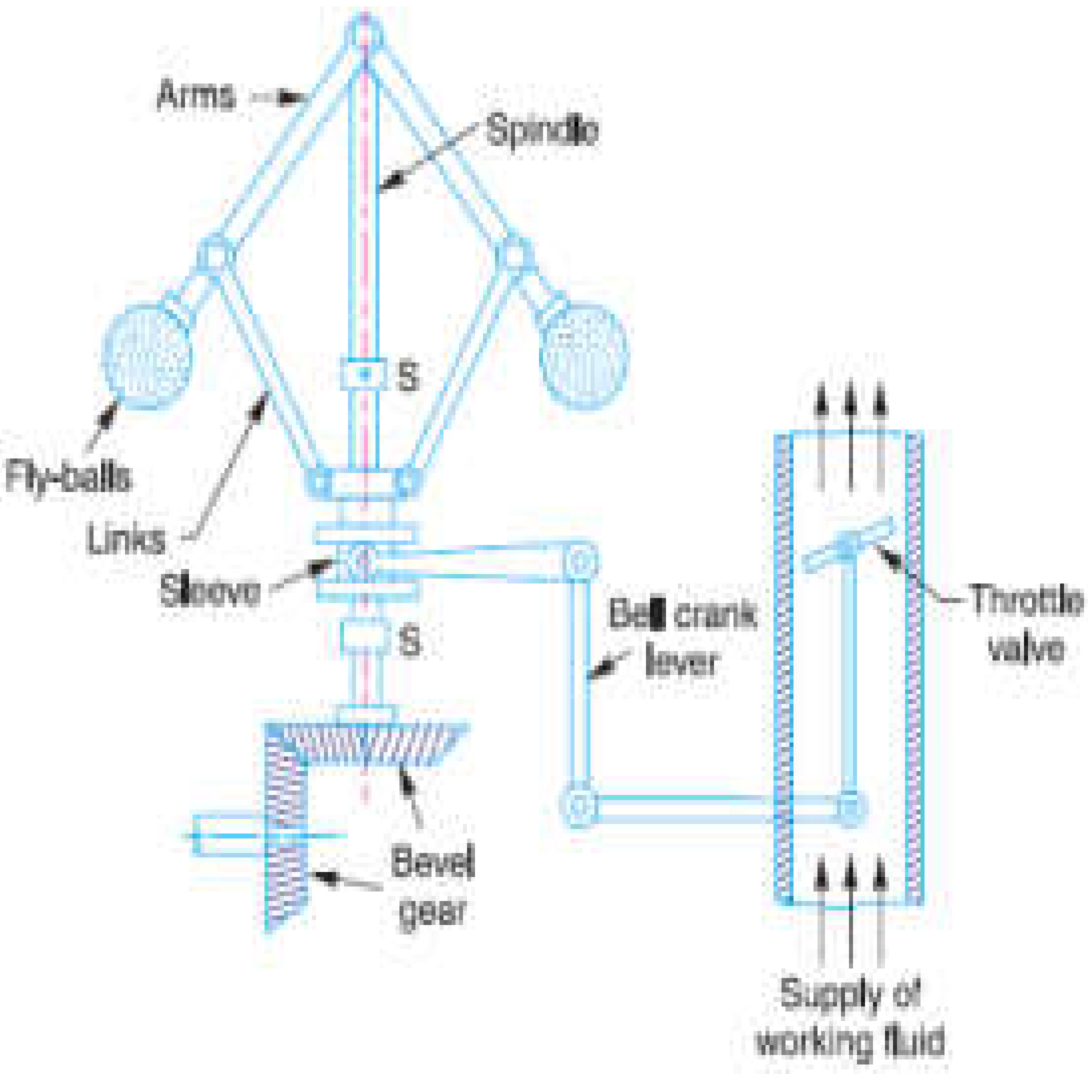

Thank you.

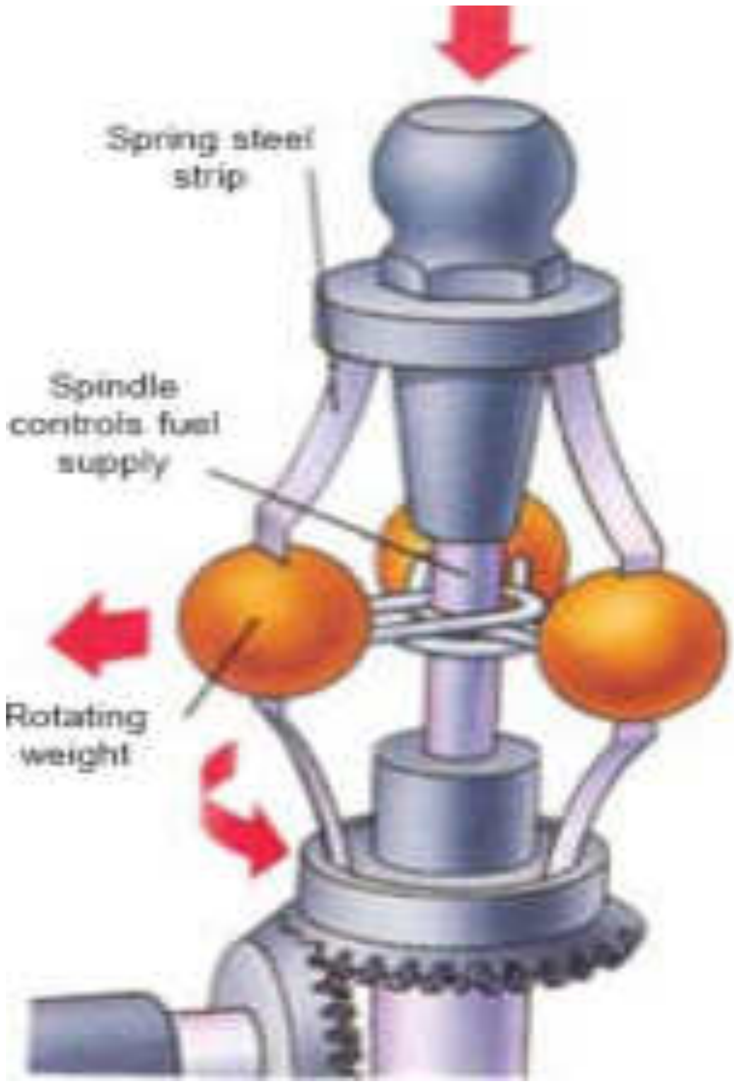
Governors

The function of a governor is to regulate the mean speed of an engine, when there are variations in the load e.g. when the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of working fluid. On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required.

The governor automatically controls the supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limits.

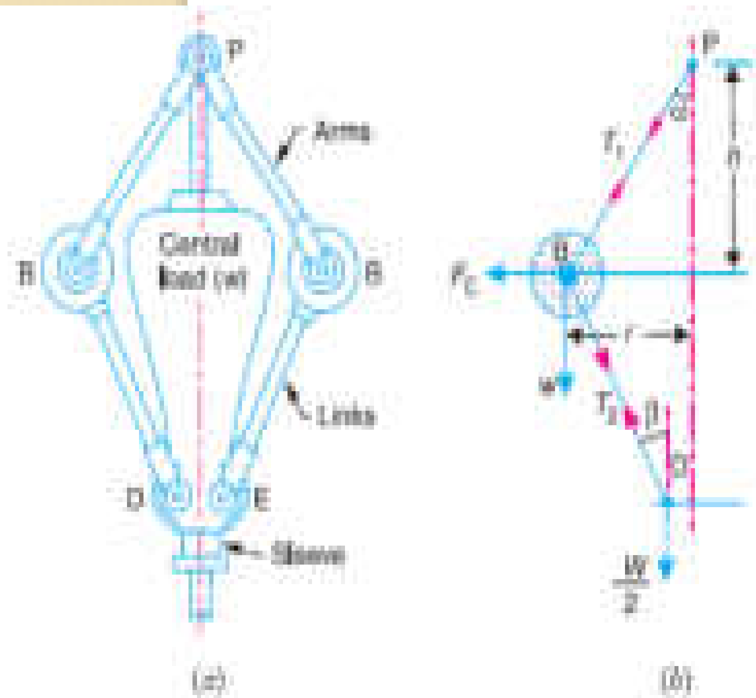
Centrifugal Governors





Porter Governor

The Porter governor is a *modification of a Watt's governor*, with central load attached to the sleeve as shown in Fig. The load moves up and down the central spindle. This additional downward force increases the speed of revolution required to enable the balls to rise to any predetermined level.



Let m = Mass of each ball in kg,

w = Wt. of each ball in newtons = *m.g*,

M = Mass of the central load in kg,

W = Weight of the central load in newtons = *M.g*,

r = Radius of rotation in metres,

h = Height of governor in metres ,

N = Speed of the balls in r.p.m .,

ω = Angular speed of the balls in rad/s = 2 π*N*/60 rad/s,

= CF_C trifugal force acting on the ball in newtons = *m.ω.r*,

T1 = Force in the arm in newtons,

T2 = Force in the link in newtons,

α = Angle of inclination of the arm (or upper link) to the vertical, and

β = Angle of inclination of the link (or lower link) to the vertical.

Method of resolution of forces

Considering the equilibrium of the forces acting at *D*, we have

$$T_2 \cos\beta = \frac{W}{2} = \frac{M.g}{2}$$

$$T_2 = \frac{M.g}{2\cos\beta} \qquad \ldots(i)$$

Again, considering the equilibrium of the forces acting on *B*. The point *B* is in equilibrium under the action of the following forces, as shown in Fig.(*b*).

(*i*) The weight of ball ($w = m.g$),
(*ii*) The centrifugal force (F_C),
(*iii*) The tension in the arm ($T1$), and
(*iv*) The tension in the link ($T2$).

Resolving the forces vertically,

$$T_1 \cos\alpha = T_2 \cos\beta + w = \frac{M.g}{2} + m.g \qquad \ldots(ii)$$

$$\ldots\left(\because\ T_2 \cos\beta = \frac{M.g}{2}\right)$$

*Resolving the forces **horizontally**,*

$$T_1 \sin\alpha + T_2 \sin\beta = F_C$$

$$T_1 \sin\alpha + \frac{M.g}{2\cos\beta} \times \sin\beta = F_C \qquad \ldots \left(\because\ T_2 = \frac{M.g}{2\cos\beta}\right)$$

$$T_1 \sin\alpha + \frac{M.g}{2} \times \tan\beta = F_C$$

$$T_1 \sin\alpha = F_C - \frac{M.g}{2} \times \tan\beta \qquad \ldots (iii)$$

Dividing equation *(iii)* by equation *(ii)*,

$$\frac{T_1 \sin\alpha}{T_1 \cos\alpha} = \frac{F_C - \frac{M.g}{2} \times \tan\beta}{\frac{M.g}{2} + m.g} \quad \text{or} \quad \frac{M.g}{2} + m.g = \frac{F_C}{\tan\alpha} - \frac{M.g}{2} \times \frac{\tan\beta}{\tan\alpha}$$

Substituting $\dfrac{\tan\beta}{\tan\alpha} = q$, and $\tan\alpha = \dfrac{r}{h}$, we have

$$\frac{M.g}{2} + m.g = m.\omega^2.r \times \frac{h}{r} - \frac{M.g}{2} \times q \qquad \ldots (\because F_C = m.\omega^2 r)$$

or

$$m.\omega^2.h = m.g + \frac{M.g}{2}(1+q)$$

$\therefore$

$$h = \left[m.g + \frac{M.g}{2}(1+q)\right]\frac{1}{m.\omega^2} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{\omega^2} \qquad \ldots (iv)$$

or

$$\left(\frac{2\pi N}{60}\right)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}$$

$\therefore$

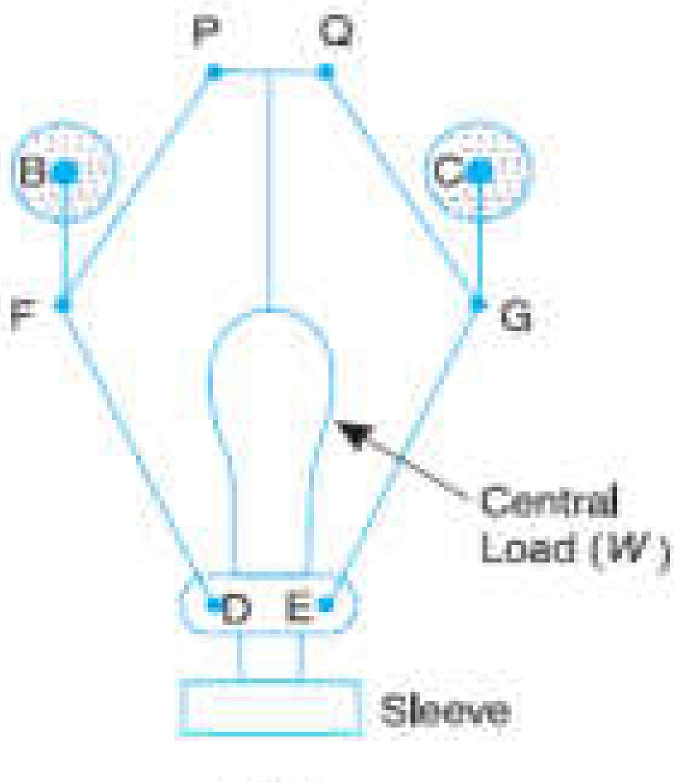
$$N^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}\left(\frac{60}{2\pi}\right)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h} \qquad \ldots (v)$$

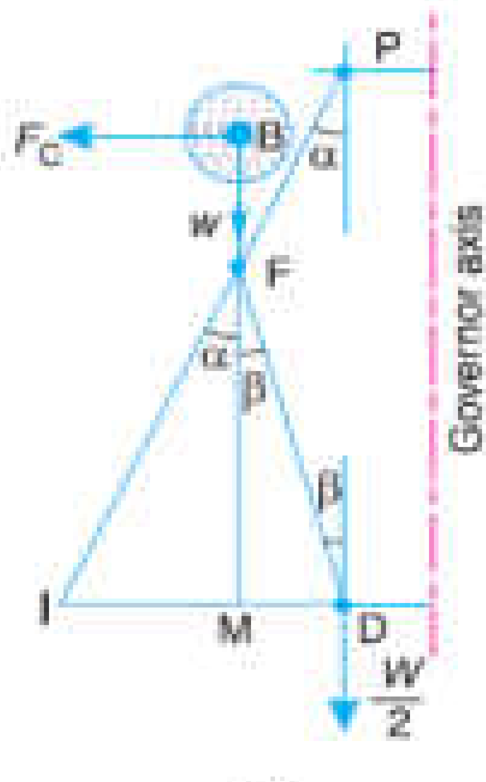
$\ldots$ (Taking $g = 9.81$ m/s^2)

Proell Governor

The Proell governor has the balls fixed at *B* and *C* to the extension of the links *DF* and *EG*, as shown in Fig.(*a*). The arms *FP* and *GQ* are pivoted at *P* and Q respectively.

Consider the equilibrium of the forces on one-half of the governor as shown in Fig.(*b*). The instantaneous centre (*I*) lies on the intersection of the line *PF* produced and the line from *D* drawn perpendicular to the spindle axis. The prependicular *BM* is drawn on *ID*.





Taking moments about *I*, using same notations as discussed in Porter governor,

$$F_C \times BM = w \times IM + \frac{W}{2} \times ID = m.g \times IM + \frac{M.g}{2} \times ID \qquad \ldots (i)$$

$$\therefore \quad F_C = m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM + MD}{BM}\right) \qquad \ldots (\because ID = IM + MD)$$

Multiplying and dividing by FM, we have

$$F_C = \frac{FM}{BM}\left[m.g \times \frac{IM}{FM} + \frac{M.g}{2}\left(\frac{IM}{FM} + \frac{MD}{FM}\right)\right]$$

$$= \frac{FM}{BM}\left[m.g \times \tan\alpha + \frac{M.g}{2}(\tan\alpha + \tan\beta)\right]$$

$$= \frac{FM}{BM} \times \tan\alpha\left[m.g + \frac{M.g}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right)\right]$$

We know that $F_C = m.\omega^2 r$; $\tan\alpha = \frac{r}{h}$ and $q = \frac{\tan\beta}{\tan\alpha}$

$$\therefore \quad m.\omega^2.r = \frac{FM}{BM} \times \frac{r}{h}\left[m.g + \frac{M.g}{2}(1+q)\right]$$

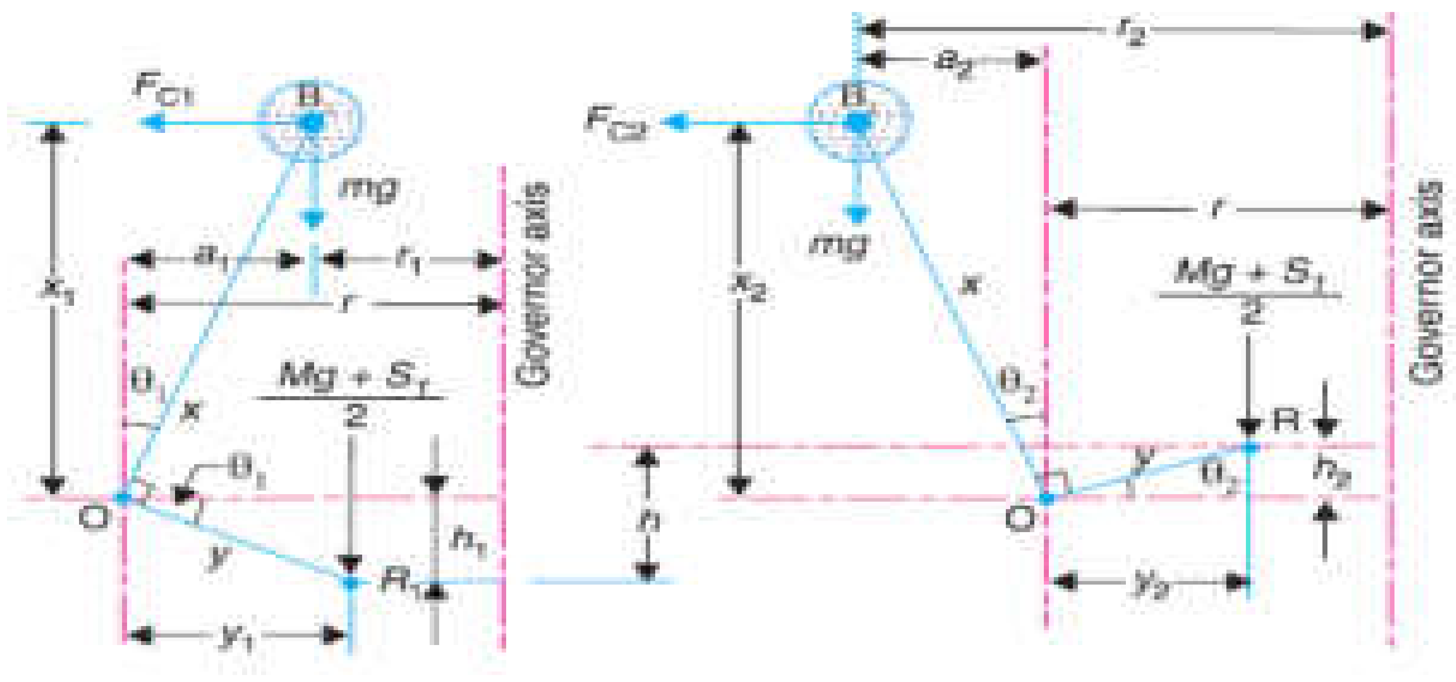
$$\omega^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{g}{h}$$

Substituting $\omega = 2\pi N/60$, and $g = 9.81 \text{ m/s}^2$, we get

$$N^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{895}{h}$$

*FC*1 = Centrifugal force at ω1 in newtons = $m(\omega_1)^2 r_1$
*FC*2 = Centrifugal force at ω2 in newtons = $m(\omega_2)^2 r_2$
s = Stiffness of the spring or force required to compress spring by one mm,
x = Length of the vertical or ball arm of the lever in metres,
y = Length of the horizontal or sleeve arm of the lever in metres, and
r = Distance of fulcrum O from the governor axis or the radius of rotation when the governor is in mid-position, in metres.

Consider the forces acting at one bell crank lever. The *minimum and maximum* position is shown in Fig. Let *h* be the compression of the spring when the radius of rotation changes from *r*1 to *r*2.



(*a*) Minimum position.   (*b*) Maximum position.

Sensitiveness of Governors

It is defined as the *ratio of the difference between the maximum and minimum equilibrium speeds* *to the mean equilibrium speed.*

Let N_1 = Minimum equilibrium speed,

N_2 = Maximum equilibrium speed, and

$$N = \text{Mean equilibrium speed} = \frac{N_1 + N_2}{2}.$$

$\therefore$ Sensitiveness of the governor

$$= \frac{N_2 - N_1}{N} = \frac{2(N_2 - N_1)}{N_1 + N_2}$$

$$= \frac{2(\omega_2 - \omega_1)}{\omega_1 + \omega_2} \quad \text{. . . (In terms of angular speeds)}$$

Hunting

A governor is said to be *hunt* if the speed of the engine fluctuates continuously above and below the mean speed. This is caused by a too sensitive governor which changes the fuel supply by a large amount when a small change in the speed of rotation takes place. *For example*, when the load on the engine increases, the engine speed decreases and, if the governor is very sensitive, the governor sleeve immediately falls to its lowest position.

This will result in the opening of the control valve wide which will supply the fuel to the engine in excess of its requirement so that the engine speed rapidly increases again and the governor sleeve rises to its highest position. Due to this movement of the sleeve, the control valve will cut off the fuel supply to the engine and thus the engine speed begins to fall once again. This cycle is repeated indefinitely.

Such a governor may admit either the *maximum or the minimum* amount of fuel. The effect of this will be to cause wide fluctuations in the engine speed or in other words, the engine will hunt.

INVERSION OF MECHANISM

Definition:-

we can obtain as many mechanisms as the number of links in a kinematic chain by fixing, in turn, different links in a kinematic chain. This method of obtaining different mechanisms by fixing different links in a kinematic chain, is known as ***inversion of the mechanism.***

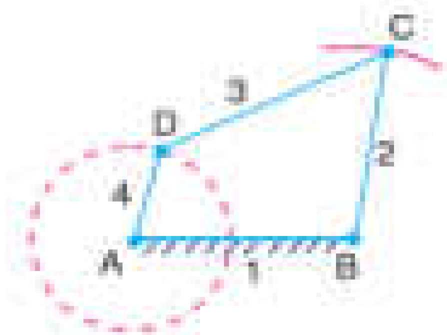
Types of Kinematic Chain

1. **Four bar chain or quadric cyclic chain.**
(4 turning pair)

2. **Single slider crank chain.**
(3 turning pair + 1 sliding pair)

3. **Double slider crank chain.**
(2 turning pair + 2 sliding pair)

Four bar chain



- It consists of four links, each of them forms a turning pair at *A, B, C and D.*
- According to **Grashof 's law for a four bar mechanism,** the sum of the shortest and longest link lengths should not be greater than the sum of the remaining two link lengths if there is continuous relative motion between the two links.
- *AD (link 4) is a* ***crank***
- link *BC (link 2) known* as ***lever or rocker or follower***
- link *CD (link 3) which connects the crank and lever is called* ***connecting rod or coupler.***
- fixed link *AB (link 1) is known as* ***frame of the mechanism.***

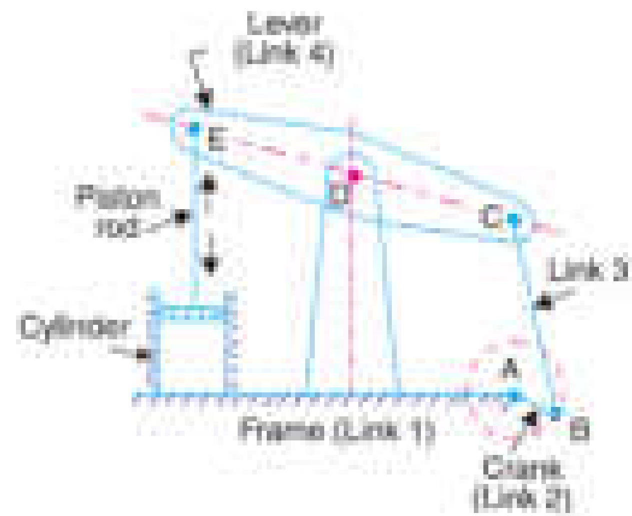
When the crank (link 4) is the driver, the mechanism is transforming rotary motion into oscillating motion.

Inversions of Four Bar Chain

- Beam engine (crank and lever mechanism).
- Coupling rod of a locomotive (Double crank mechanism).
- Watt's indicator mechanism (Double lever mechanism).

Beam engine (crank and lever mechanism).

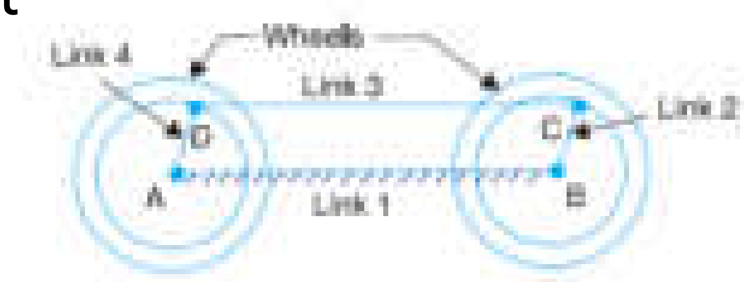
- It consists of four links
- when the crank rotates about the fixed centre *A*, *the lever oscillates about* a fixed centre *D*
- The end *E of the lever CDE is* connected to a piston rod which reciprocates due to the rotation of the crank.



In other words, the purpose of this mechanism is to convert rotary motion into reciprocating motion.

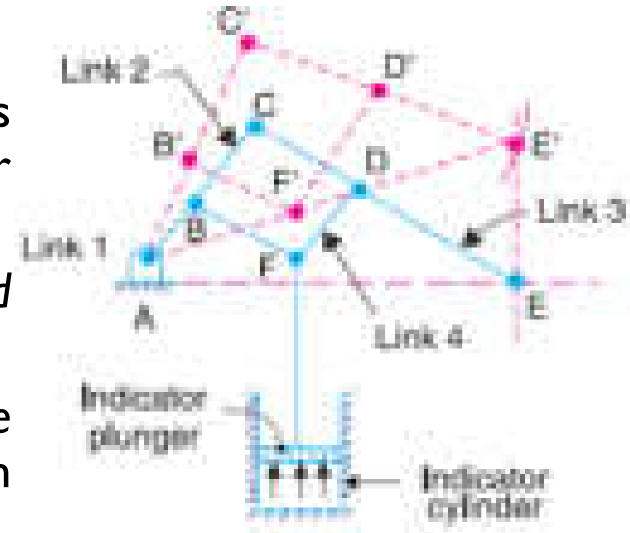
Coupling rod of a locomotive (Double crank mechanism)

- The mechanism of a coupling rod of a locomotive (also known as double crank mechanism) which consists of four links.
- In this mechanism, the links *AD and BC (having equal length) act as cranks.*
- The link *CD acts as a coupling rod and the link AB is fixed in order* to maintain a constant centre to centre distance between them.
- This mechanism is meant [illegible] on from one wheel to the ot

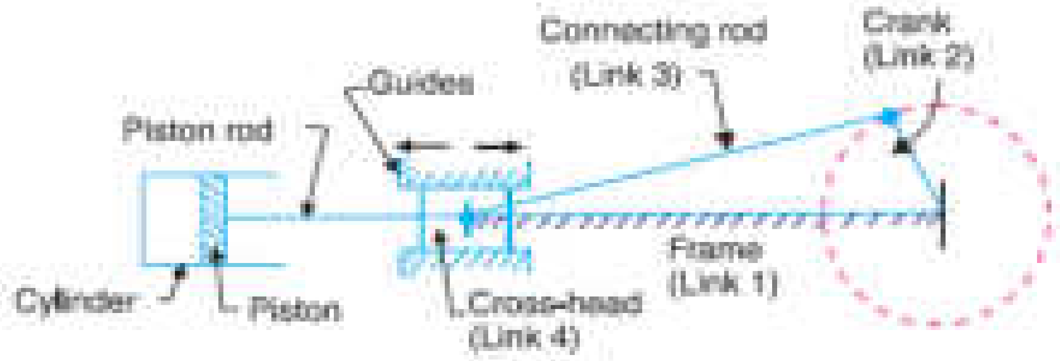


Watt's indicator mechanism (Double lever mechanism)

- Watt's indicator mechanism (also known as Watt's straight line mechanism or double lever mechanism).
- The four links are : fixed link at *A, link AC, link CE and link BFD.*
- It may be noted that *BF and FD form one link because these two parts have no* relative motion between them.
- The links *CE and BFD act* as levers.
- The displacement of the link *BFD is directly* proportional to the pressure of gas or steam which acts on the indicator plunger.
- On any small displacement of the mechanism, the tracing point *E at the end of the link CE* traces out approximately a straight line.



Single slider crank chain



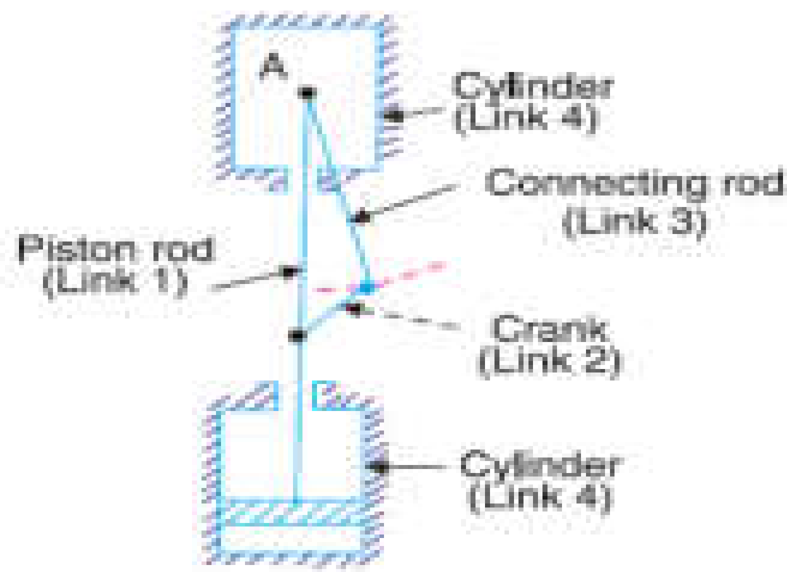
- This type of mechanism converts rotary motion into reciprocating motion and vice versa.
- the links 1 and 2, links 2 and 3, and links 3 and 4 form three turning pairs while the links 4 and 1 form a sliding pair.
- The link 1 corresponds to the frame of the engine, which is fixed. The link 2 corresponds to the crank ; link 3 corresponds to the connecting rod and link 4 corresponds to cross-head. As the crank rotates, the cross-head reciprocates in the guides and thus the piston reciprocates in the cylinder.

Inversions of Single Slider Crank Chain

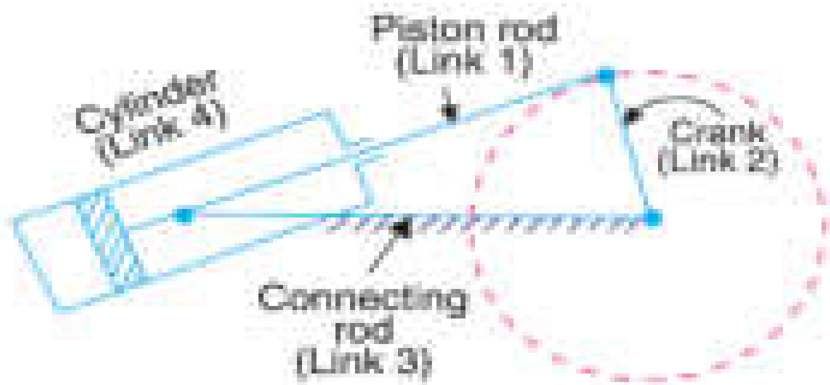
- Pendulum pump or Bull engine.
- Oscillating cylinder engine.
- Rotary internal combustion engine or Gnome engine.
- Crank and slotted lever quick return motion mechanism.
- Whitworth quick return motion mechanism.

Pendulum pump or Bull engine

- In this mechanism, the inversion is obtained by fixing the cylinder or link 4 (*i.e. sliding pair*).
- when the crank (link 2) rotates, the connecting rod (link 3) oscillates about a pin pivoted to the fixed link 4 at *A and the piston* attached to the piston rod (link 1) reciprocates.
- The pump is used to supply feed water to boilers.



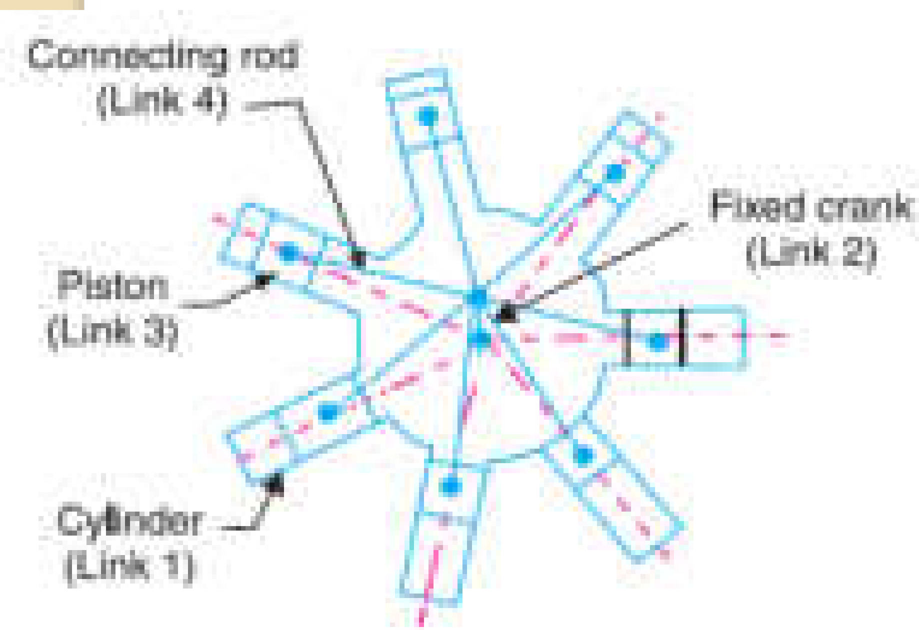
Oscillating cylinder engine



- Is used to convert reciprocating motion into rotary motion.
- In this mechanism, the link 3 forming the turning pair is fixed.
- The link 3 corresponds to the connecting rod of a reciprocating steam engine mechanism.
- When the crank (link 2) rotates, the piston attached to piston rod (link 1) reciprocates and the cylinder (link 4) oscillates about a pin pivoted to the fixed link at *A*.

Rotary internal combustion engine or Gnome engine

- the crank (link 2) is fixed.
- In this mechanism, when the connecting rod (link 4) rotates, the piston (link 3) reciprocates inside the cylinders forming link 1.



Crank and slotted lever quick return motion mechanism.

- This mechanism is mostly used in shaping machines, slotting machines.
- In this mechanism, the link *AC (i.e. link 3) forming the turning pair is fixed*
- The link 3 corresponds to the connecting rod of a reciprocating steam engine.
- The driving crank *CB revolves with uniform angular speed about the fixed centre C. A sliding block attached to the crank*
- A pin at *B slides along the slotted bar AP and thus causes AP to oscillate about the pivoted point A. A* short link *PR transmits the motion from AP to the ram which carries the tool and reciprocates along the* line of stroke *R1R2. The line of stroke of the ram (i.e. R1R2) is perpendicular to AC produced.*

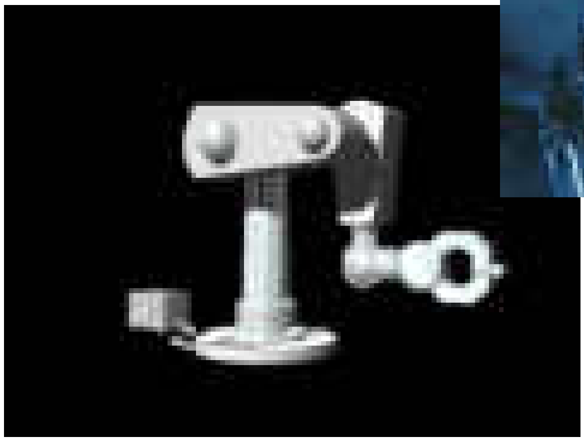
Kinematics

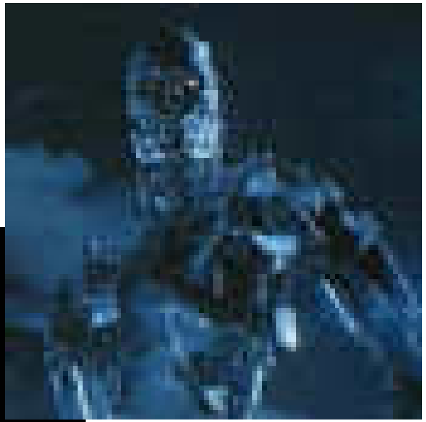
Advanced Graphics (and Animation)

Spring 2002

Kinematics

- The study of object movements irrespective of their speed or style of movement





Degrees of Freedom (DOFs)

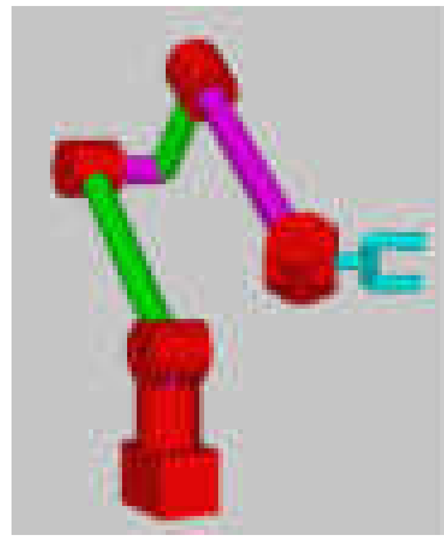
- The variables that affect an object's orientation
- How many degrees of freedom when flying?

- So the kinematics of this airplane permit movement anywhere in three dimensions

- Six
  - x, y, and z positions
  - roll, pitch, and yaw

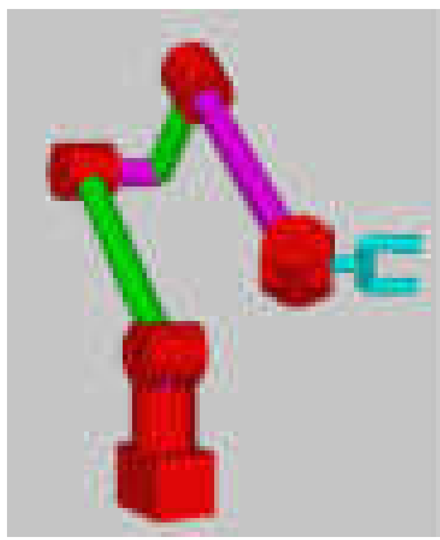
Degrees of Freedom

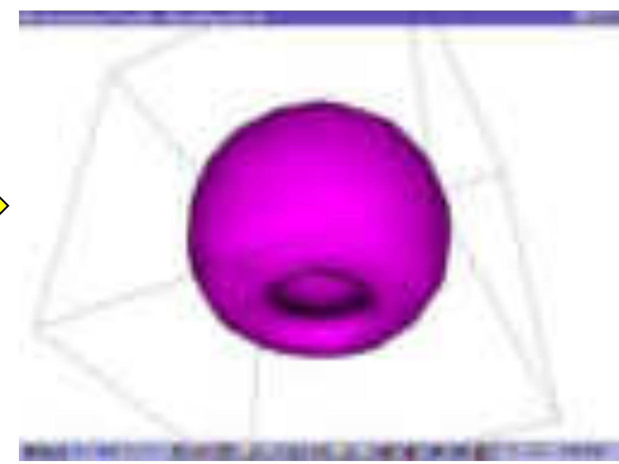
- How about this robot arm?



- Six again
  - 2-base, 1-shoulder, 1-elbow, 2-wrist

Configuration Space



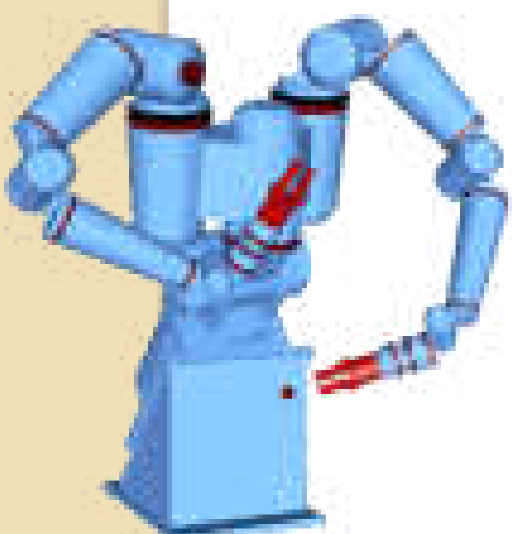


Work Space vs. Configuration Space

- Work space
  - The space in which the object exists
  - Dimensionality
    - R^3 for most things, R^2 for planar arms
- Configuration space
  - The space that defines the possible object configurations
  - Degrees of Freedom
    - The number of parameters that necessary and sufficient to define position in configuration

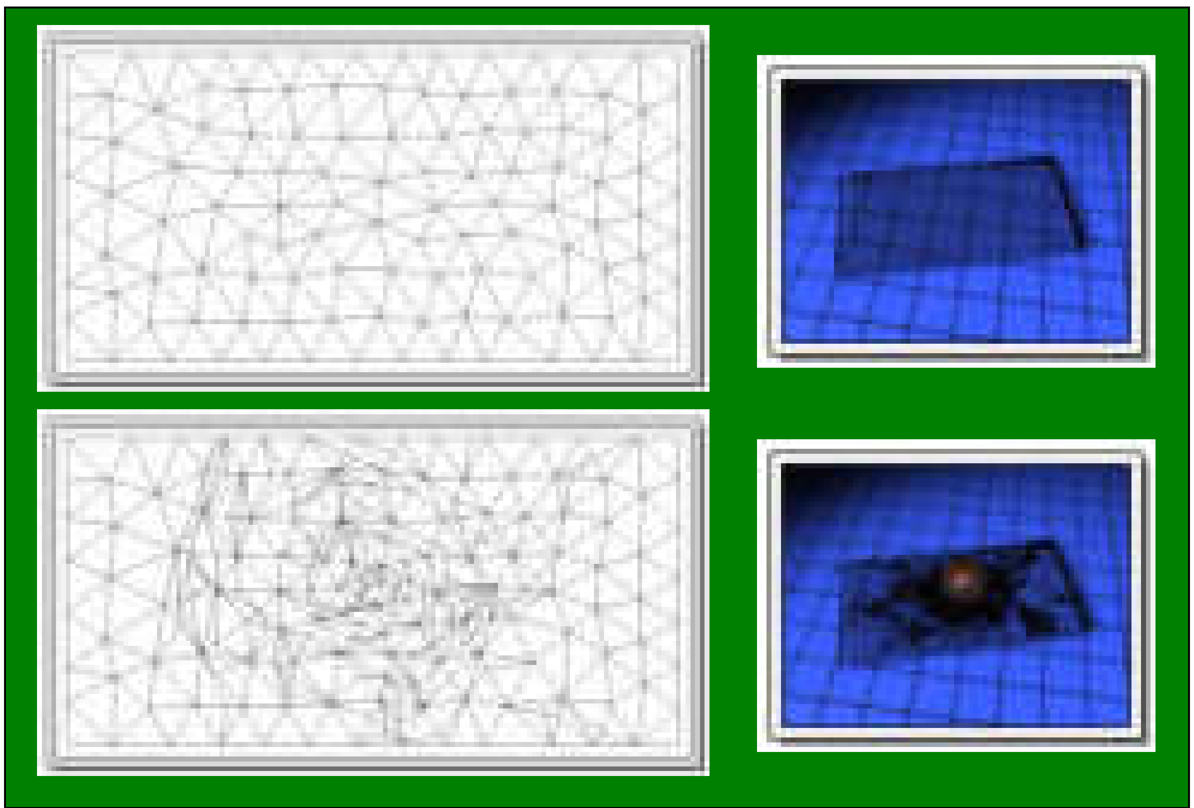
More examples

- A point on a plane
- A point in space
- A point moving on a line in space



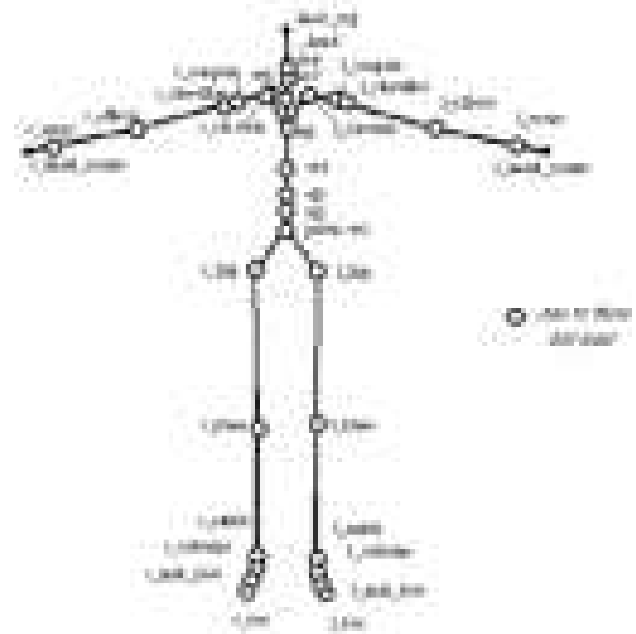
Controlled DOFs

- DOFs that you can actually control (position explicitly)



Hierarchical Kinematic Modeling

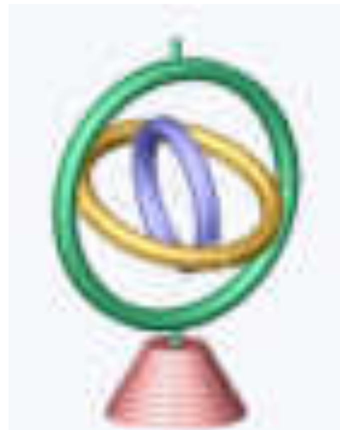
- A family of parent-child spatial relationships are functionally defined
  - Moon/Earth/Sun movements
  - Articulations of a humanoid
- Limb connectivity is built into model (joints) and animation is easier



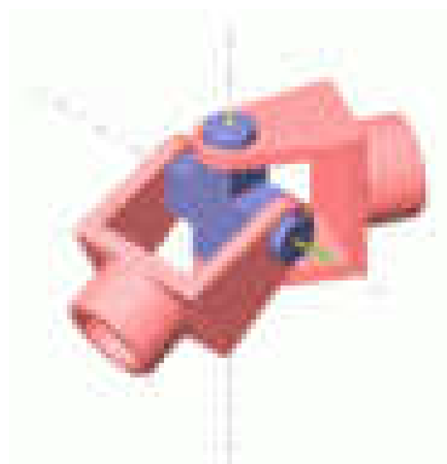
Robot Parts/Terms

- Links
- End effector
- Frame
- Revolute Joint
- Prismatic Joint

More Complex Joints



- 3 DOF joints
  - Gimbal
  - Spherical (doesn't possess singularity)
- 2 DOF joints
  - Universal



Hierarchy Representation

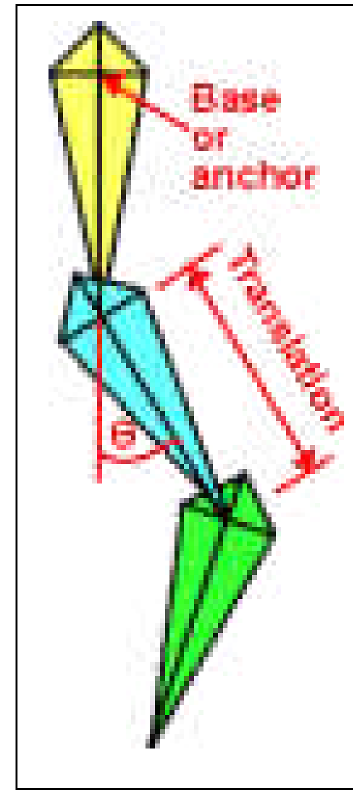
- Model bodies (links) as nodes of a tree
- All body frames are local (relative to parent)
  - Transformations affecting root affect all children
  - Transformations affecting any node affect all its children

Forward vs. Inverse Kinematics

- Forward Kinematics
  - Compute configuration (pose) given individual DOF values
- Inverse Kinematics
  - Compute individual DOF values that result in specified end effector position

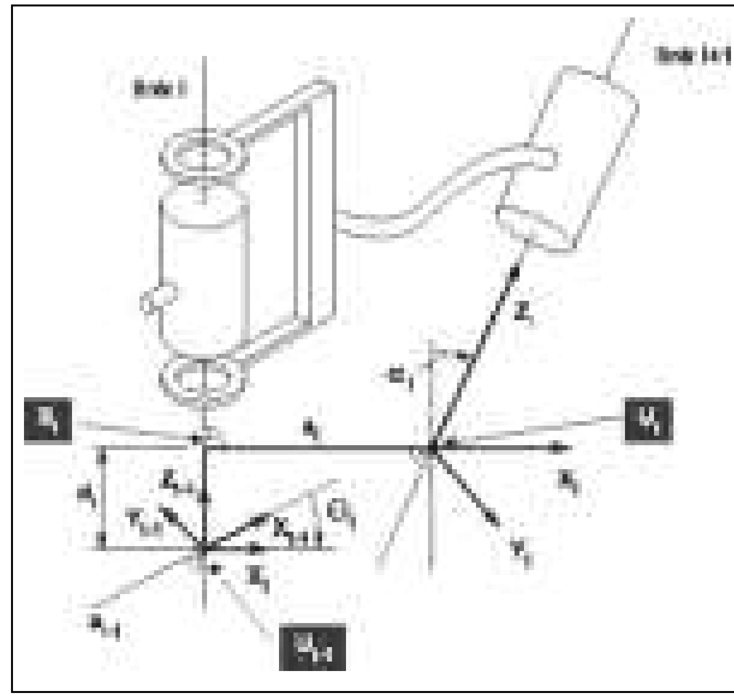
Forward Kinematics

- Traverse kinematic tree and propagate transformations downward
  - Use stack
  - Compose parent transformation with child's
  - Pop stack when leaf is reached
- High DOF models are tedious to control this way



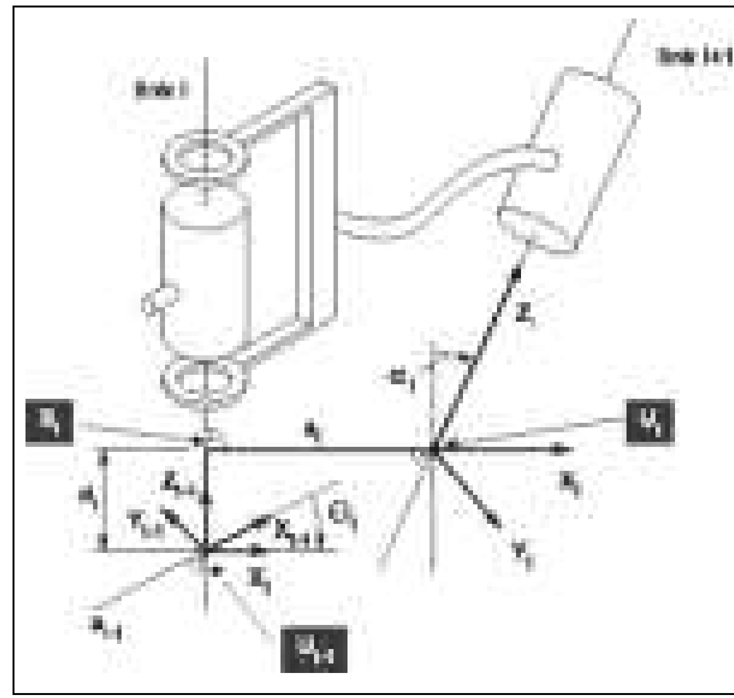
Denavit-Hartenberg (DH) Notation

- A kinematic representation (convention) inherited from robotics
- Z-axis is aligned with joint
- X-axis is aligned with outgoing limb
- Y-axis is orthogonal



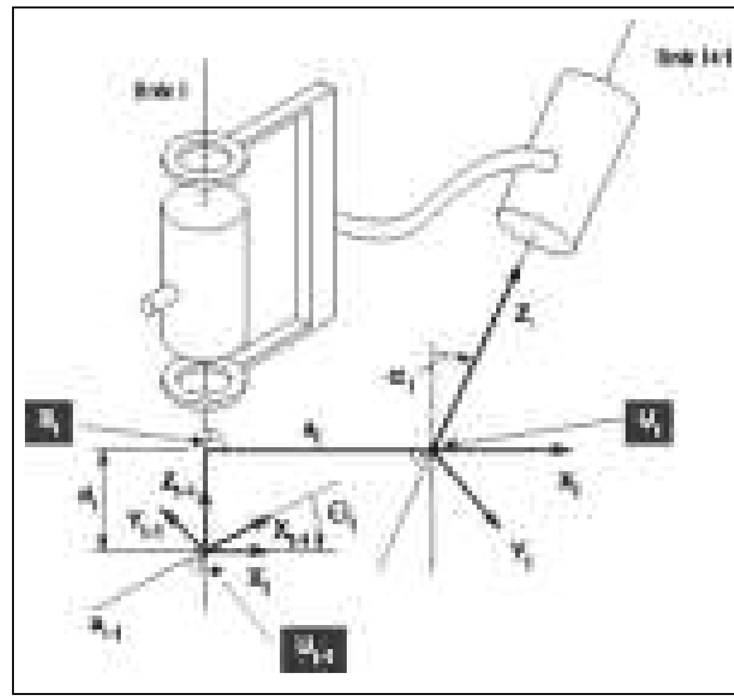
DH Notation

- Joints are numbered to represent hierarchy
- U_{i-1} is parent of U_i
- Parameter a_{i-1} is outgoing limb length of joint U_{i-1}
- Joint angle, θ_{i-1}, is rotation of i-1 x-axis, x_{i-1}, about z_{i-1}, relative to i-2th frame's x-axis direction, x_{i-2}



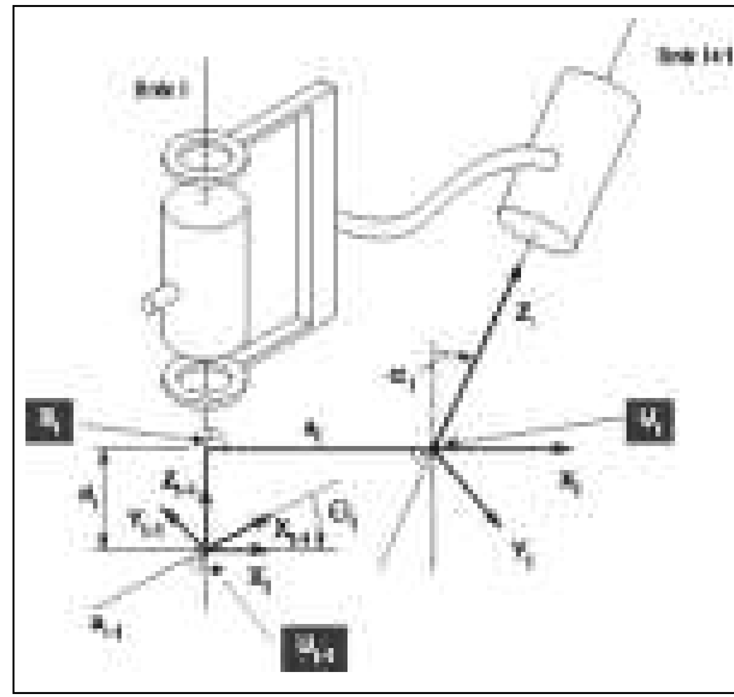
DH Notation

- If nonplanar
- X-axis of i-1[th] joint is line perpendicular to z-axes of i-1 and i frames
- Link twist, α_{i-1}, is the rotation of i[th] z-axis about x_{i-1}-axis relative to z-axis of i-1[th] frame



DH Notation

- Link offset, d_{i-1}, specifies the distance along the z_{i-1}-axis (rotated by α_{i-1}) of the i^{th} frame from the $i-1^{th}$ x-axis to the i^{th} x-axis



DH Notation

- Not all i^{th} variables relate to i and i-1
- Link offset (d_i):Distance from x_{i-1} to x_i along z_i
- Joint angle (θ_i): angle between x_{i-1} and x_i about z_i
- Link length (a_i): distance from z_i to z_{i+1} along x_i
- Link Twist (α_i): angle between z_i and z_{i+1} about x_i

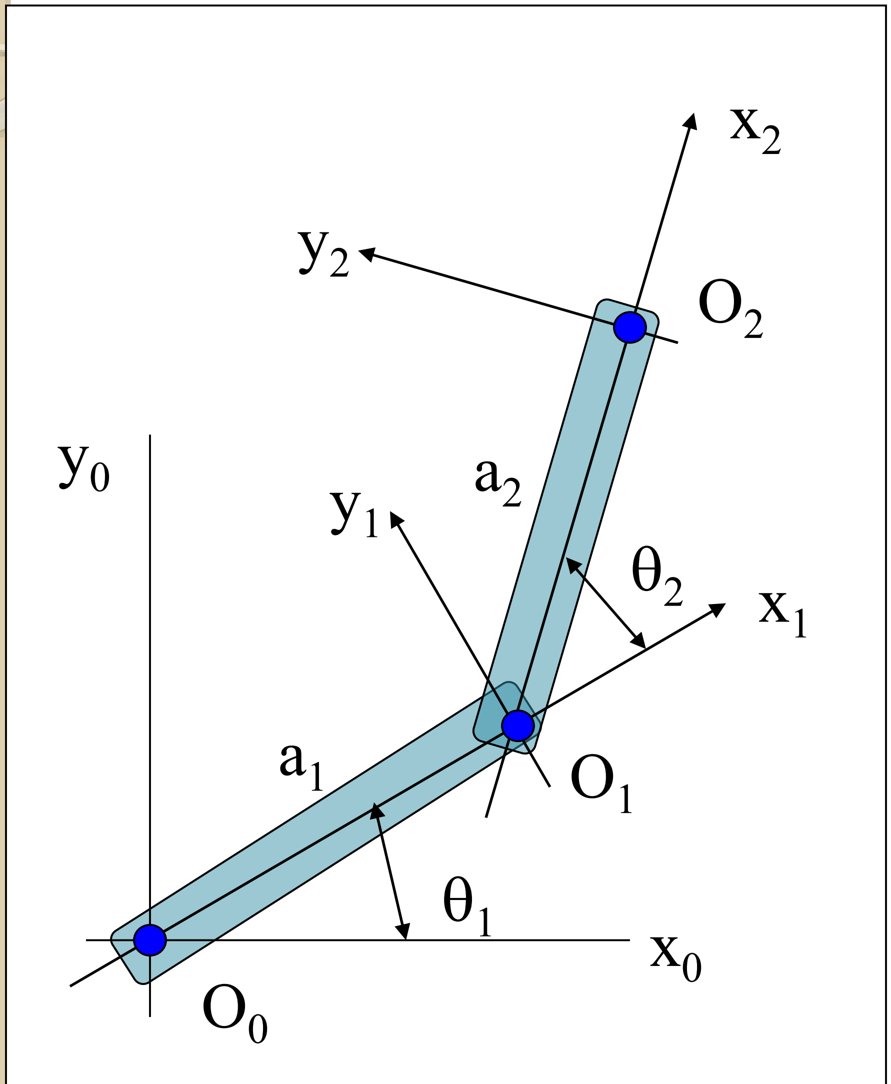
Screw Transformations

- No, I'm not mad at them
- Relationship between i+1 frame and i frame are a combination
  - i^{th} joint parameters
  - i+1 joint parameters
- Call this relationship screw transformations
  - Two (translation, rotation) pairs each relative to specific axis of i^{th} and i+1 frames

Screw Transformations

- Offset (d_{i+1}) and angle (θ_{i+1}) are translation and rotation of i+1 joint relative to i^{th} joint w.r.t. z_i-axis
- Length (a_i) and twist (α_i) are translation and rotation w.r.t. x_i-axis

Planar Example



Ball and Socket

- Model as 3 revolute joints with zero-length links between them
- If all angles are set to 0, we are in gimbal lock situation (z-axes of two joints are colinear)
- Instead, initialize middle joint angle to 90 degrees
- … or represent using quaternions

Inverse Kinematics (IK)

- Given end effector position, compute required joint angles
- In simple case, analytic solution exists
  - Use trig, geometry, and algebra to solve

$$x^2 + y^2 = a_1^2 + a_2^2 - 2a_1a_2\cos(\pi - \theta_2)$$

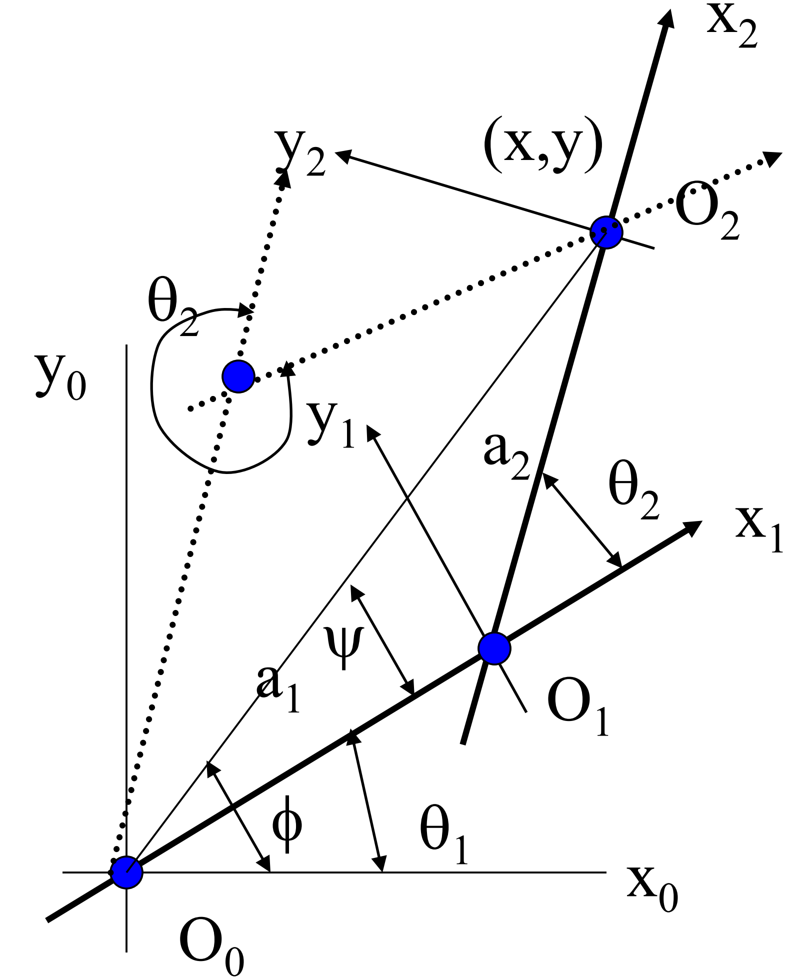
$$\cos\theta_2 = \frac{x^2 + y^2 - a_1^2 - a_2^2}{2a_1a_2}$$

for greater accuracy

$$\tan^2\frac{\theta_2}{2} = \frac{1-\cos\theta}{1+\cos\theta} = \frac{2a_1a_2 - x^2 - y^2 + a_1^2 + a_2^2}{2a_1a_2 + x^2 + y^2 - a_1^2 - a_2^2}$$

$$= \frac{\left(a_1^2 + a_2^2\right)^2 - \left(x^2 + y^2\right)}{\left(x^2 + y^2\right) - \left(a_1^2 - a_2^2\right)^2}$$

$$\theta_2 = \pm 2\tan^{-1}\sqrt{\frac{\left(a_1^2 + a_2^2\right)^2 - \left(x^2 + y^2\right)}{\left(x^2 + y^2\right) - \left(a_1^2 - a_2^2\right)^2}}$$



Iterative IK Solutions

- Frequently analytic solution is infeasible
- Use Jacobian
- Derivative of function output relative to each of its inputs
- If y is function of three inputs and one output

$$y = f(x_1, x_2, x_3)$$

$$\delta y = \frac{\delta f}{\partial x_1} \cdot \delta x_1 + \frac{\delta f}{\partial x_2} \cdot \delta x_2 + \frac{\delta f}{\partial x_3} \cdot \delta x_3$$

Jacobian

- In another situation, end effector has 6 DOFs and robotic arm has 6 DOFs
- $f(x_1, \ldots, x_6) = (x, y, z, r, p, y)$
- Therefore $J(X) =$ 6x6 matrix

$$\begin{bmatrix} \frac{\partial f_x}{\partial x_1} & \frac{\partial f_y}{\partial x_1} & \frac{\partial f_z}{\partial x_1} & \frac{\partial f_r}{\partial x_1} & \frac{\partial f_p}{\partial x_1} & \frac{\partial f_y}{\partial x_1} \\ \frac{\partial f_x}{\partial x_2} & & & & & \\ \frac{\partial f_x}{\partial x_3} & & & & & \\ \frac{\partial f_x}{\partial x_4} & & & & & \\ \frac{\partial f_x}{\partial x_5} & & & & & \\ \frac{\partial f_x}{\partial x_6} & & & & & \end{bmatrix}$$

Jacobian

- Relates velocities in parameter space to velocities of outputs

$$\dot{Y} = J(X) \cdot \dot{X}$$

- If we know $Y_{current}$ and $Y_{desired}$, then we subtract to compute Y_{dot}
- Invert Jacobian and solve for X_{dot}