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Presentation On

Solving Statically Determinate Structures

Static Equilibrium

A system of particles is in static equilibrium when all the particles of the system are at rest and the total force on each particle is permanently zero.

Statically Determinate

A member or structure that can be analyzed and the reactions and forces determined from the equations of equilibrium.

Statically Indeterminate

A member or structure that cannot be analyzed by the equations of statics. It contains unknowns in excess of the number of equilibrium equations available.

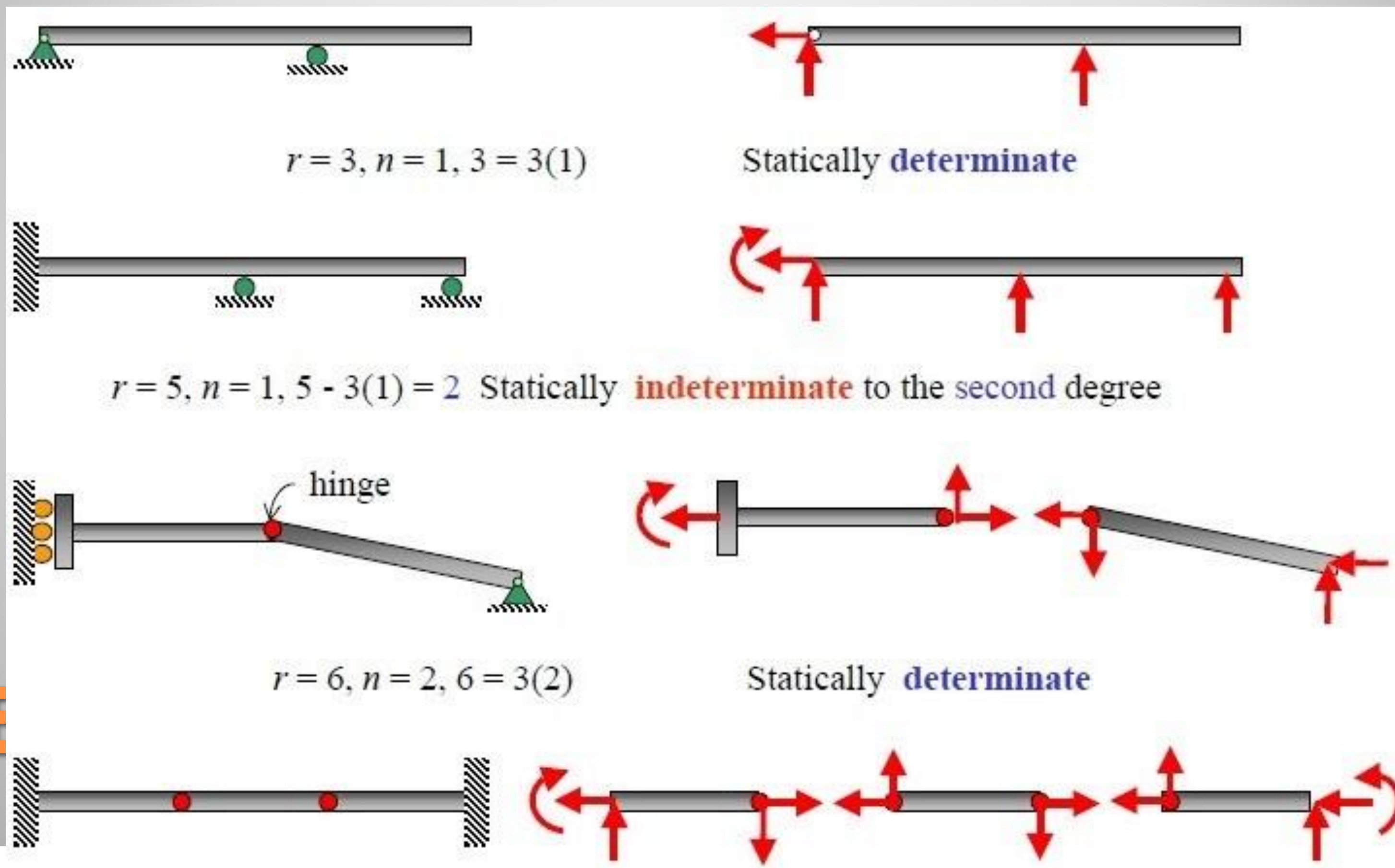
Determinacy

- **r = 3n** , statically **determinate**
- **r > 3n** , statically **indeterminate**

where,

n = the total parts of structure members

r = the total number of unknown reactive forces and moment components



r = 3, n = 1, 3 = 3(1)
Statically determinate
r = 5, n = 1, 5 - 3(1) = 2 Statically indeterminate to the second degree
hinge
r = 6, n = 2, 6 = 3(2)
Statically determinate

Redundants

- The excess members or reactions of an indeterminate structures are called redundants.
- Redundant forces are chosen so that the structure is stable and statically determinate when they are removed.

How do we make an indeterminate structure statically determinate?

- If there is two degrees of indeterminacy, we have to remove two reactive forces, remove three for three degrees and so forth.

- Byremoving excess supports.

- Byintroducing hinges.

What are the advantages of statically indeterminate structures over determinate structures?

There are several advantages in designing indeterminate structures. These include the design of lighter and more rigid structures. With added redundancy in the structural system, there is an increase in the overall factor of safety.


Thank You!

Structural Design and Inspection-Deflection and Slope of Beams

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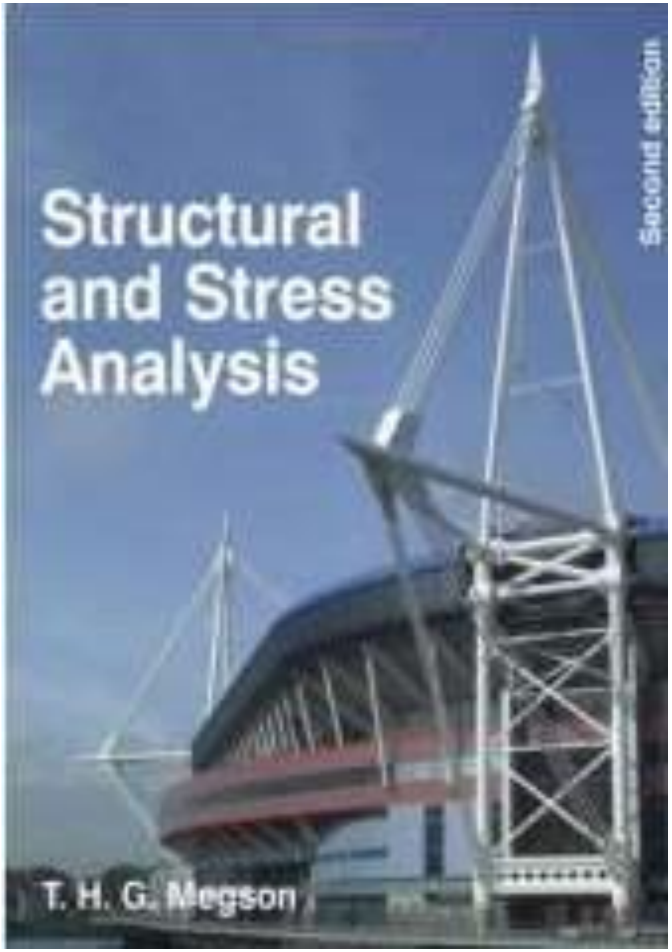
Suggested Readings

Reference 1
Chapter 16



Objective

- To obtain slope and deflection of beam and frame structures using slope-deflection method

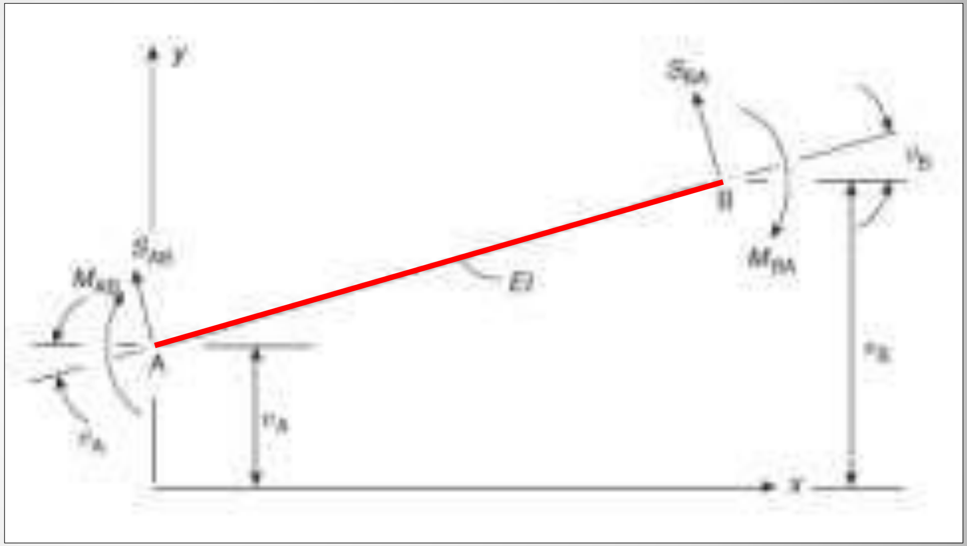
Introduction

- Structural analysis method for beams and frames introduced in 1914 by George A. Maney

- This method was later replaced by moment distribution method which is more advanced and useful (students are encouraged) to study this separately

Slope-Deflection Method

- Sign convention:
  - Moments, slopes, displacements, shear are all in positive direction as shown
- Axial forces are ignored



$$EI\frac{d^2v}{dx^2} = M_{AB} + S_{AB}x \rightarrow$$

$$\begin{cases} EI\dfrac{dv}{dx} = M_{AB}x + S_{AB}\dfrac{x^2}{2} + C_1 \\ EIv = M_{AB}\dfrac{x^2}{2} + S_{AB}\dfrac{x^3}{6} + C_1x + C_2 \end{cases} \rightarrow$$

Slope-Deflection Method

$$@\, x = 0 \Rightarrow \frac{dv}{dx} = \theta_A,\; v = v_A \rightarrow$$

$$\begin{cases} EI\dfrac{dv}{dx} = M_{AB}x + S_{AB}\dfrac{x^2}{2} + EI\theta_A \\ EIv = M_{AB}\dfrac{x^2}{2} + S_{AB}\dfrac{x^3}{6} + EI\theta_A x + EIv_A \end{cases} \rightarrow$$

$$@\, x = L \Rightarrow \frac{dv}{dx} = \theta_B,\; v = v_B \rightarrow$$

$$\begin{cases} EI\theta_B = M_{AB}L + S_{AB}\dfrac{L^2}{2} + EI\theta_A \\ EIv_B = M_{AB}\dfrac{L^2}{2} + S_{AB}\dfrac{L^3}{6} + EI\theta_A L + EIv_A \end{cases} \rightarrow$$

Solving for M_{AB} and S_{AB}

$$M_{AB} = -\frac{2EI}{L}\left[2\theta_A + \theta_B + \frac{3}{L}(v_A - v_B)\right]$$

$$S_{AB} = \frac{6EI}{L^2}\left[\theta_A + \theta_B + \frac{2}{L}(v_A - v_B)\right]$$

$$M_{BA} = -\frac{2EI}{L}\left[\theta_A + 2\theta_B + \frac{3}{L}(v_A - v_B)\right]$$

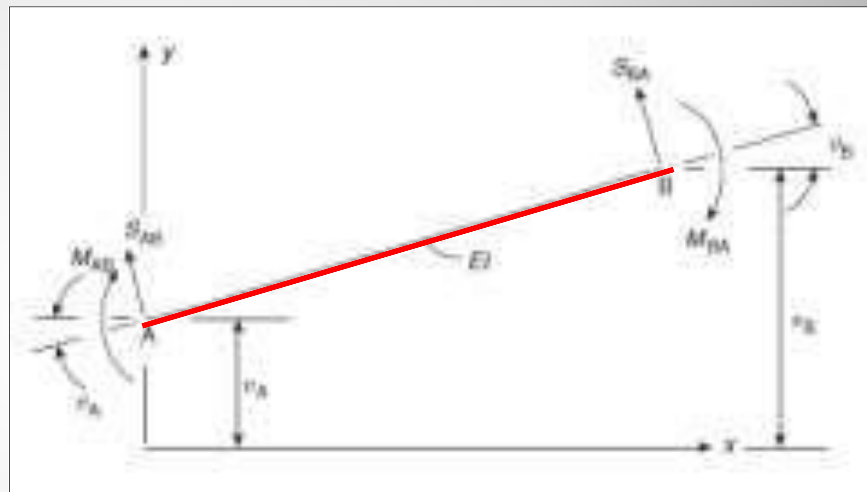
$$S_{BA} = -\frac{6EI}{L^2}\left[\theta_A + \theta_B + \frac{2}{L}(v_A - v_B)\right]$$

Slope-Deflection Method

- Last 4 equations obtained in previous slide are called slope-deflection equations
- They establish force-displacement relationship
- This method can find exact solution to indeterminate structures

Slope-Deflection Method

- The beam we considered so far did not have any external loading from A to B



- In the presence of mid-span loading (common engineering problems) the equations become:

$$M_{AB} = -\frac{2EI}{L}\left[2\theta_A + \theta_B + \frac{3}{L}(v_A - v_B)\right] + M_{AB}^F$$

$$S_{AB} = \frac{6EI}{L^2}\left[\theta_A + \theta_B + \left(\frac{2}{L}\right)(v_A - v_B)\right] + S_{AB}^F$$

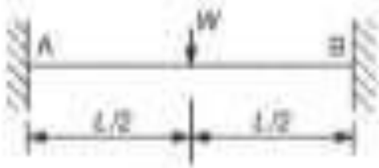
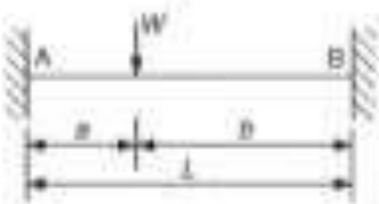
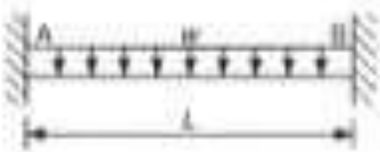
$$M_{BA} = -\frac{2EI}{L}\left[\theta_A + 2\theta_B + \frac{3}{L}(v_A - v_B)\right] + M_{BA}^F$$

$$S_{BA} = -\frac{6EI}{L^2}\left[\theta_A + \theta_B + \left(\frac{2}{L}\right)(v_A - v_B)\right] + S_{BA}^F$$

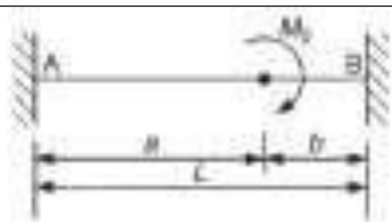
Fixed End Moment/Shear

- M_{AB}^F, M_{BA}^F are fixed end moments at nodes A and B, respectively.
  - Moments at two ends of beam when beam is clamped at both ends under external loading (see next slides)
- S_{AB}^F, S_{BA}^F are fixed end shears at nodes A and B, respectively.
  - Shears at two ends of beam when beam is clamped at both ends under external loading (see next slides)

Fixed End Moment/Shear

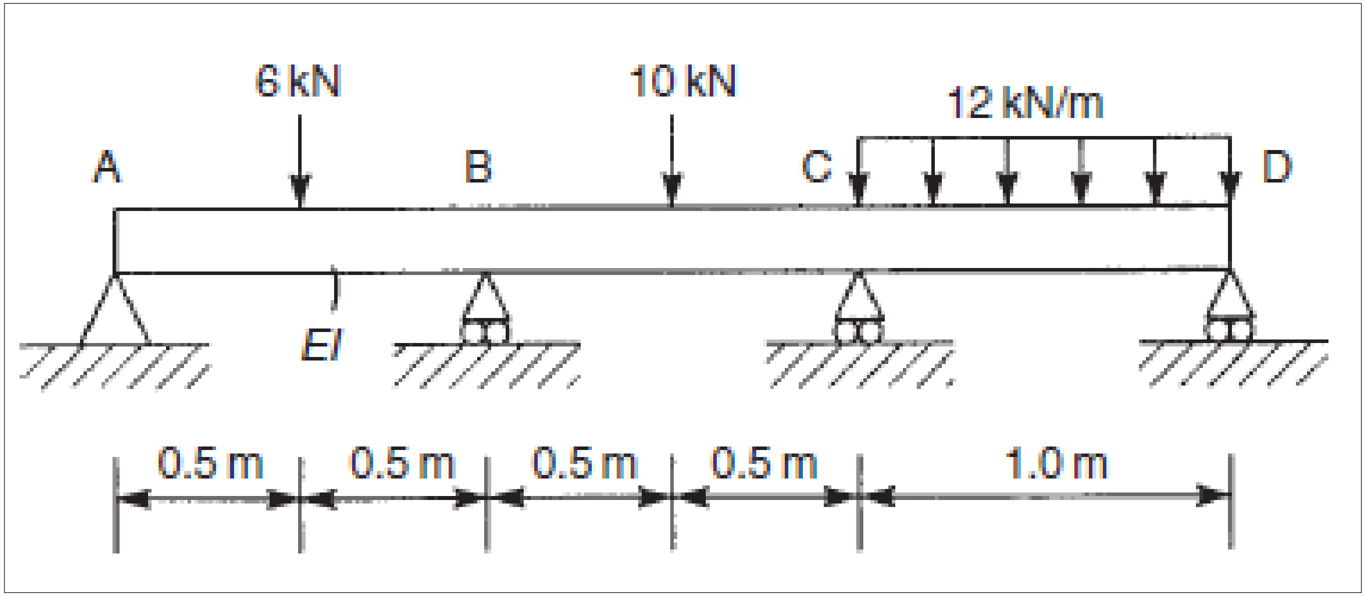
Load case	FEMs (+ ↻, − ↺) M^F_{AB}	M^F_{BA}
	$-\frac{WL}{8}$	$+\frac{WL}{8}$
	$-\frac{Wab^2}{L^2}$	$+\frac{Wa^2b}{L^2}$
	$-\frac{wL^2}{12}$	$+\frac{wL^2}{12}$
	$-\frac{w}{L^2}\left[\frac{L^2}{2}(b^2-a^2)-\frac{2}{3}L(b^3-a^3)+\frac{1}{4}(b^4-a^4)\right]$	$+\frac{wb^3}{L^2}\left(\frac{L}{3}-\frac{b}{4}\right)$

Fixed End Moment/Shear

Load case	FEMs (+ ↻, − ↺) M^F_{AB}	M^F_{BA}
	$+\frac{M_0 b}{L^2}(2a-b)$	$+\frac{M_0 a}{L^2}(2b-a)$
	$-\frac{6EI\delta}{L^2}$	$-\frac{6EI\delta}{L^2}$
	0	$-\frac{3EI\delta}{L^2}$

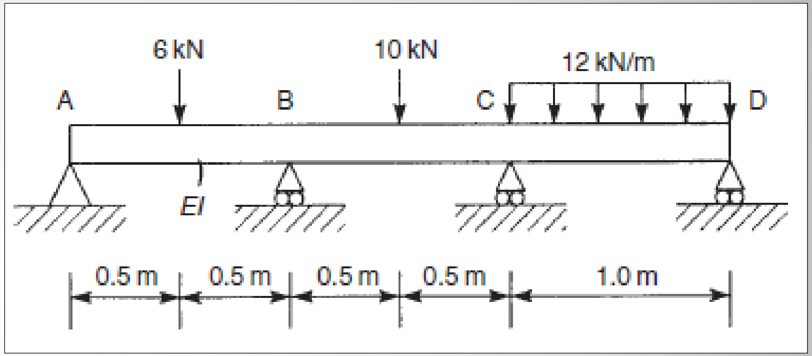
Example

- Find support reactions.



Solution

- This beam has 2 degrees of indeterminacy



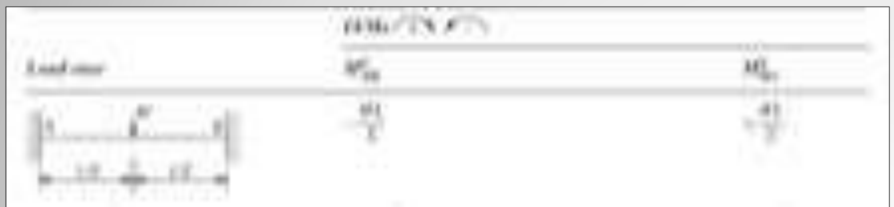
1- Assume all beams are fixed & calculate FEM

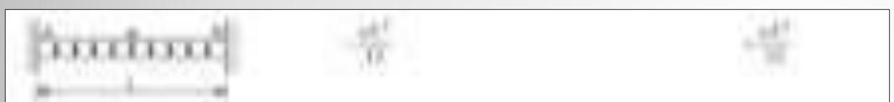
2- Establish Slope-Deflection equations

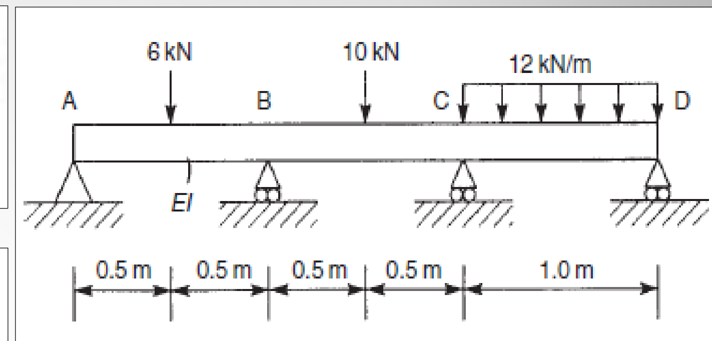
3- Enforce boundary conditions & equilibrium conditions at joints

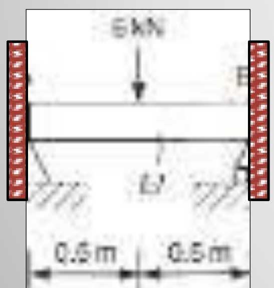
4- Solve simultaneous equations to get slopes/deflections

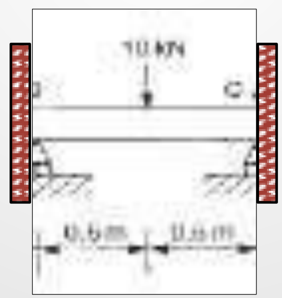
Solution

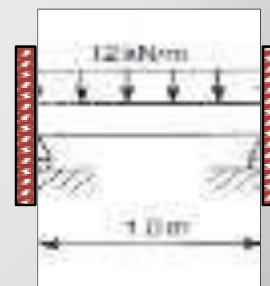












$$M^F_{AB} = -M^F_{BA} = -\frac{6\times 1.0}{8} = -0.75 kNm$$

$$M^F_{BC} = -M^F_{CB} = -\frac{10\times 1.0}{8} = -1.25 kNm$$

$$M^F_{CD} = -M^F_{DC} = -\frac{12\times 1.0^2}{12} = -1.00 kNm$$

Solution

$$M_{AB} = -\frac{2EI}{1.0}(2\theta_A + \theta_B) - 0.75$$

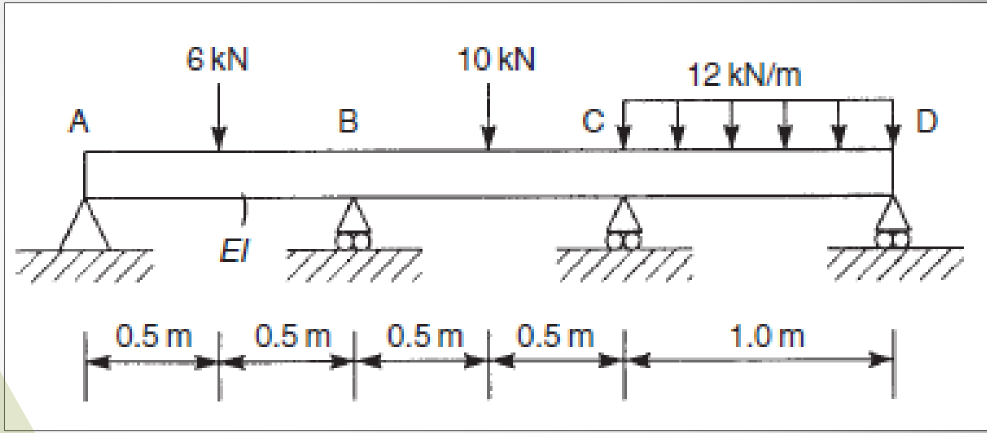
$$M_{BA} = -\frac{2EI}{1.0}(2\theta_B + \theta_A) + 0.75$$

$$M_{BC} = -\frac{2EI}{1.0}(2\theta_B + \theta_C) - 1.25$$

$$M_{CB} = -\frac{2EI}{1.0}(2\theta_C + \theta_B) + 1.25$$

$$M_{CD} = -\frac{2EI}{1.0}(2\theta_C + \theta_D) - 1.0$$

$$M_{DC} = -\frac{2EI}{1.0}(2\theta_D + \theta_C) + 1.0$$



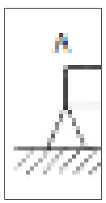
$$M_{ij} = -\frac{2EI}{L}\left[2\theta_i + \theta_j + \frac{3}{L}\left(v_i - v_j\right)\right] + M_{ij}^F$$

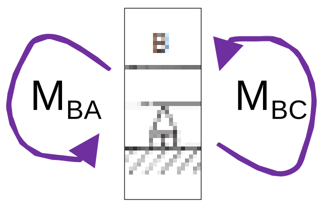
$$M_{ji} = -\frac{2EI}{L}\left[\theta_i + 2\theta_j + \frac{3}{L}\left(v_i - v_j\right)\right] + M_{ji}^F$$

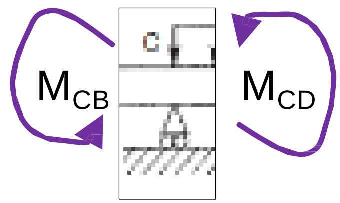
v_i=0 and v_j=0 for all cases

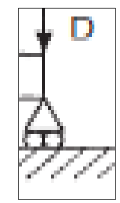
Solution

- Equilibrium moments at the joints









$$M_{AB} = 0$$

$$\sum M_B = 0 \rightarrow$$
$$M_{BA} + M_{BC} = 0$$

$$\sum M_C = 0 \rightarrow$$
$$M_{CB} + M_{CD} = 0$$

$$M_{DC} = 0$$

Solution

- Substitution into slope deflection equations gives 4 equations and 4 unknown slopes.

$$4EI\theta_A + 2EI\theta_B + 0.75 = 0$$

$$2EI\theta_A + 8EI\theta_B + 2EI\theta_C + 0.5 = 0$$

$$2EI\theta_B + 8EI\theta_C + 2EI\theta_D - 0.25 = 0$$

$$4EI\theta_D + 2EI\theta_C - 1.0 = 0$$

- By simultaneously solving the equations

$$EI\theta_A = -0.183 \quad EI\theta_B = -0.008 \quad EI\theta_C = -0.033 \quad EI\theta_D = +0.267$$

Solution

- Simply operation of substitution:

$$M_{AB} = -\frac{2EI}{1.0}(2\theta_A + \theta_B) - 0.75$$

$$M_{BA} = -\frac{2EI}{1.0}(2\theta_B + \theta_A) + 0.75$$

$$M_{BC} = -\frac{2EI}{1.0}(2\theta_B + \theta_C) - 1.25$$

$$M_{CB} = -\frac{2EI}{1.0}(2\theta_C + \theta_B) + 1.25$$

$$M_{CD} = -\frac{2EI}{1.0}(2\theta_C + \theta_D) - 1.0$$

$$M_{DC} = -\frac{2EI}{1.0}(2\theta_D + \theta_C) + 1.0$$

$$EI\theta_A = -0.183$$

$$EI\theta_B = -0.008$$

$$EI\theta_C = -0.033$$

$$EI\theta_D = +0.267$$

$$M_{AB} = 0$$

$$M_{BA} = 1.15$$

$$M_{BC} = -1.15$$

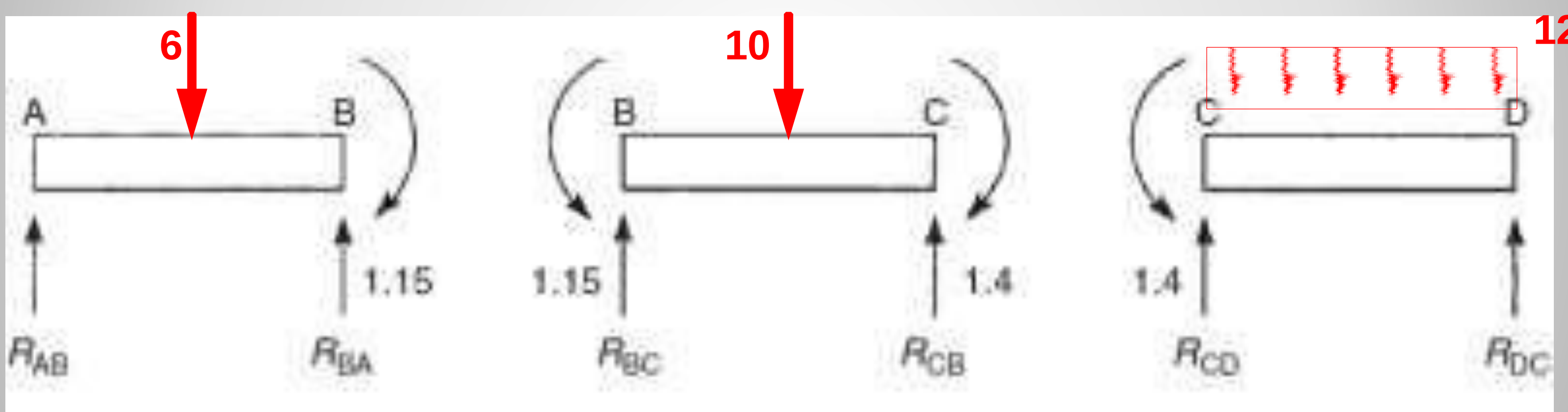
$$M_{CB} = 1.4$$

$$M_{CD} = -1.4$$

$$M_{DC} = 0$$

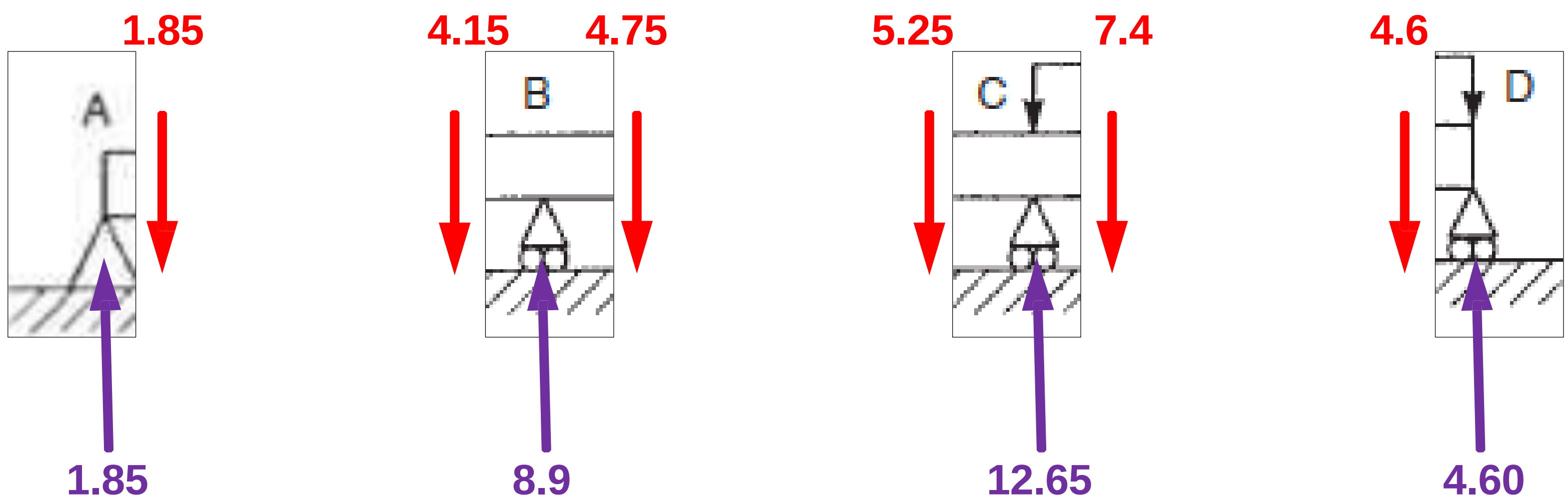
Solution

- Now support reactions can easily be calculated as

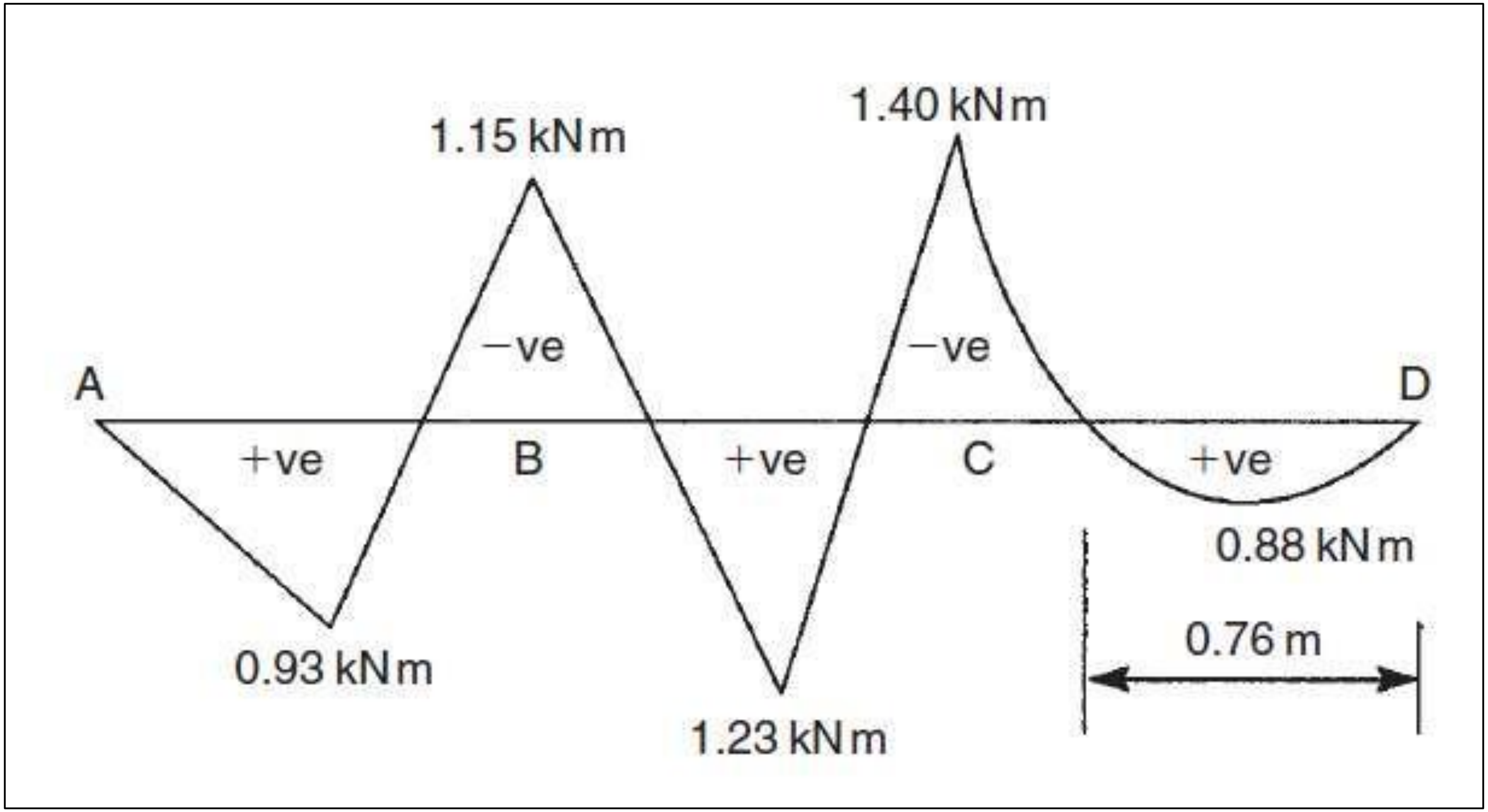


$R_{AB} = \frac{6\times 0.5 - 1.15}{1.0} = 1.85$ $R_{BA} = 6 - 1.85 = 4.15$	$R_{BC} = \frac{10\times 0.5 + 1.15 - 1.4}{1.0} = 4.75$ $R_{CB} = 10 - 4.75 = 5.25$	$R_{CD} = \frac{12\times 0.5 + 1.4}{1.0} = 7.4$ $R_{DC} = 12 - 7.4 = 4.6$

Solution



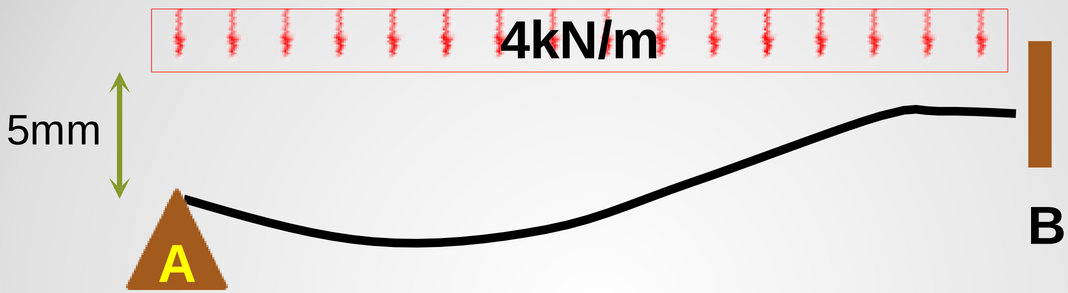
Solution



Example 2

- Obtain moment reaction at the clamped support *B* for 6m long beam if support *A* settles down by 5mm. $EI = 17\times10^{12}$ Nmm^2

Solution



$$M_{ij} = -\frac{2EI}{L}\left[2\theta_i + \theta_j + \frac{3}{L}\left(v_i - v_j\right)\right] + M_{ij}^F$$

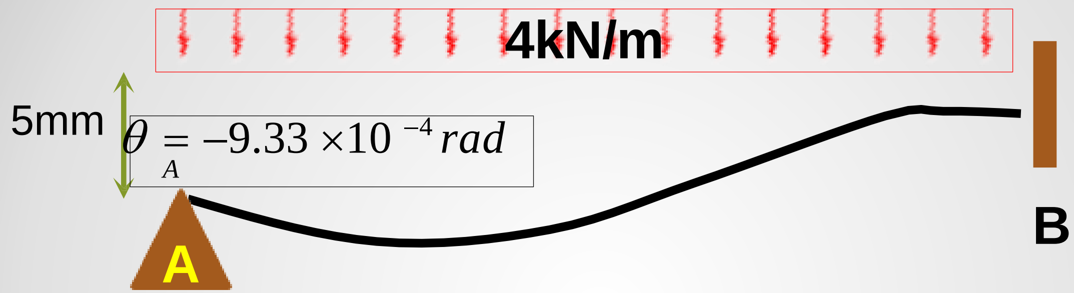
$$M_{ji} = -\frac{2EI}{L}\left[\theta_i + 2\theta_j + \frac{3}{L}\left(v_i - v_j\right)\right] + M_{ji}^F$$

$$M_{AB} = -\frac{2EI}{L}\left[2\theta_A + \theta_B + \frac{3}{L}\left(v_A - v_B\right)\right] + M_{AB}^F \rightarrow$$

$$M_{AB} = 0 = -\frac{2EI}{L}\left[2\theta_A + 0 + \frac{3}{L}\left(0.005 - 0\right)\right] \quad \frac{4000 \times L^2}{12} \rightarrow$$

$$\theta_A = \left(-\frac{4000}{24EI}L^3 + \frac{0.0015}{L}\right) \Big/ 2 = -9.33\times10^{-4}\, rad$$

Solution



$$M_{ij} = -\frac{2EI}{L}\left[2\theta_i + \theta_j + \frac{3}{L}(v_i - v_j)\right] + M_{ij}^F$$

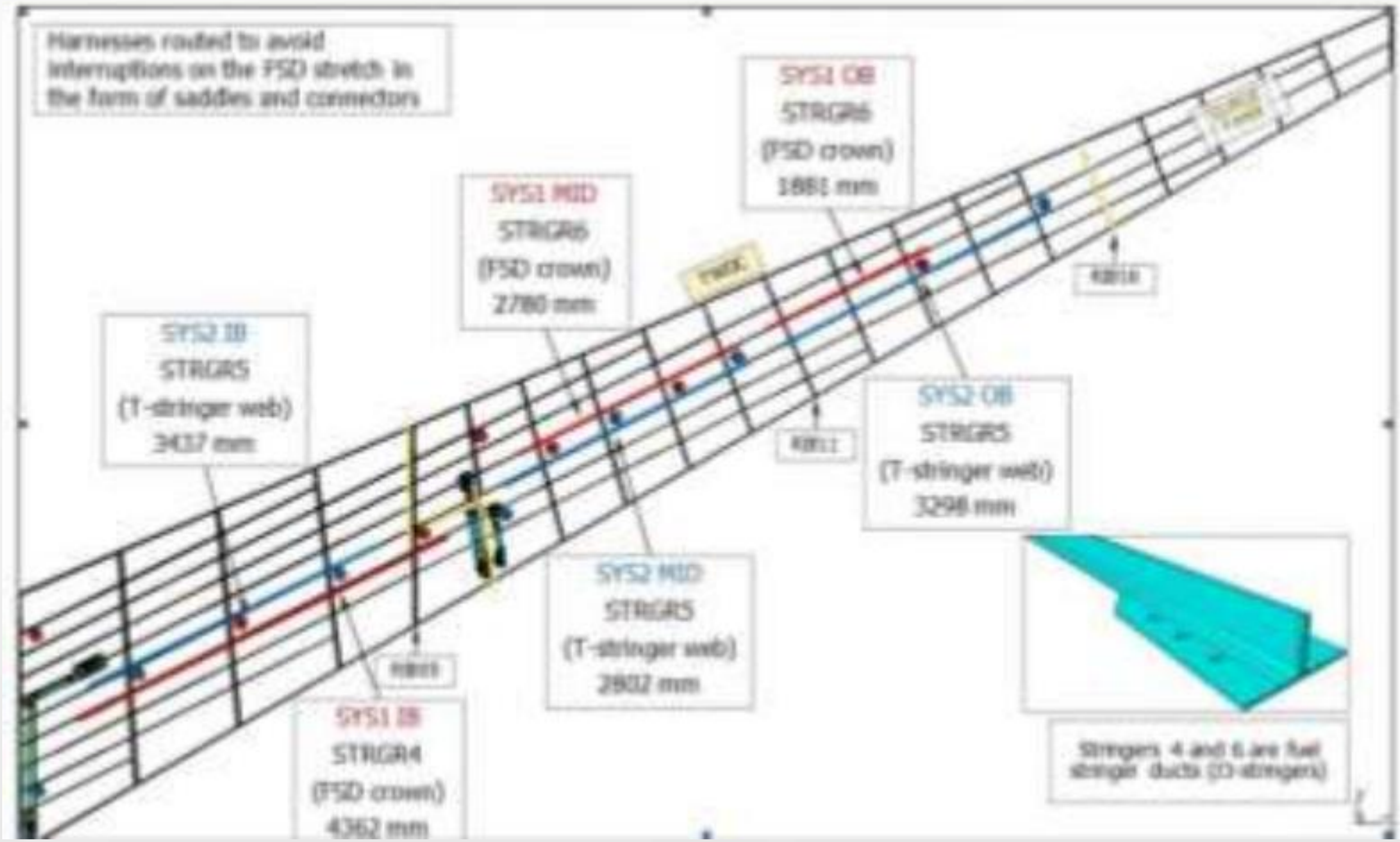
$$M_{ji} = -\frac{2EI}{L}\left[\theta_i + 2\theta_j + \frac{3}{L}(v_i - v_j)\right] + M_{ji}^F$$

$$M_{BA} = -\frac{2EI}{L}\left[2\theta_B + \theta_A + \frac{3}{L}(v_A - v_B)\right] + M_{BA}^F \rightarrow$$

$$M_{BA} = -\frac{2EI}{L}\left[0 - 9.33 \times 10^{-4} + \frac{3}{L}(-0.005 - 0)\right] + \frac{4000 \times L^2}{12} \rightarrow$$

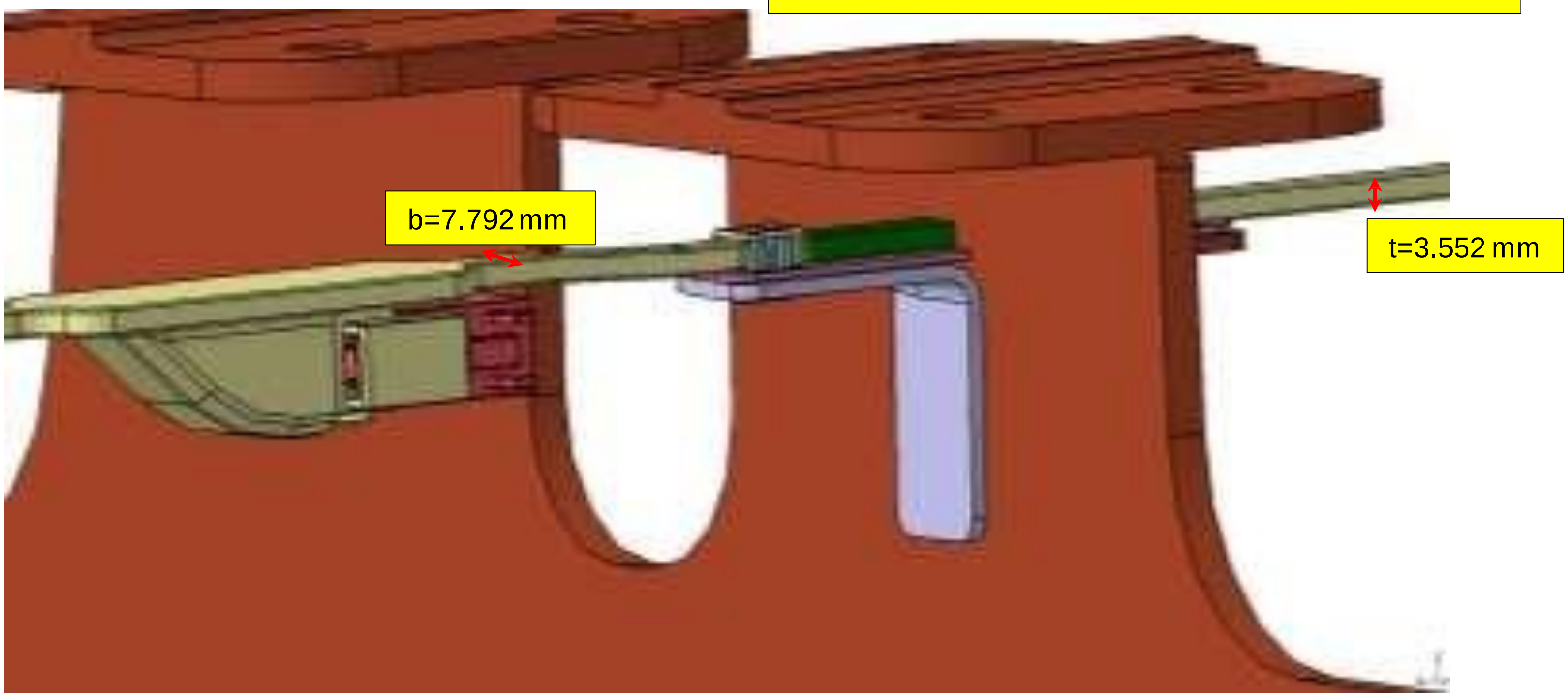
$$M_{BA} = 18705.3 N.m$$

Case study

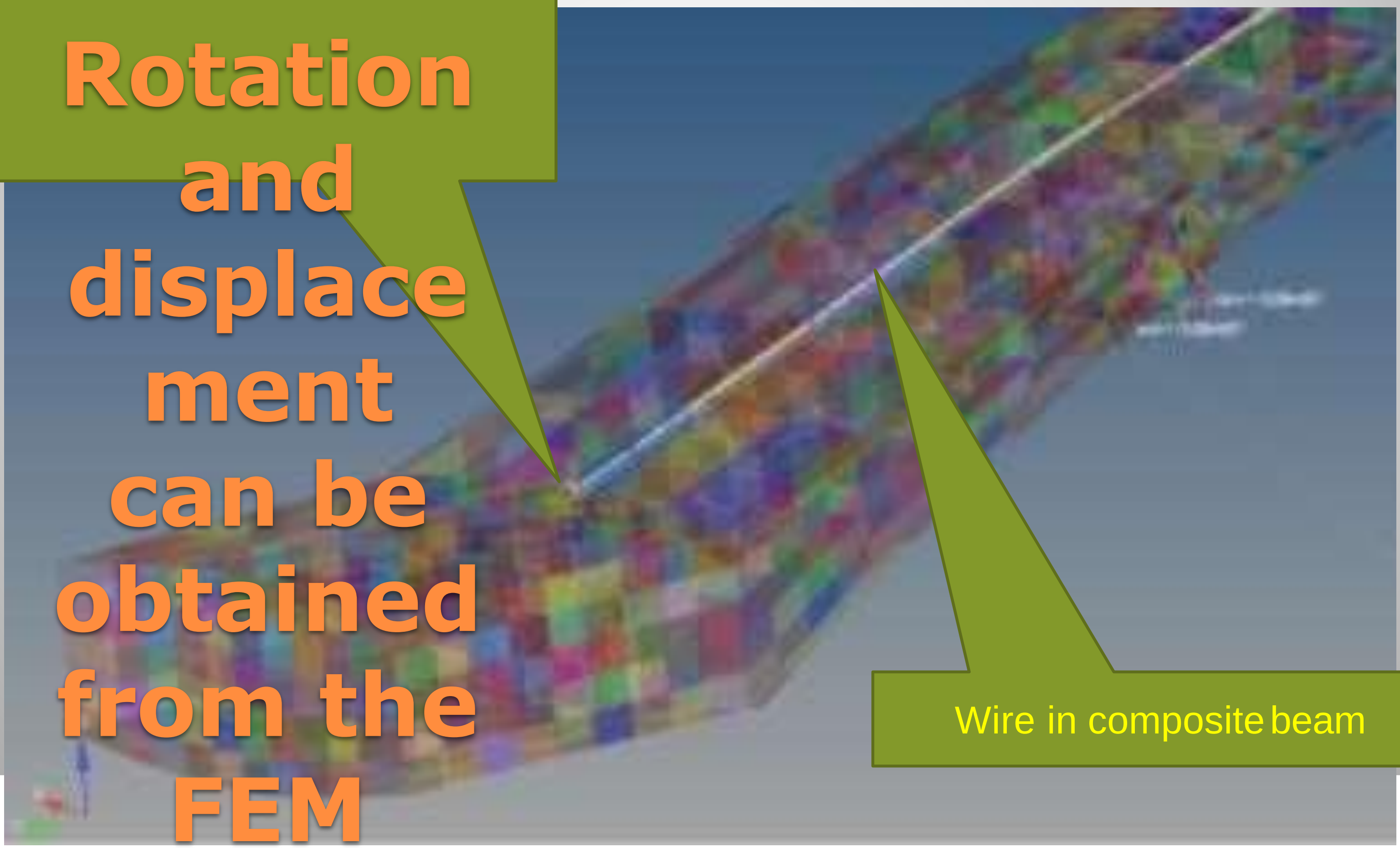

Harnesses routed to avoid interruptions on the FSD stretch in the form of saddles and connectors
SYS1 OB
STRGR6
(FSD crown)
1881 mm
SYS1 MID
STRGR6
(FSD crown)
2780 mm
SYS2 IB
STRGR5
(T-stringer web)
3437 mm
SYS2 OB
STRGR5
(T-stringer web)
3298 mm
SYS2 MID
STRGR5
(T-stringer web)
2802 mm
SYS1 IB
STRGR4
(FSD crown)
4362 mm
Stringers 4 and 6 are fuel stringer ducts (D-stringers)

Case study

$$M_{ij} = -\frac{2EI}{L}\left[2\theta_i + \theta_j + \left(\frac{3}{L}\right)(v_i - v_j)\right] + M_{ij}^{F}$$



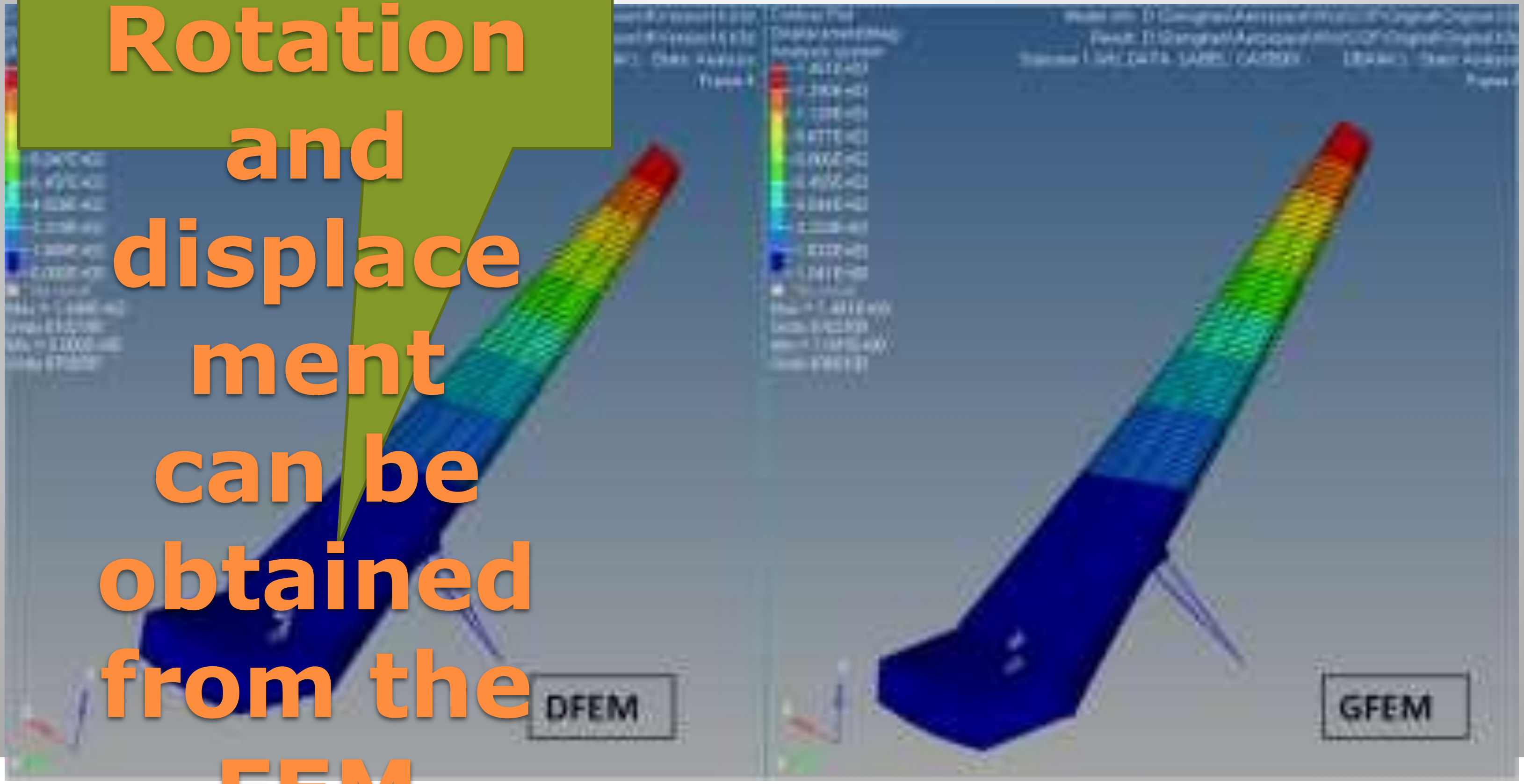
Case study



Rotation and displacement can be obtained from the FEM

Wire in composite beam

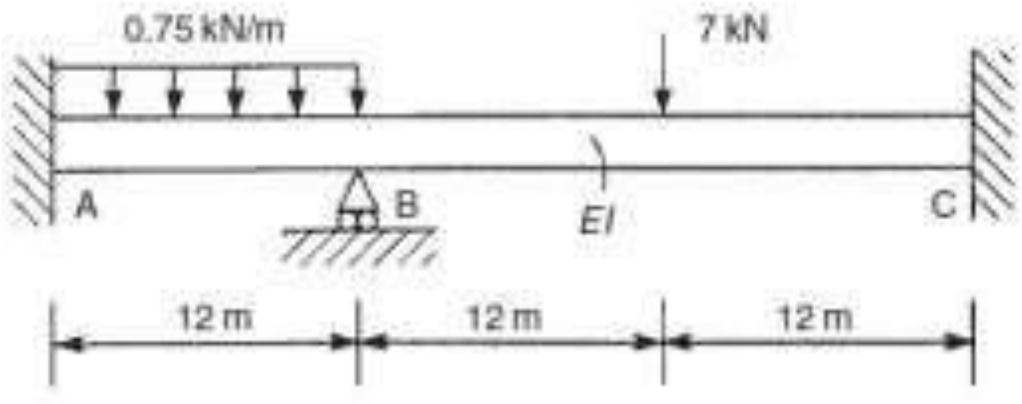
Case study



Rotation and displacement can be obtained from the FEM

Q1

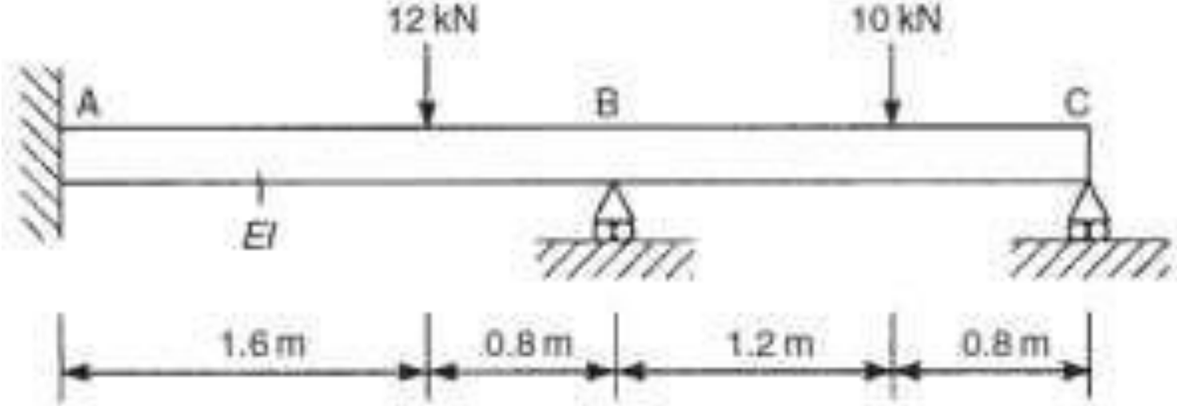
- Determine the support reactions in the beam shown below.



Ans. $R_A = 3.5$ kN $R_B = 9.0$ kN $R_C = 3.5$ kN $M_A = 7$ kN m (hogging) $M_C = -19$ kN m (hogging).

Q2

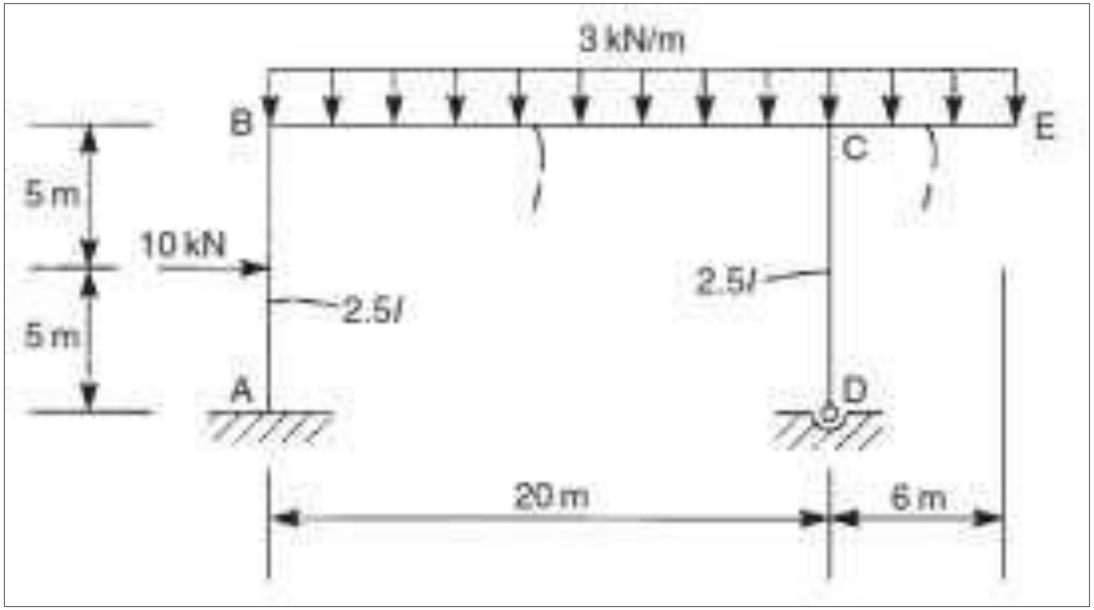
- Calculate the support reactions in the beam shown below.



Ans. $R_A = 3.3$ kN $R_B = 14.7$ kN $R_C = 4.0$ kN $M_A = 2.2$ kN m (hogging).

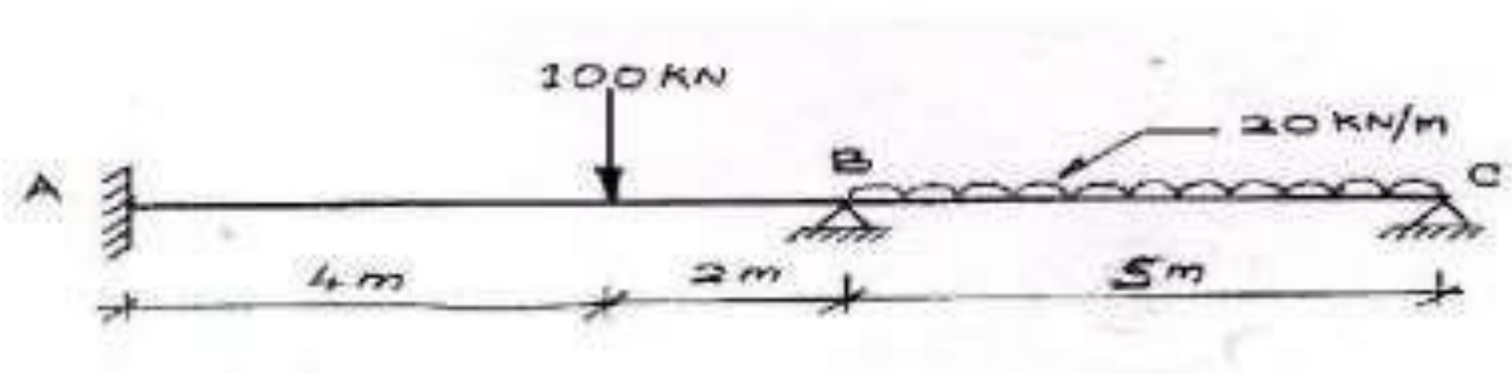
Q3

- Determine the end moments in the members of the portal frame shown. The second moment of area of the vertical members is $2.5I$ while that of the horizontal members is I.



Q4

- Analyze two span continuous beam ABC by slope deflection method. Then draw Bending moment & Shear force diagram. Take EI constant.



Outline of the presentation

- Introduction to moment distribution method.
- Important terms.
- Sign conventions.
- Fixed end moments (FEM)
- Examples;
- (A) example of simply supported beam
- (B) example of fixed supported beam with sinking of support.

Introduction

- The moment distribution method was first introduced by Prof. Hardy Cross of Illinois University in 1930.
- This method provides a convinient means of analysing statically indeterminate beams and rigid frames.
- It is used when number of reduntants are large and when other method becomes very tedious.

Important terms

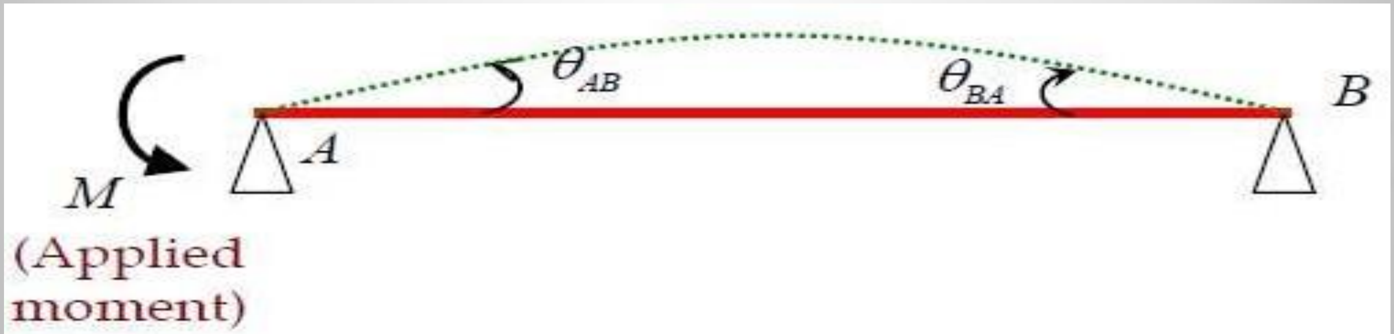
1. Stiffness

The moment required to produce a unit rotation (slope) at a simply supported end of a member is called Stiffness. It is denoted by 'K'.

A) Stiffness when both ends are hinged.

B) Stiffness when both ends are fixed.

A) Beam hinged at both ends:



$$\frac{M_{AB}}{\theta_{AB}} = \frac{3EI}{L}$$

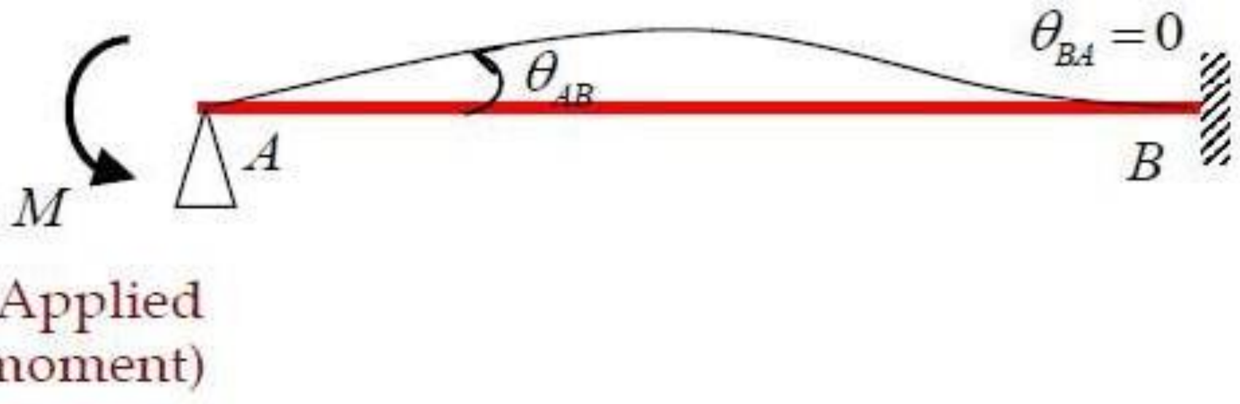
i.e., the moment required at A to induce a unit rotation at A is $\frac{3EI}{L}$

(when the far end B is free to rotate)

This moment, i.e., moment required to induce a unit rotation, is called stiffness (denoted by k).

Cont..

B) Beam hinged at near end and fixed at far end:



$$M_{AB} = \frac{2EI}{L}(2\theta_{AB} + 0) \qquad \Rightarrow \frac{M_{AB}}{\theta_{AB}} = \frac{4EI}{L}$$

i.e., the moment required at A to induce a unit rotation at A is $\frac{4EI}{L}$

(when the far end B is fixed against rotation)

Cont..

Carry over factor (C.O.F):

A moment applied at the near end induces at a fixed far end a moment equal to half its magnitude, in the same direction.

Half of moment applied at the near end is carried over to the fixed far end.

Carry over factor is 1/2.

Cont..

- Distribution factor (D.F.)

- The factor by which the applied moment is distributed to the member is known as the distribution factor.

- - far-end pined (DF = 1)

- far-end fixed (DF = 0)

- Figure:

-

Several members meeting at a joint

$$M_1 = \frac{3E_1 I_1}{L_1}\theta = k_1\theta$$

$$M_2 = \frac{4E_2 I_2}{L_2}\theta = k_2\theta$$

$$M_3 = \frac{3E_3 I_3}{L_3}\theta = k_3\theta$$

$$M_4 = \frac{4E_4 I_4}{L_4}\theta = k_4\theta$$

$$M_1 : M_2 : M_3 : M_4 :: k_1 : k_2 : k_3 : k_4$$

Cont. ...

$$M_1 = \frac{k_1}{k_1 + k_2 + k_3 + k_4} M = \frac{k_1}{\sum k} M$$

$$M_i = \frac{k_i}{\sum k} M$$

A moment applied at a joint, where several members meet, will be distributed amongst the members in proportion to their stiffness.

$$M_i = \frac{k_i}{\sum k} M$$

distribution factor

Sign Conventions

A)Support moments :

clockwise moment = +ve

anticlockwise moment = -ve

B)Rotation (slope):

clockwise moment = +ve

anticlockwise moment = -ve

Cont.

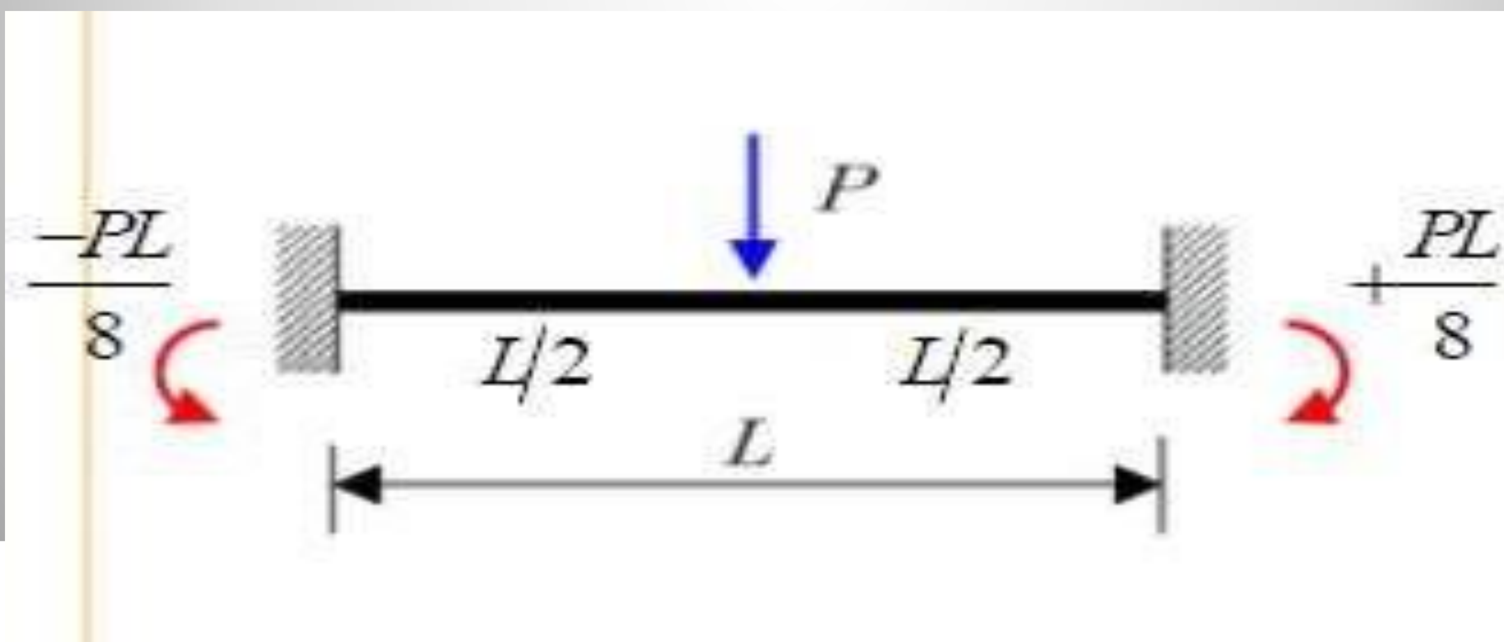
C) Sinking (settlement) :

- The settlement will be taken as +ve, if it rotates the beam as a whole in clockwise direction.

- The settlement will be taken as -ve, if it rotates the beam as a whole in anti-clockwise direction.

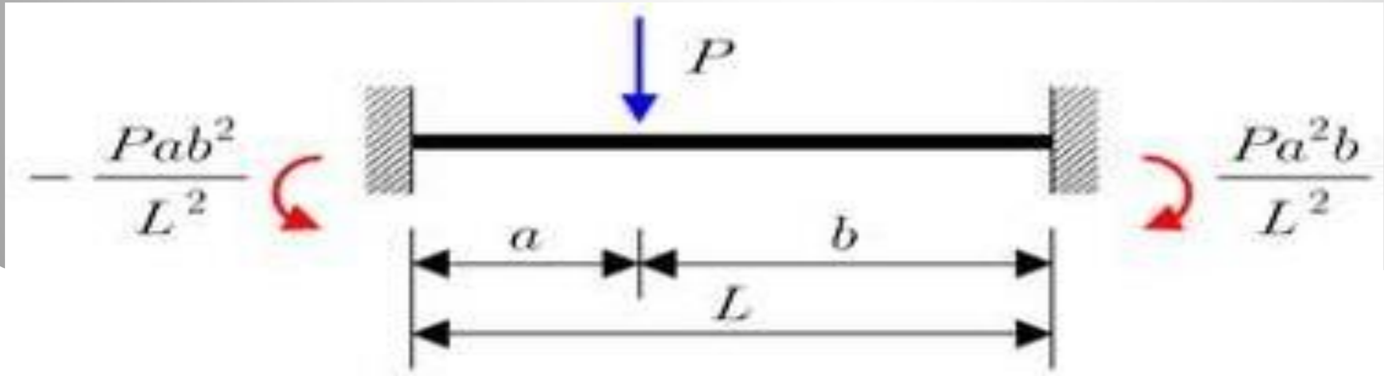
Fixed End Moments

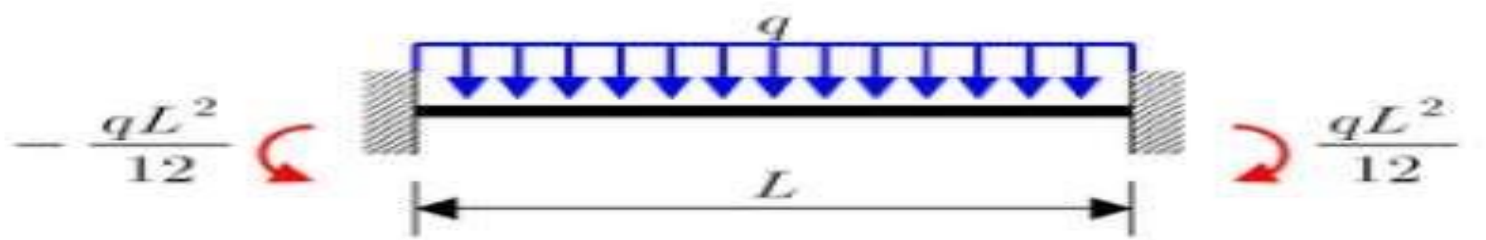
- The fixed end moments for the various load casesis as shown in figure;
- a) for centric loading;



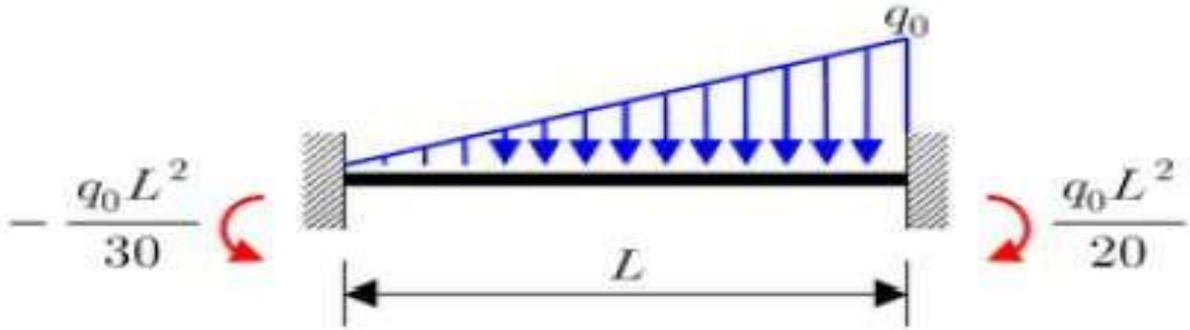
Cont..

b) for eccentric loading, udl,rotation,sinking of supports & uvl

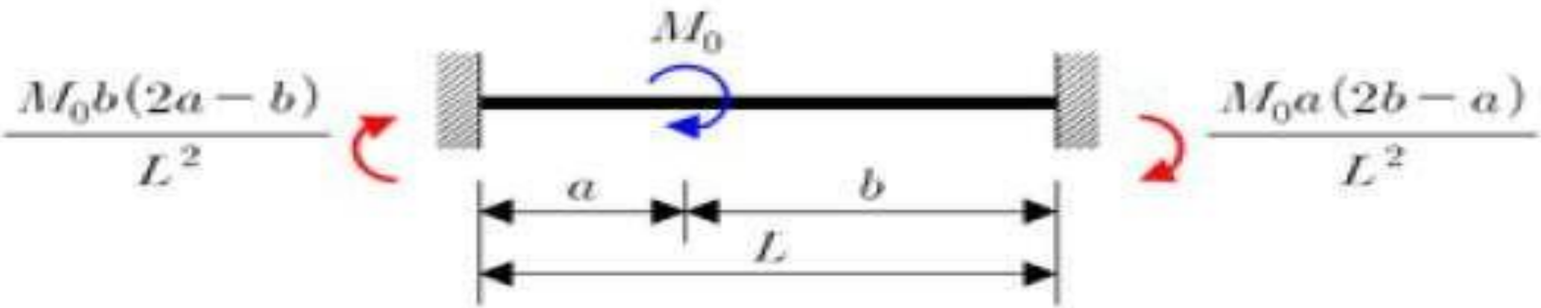




Cont
..



q_0
$-\frac{q_0 L^2}{30}$
$\frac{q_0 L^2}{20}$
L

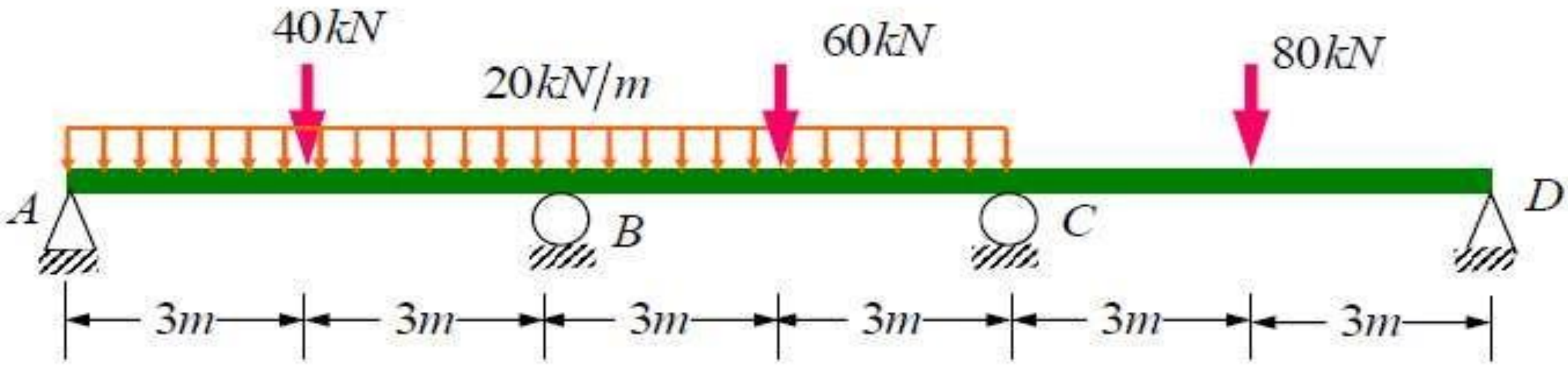


M_0
$\frac{M_0 b(2a-b)}{L^2}$
$\frac{M_0 a(2b-a)}{L^2}$
a
b
L

Cont.
..

- Fixed end moment for sinking of supports: $-\frac{6EI\delta}{L^2}$

Examp le



Fixed end moments

$$-FEM_{AB} = FEM_{BA} = \frac{wl^2}{12} + \frac{Pl}{8} = \frac{20\times 6^2}{12} + \frac{40\times 6}{8} = 60 + 30 = 90\,kNm$$

$$-FEM_{BC} = FEM_{CB} = \frac{wl^2}{12} + \frac{Pl}{8} = \frac{20\times 6^2}{12} + \frac{60\times 6}{8} = 60 + 45 = 105\,kNm$$

$$-FEM_{CD} = FEM_{DC} = \frac{Pl}{8} = \frac{80\times 6}{8} = 60\,kNm$$

Joint	Member	K (Stifness)	ΣK (Total stifness)	D.F
A	—	—	—	1
B	BA	$\frac{3EI}{6} = 0.5EI$	$1.16EI$	0.43
	BC	$4EI/6 = 0.66EI$		0.57
C	CB	$\frac{4EI}{6} = 0.66EI$	$1.16EI$	0.57
	CD	$3EI/6 = 0.5EI$		0.43
D	—	—	—	1

Cont

A	B		C		D	
	0.429	0.571	0.571	0.429		Distribution factors
-90	+90	-105	+105	-60	+60	Fixed End Moments
+90	+45			-30	-60	Release A& D, and carry over
0	**+135**	**-105**	**+105**	**-90**	**0**	Initial moments
	-12.87	-17.13	-8.565	-6.435		Distribution
		-4.283	-8.565			Carry over
	+1.837	+2.445	4.89	3.674		Distribution
		+2.445	1.223			Carry over
	-1.049	-1.396	-0.698	-0.524		Distribution
0	**+122.92**	**-122.92**	**+93.29**	**- 93.29**	**0**	Final Moments

Example-5 : Analyse the beam shown in figure by moment distribution method and draw SFD and BMD. **(G.U., Dec. 2006)**

Solution :

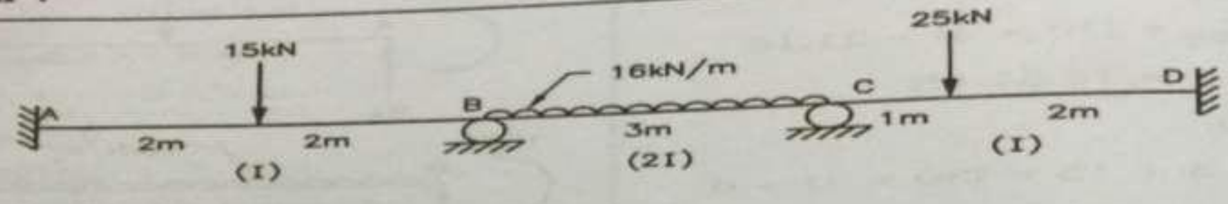


FIG. 3.21

(a) Fixed End Moments (FEM) :

$$M_f\ AB = -\frac{Wl}{8} = -\frac{15 \times 4}{8} = -7.5 \text{ kN.m}$$

$$M_f\ BA = +\frac{Wl}{8} = 7.5 \text{ kN.m}$$

$$M_f\ BC = -\frac{wl^2}{12} = -\frac{16 \times 3^2}{12} = -12 \text{ kN.m}$$

$$M_f\ CB = +\frac{wl^2}{12} = 12 \text{ kN.m}$$

$$M_f\ CD = -\frac{W\ ab^2}{l^2} = -\frac{25 \times 1 \times 2^2}{3^2} = -11.11 \text{ kN.m}$$

$$M_f\ DC = +\frac{W\ ba^2}{l^2} = \frac{25 \times 2 \times 1^2}{3^2} = 5.55 \text{ kN.m}$$

(b) Distribution Factors (D.F.) :

Sr.No.	Joint	Member	k	Σk	D.F. = $\frac{k}{\Sigma k}$
1.	B	BA	$\frac{4\ EI}{4} = 1.0\ EI$	3.67 EI	0.27
		BC	$\frac{4\ E\ (2I)}{3} = 2.67\ EI$		0.73
2.	C	CB	$\frac{4\ E\ (2I)}{3} = 2.67\ EI$	4.0 EI	0.67
		CD	$\frac{4\ EI}{3} = 1.33\ EI$		0.33
3.	A	–	–	–	0
4.	D	–	–	–	0

(c) Moment Distribution (M.D.) :

A		B		C		D	
0	0.27	0.73	0.67	0.33	0	D.F.
−7.5	7.5	−12	12	−11.11	5.55	F.E.M.
0	1.21	3.28	−0.59	−0.30	0	Balance
0.60	0	−0.30	1.64	0	−0.15	C.O.
0	0.08	0.22	−1.10	−0.54	0	Balance
0.04	0	−0.55	0.11	0	−0.27	C.O.
0	0.15	0.40	−0.07	−0.04	0	Balance
0.075	0	−0.035	0.20	0	−0.02	C.O.
0	0.009	0.026	−0.13	−0.07	0	Balance
−6.79	8.95	−8.95	12.06	−12.06	5.11	Final moments

↓ Sum

$M_{AB} = -6.79$ kN.m.

$M_{BA} = 8.95$ kN.m. $M_{BC} = -8.95$ kN.m

$M_{CB} = 12.06$ kN.m. $M_{CD} = -12.06$ kN.m

$M_{DC} = 5.11$ kN.m

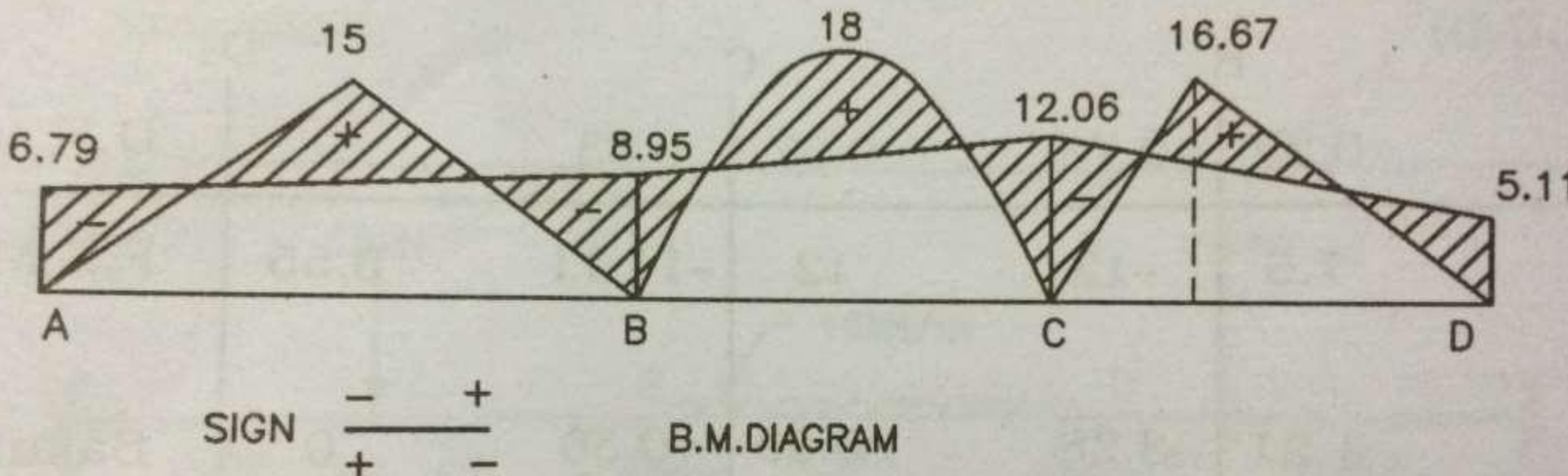
(d) B.M. diagram :

Simply supported moments.

Span AB. $M = \dfrac{Wl}{4} = \dfrac{15 \times 4}{4} = 15$ kN.m

Span BC. $M = \dfrac{wl^2}{8} = \dfrac{16 \times 3^2}{8} = 18$ kN.m

Span CD. $M = \dfrac{W\,ab}{l} = \dfrac{25 \times 1 \times 2}{3} = 16.67$ kN.m


15
18
16.67
12.06
6.79
8.95
5.11
A
B
C
D
SIGN
B.M.DIAGRAM

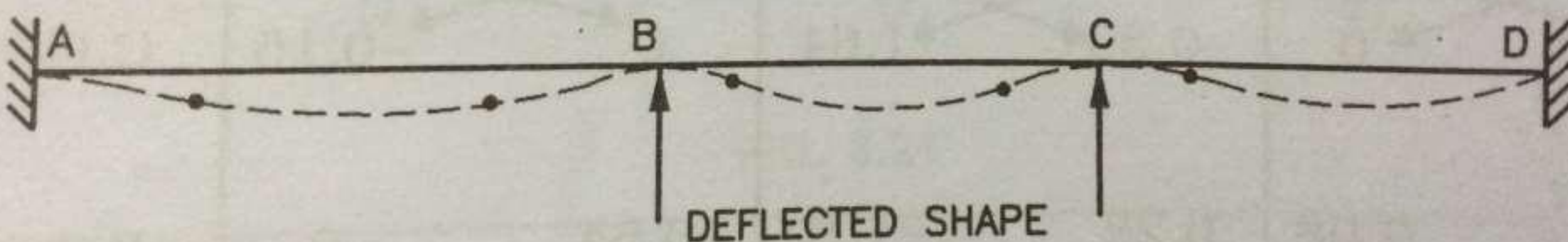
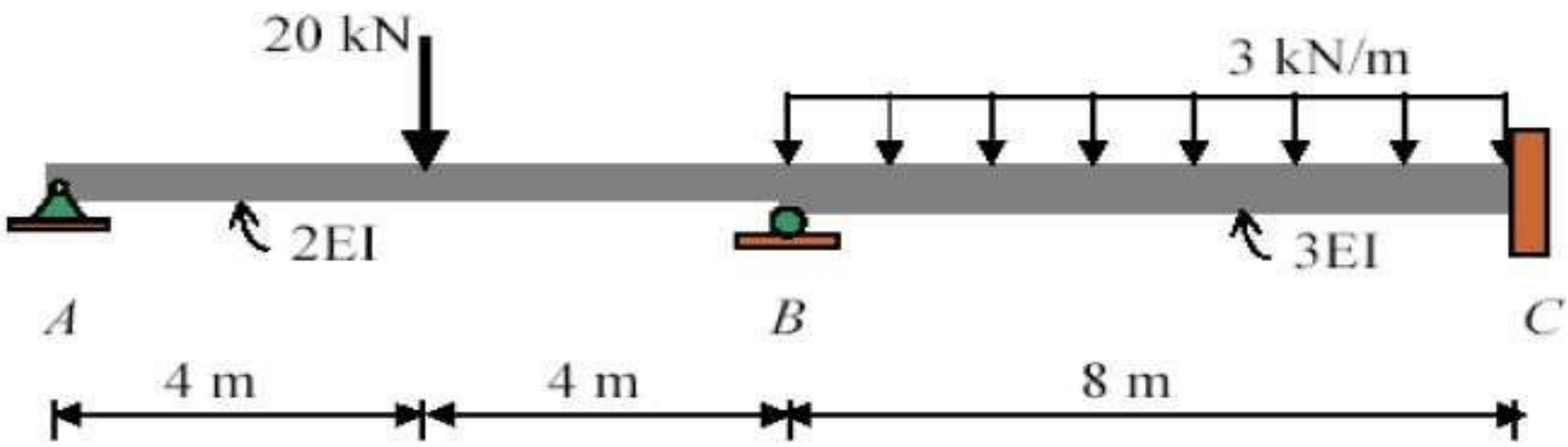

A
B
C
D
DEFLECTED SHAPE

FIG. 3.22

Example 3

Support B settles by 10 mm. $E = 200GPa, \quad I = 50\times10^6 mm^4$



$$DF_{BA} = \frac{3(2EI/8)}{3(2EI/8)+4(3EI/8)} = 0.333$$

$$DF_{BC} = \frac{4(3EI/8)}{3(2EI/8)+4(3EI/8)} = 0.6$$

$$\frac{Pl}{8}=\frac{20\times 8}{8}=20kNm \qquad \frac{wl^2}{12}=\frac{3\times 8^2}{12}=16kNm$$

$$FEM_{AB}=\frac{-Pl}{8}-\frac{6EI\delta}{L^2}$$

$$=-20-\frac{6\times 2\times 200\times 10^6\times 50\times 10^6\times 10^{-12}\times 10\times 10^{-3}}{8^2}$$

$$=-20-18.75=-38.75\ kNm$$

$$FEM_{BA}=\frac{Pl}{8}-\frac{6EI\delta}{L^2}$$

$$=20-\frac{6\times 2\times 200\times 10^6\times 50\times 10^6\times 10^{-12}\times 10\times 10^{-3}}{8^2}$$

$$=20-18.75=1.25\ kNm$$

$$FEM_{BC} = \frac{-wl^2}{12} + \frac{6EI\delta}{L^2}$$

$$= -16 + \frac{6 \times 3 \times 200 \times 10^6 \times 50 \times 10^6 \times 10^{-12} \times 10 \times 10^{-3}}{8^2}$$

$$= -16 + 28.125 = 12.125\ kNm$$

$$FEM_{CB} = \frac{wl^2}{12} + \frac{6EI\delta}{L^2}$$

$$= 16 + \frac{6 \times 3 \times 200 \times 10^6 \times 50 \times 10^6 \times 10^{-12} \times 10 \times 10^{-3}}{8^2}$$

$$= 16 + 28.125 = 44.125\ kNm$$

A	B		C	
1	0.333	0.667	0	
-38.75	+1.25	12.125	44.125	Fixed End Moments
38.75	19.375			Release A, and carry over
0.0	**20.625**	**12.125**	**44.125**	Initial Moments
	-10.906	-21.844	0	Distribution
			-10.922	Carry over
				Distribution
0.0	**+ 9.719**	**-9.719**	**+33.203**	Final Moments


Thank you!

Structural Analysis - II

Approximate Methods

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Module

Approximate Methods of Analysis of Multi-storey Frames

- **Analysis for vertical loads** - Substitute frames-Loading conditions for maximum positive and negative bending moments in beams and maximum bending moment in columns
- **Analysis for lateral loads** - Portal method–Cantilever method– Factor method.

Why approximate

- Rapid check on computer aided analysis
- Preliminary dimensioning before exact analysis

Advantage
- Faste

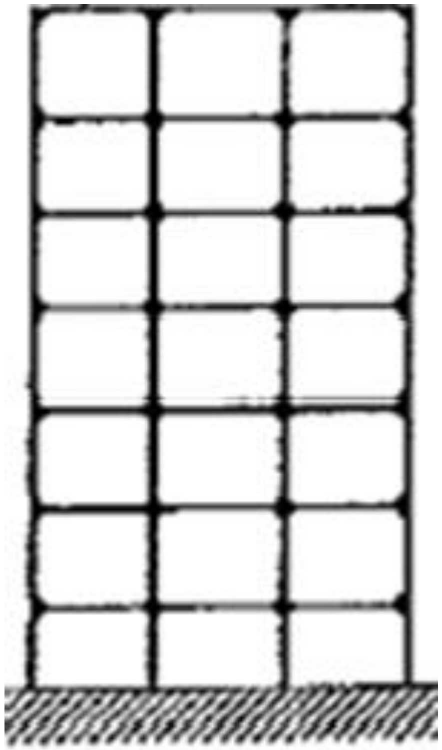
Disadvantage
- Results are

•Approximate methods are particularly useful for multi-storey frames taller than 3 storeys.

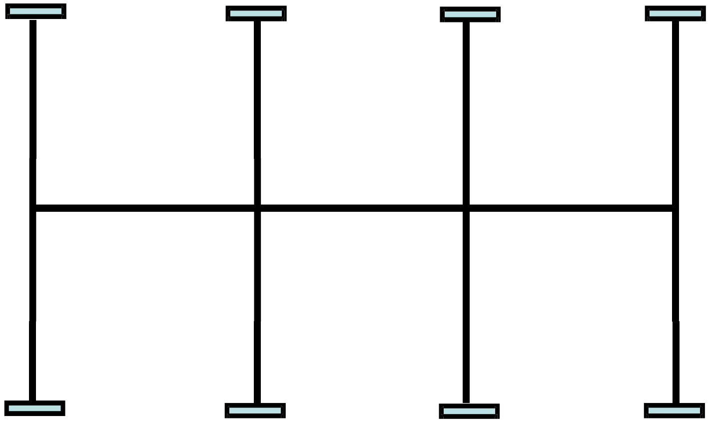
Approximate analysis for Vertical

SUBSTITUTE FRAME METHOD

- **Analyse only a part of the frame – substitute frame**
- **Carry out a two-cycle moment distribution**



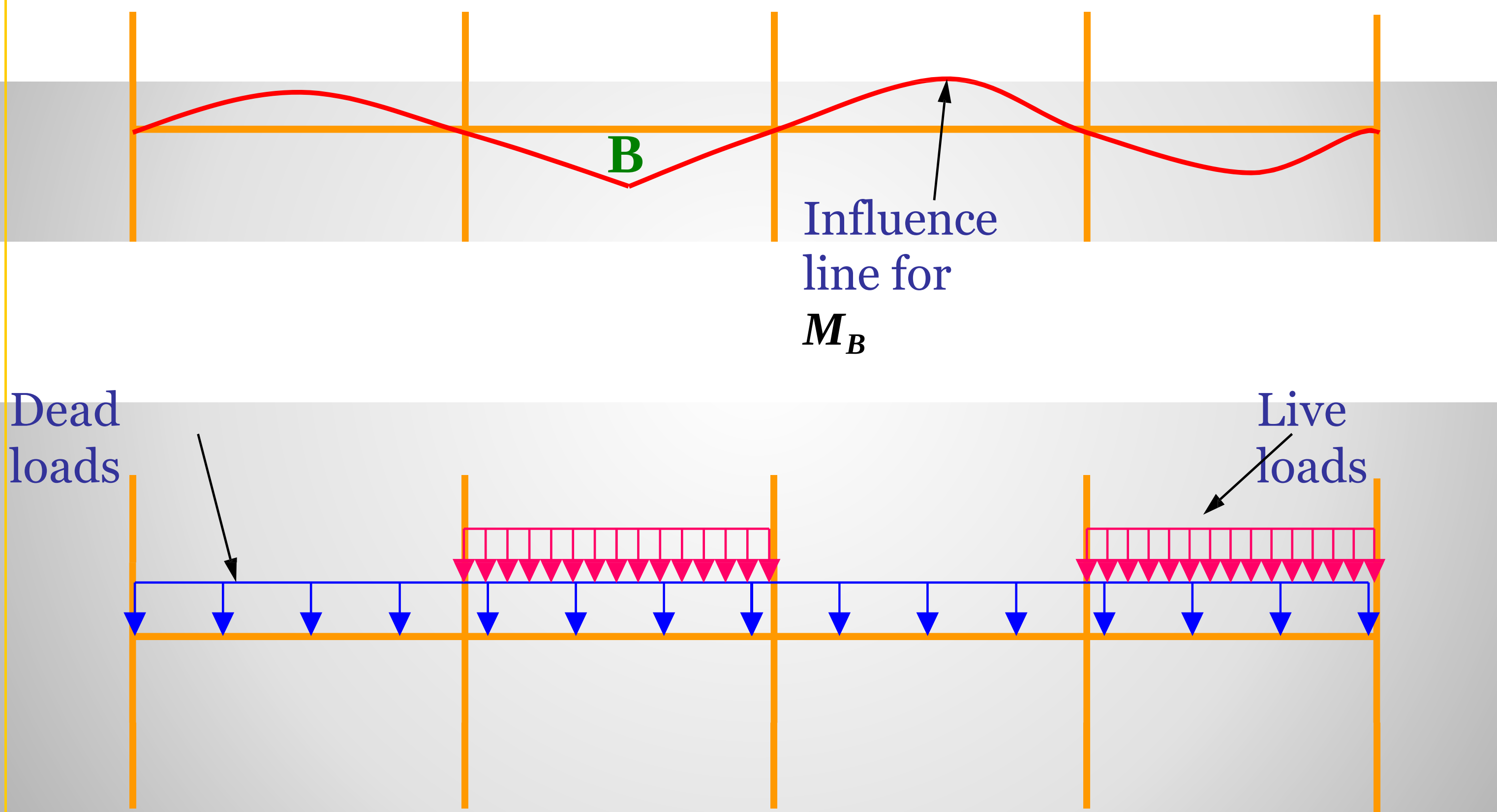
Actual frame



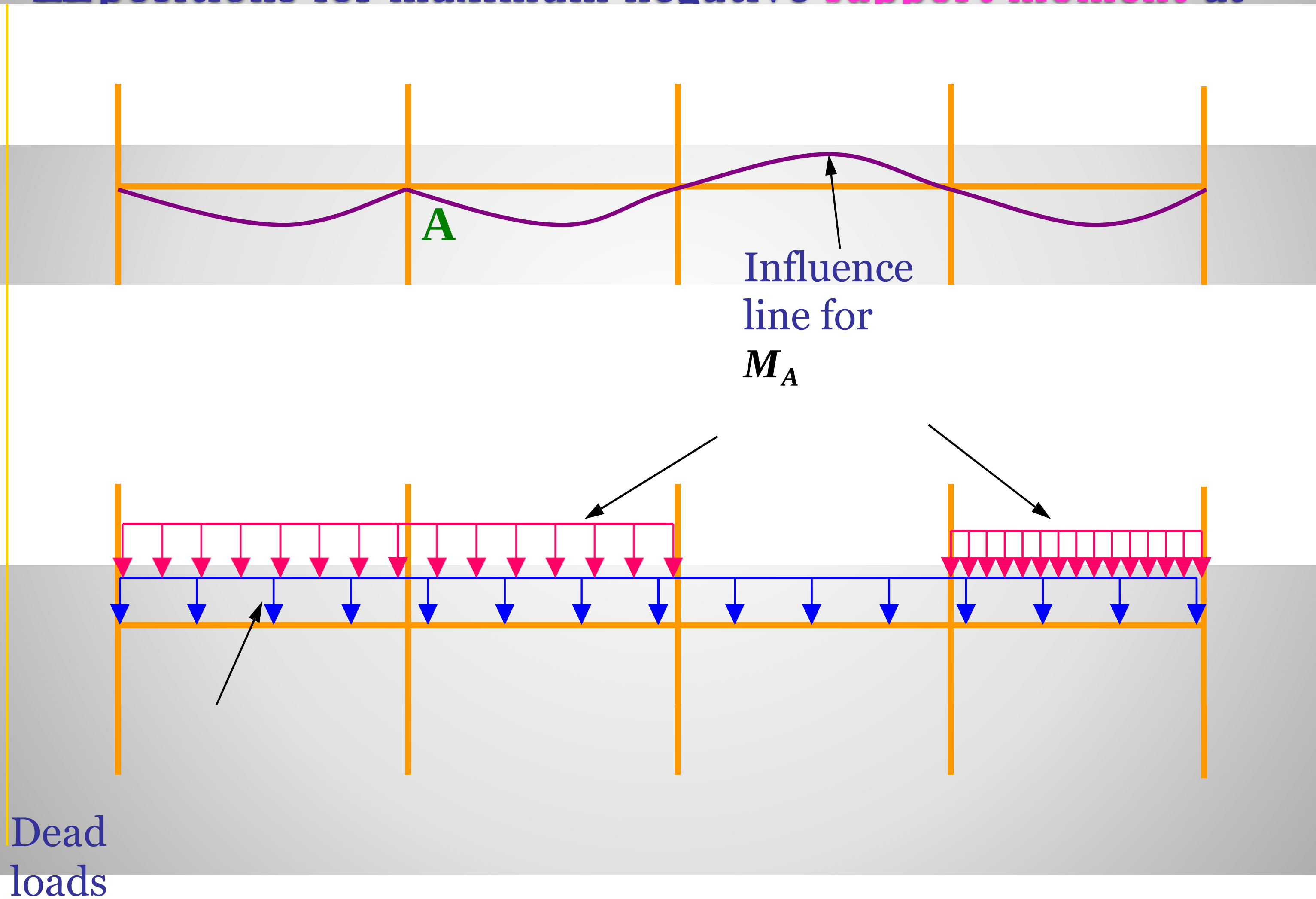
Substitute frame

- **Analysis done for:**
    - **Beam span moments**
    - **Beam support moments**
    - **Column moments**
- **Liveload positioning for the worst condition**
- **For the same frame, liveload positions for maximum span moments, support moments and column moments may be different**
- **For maximum moments at different points, liveload positions may de different**

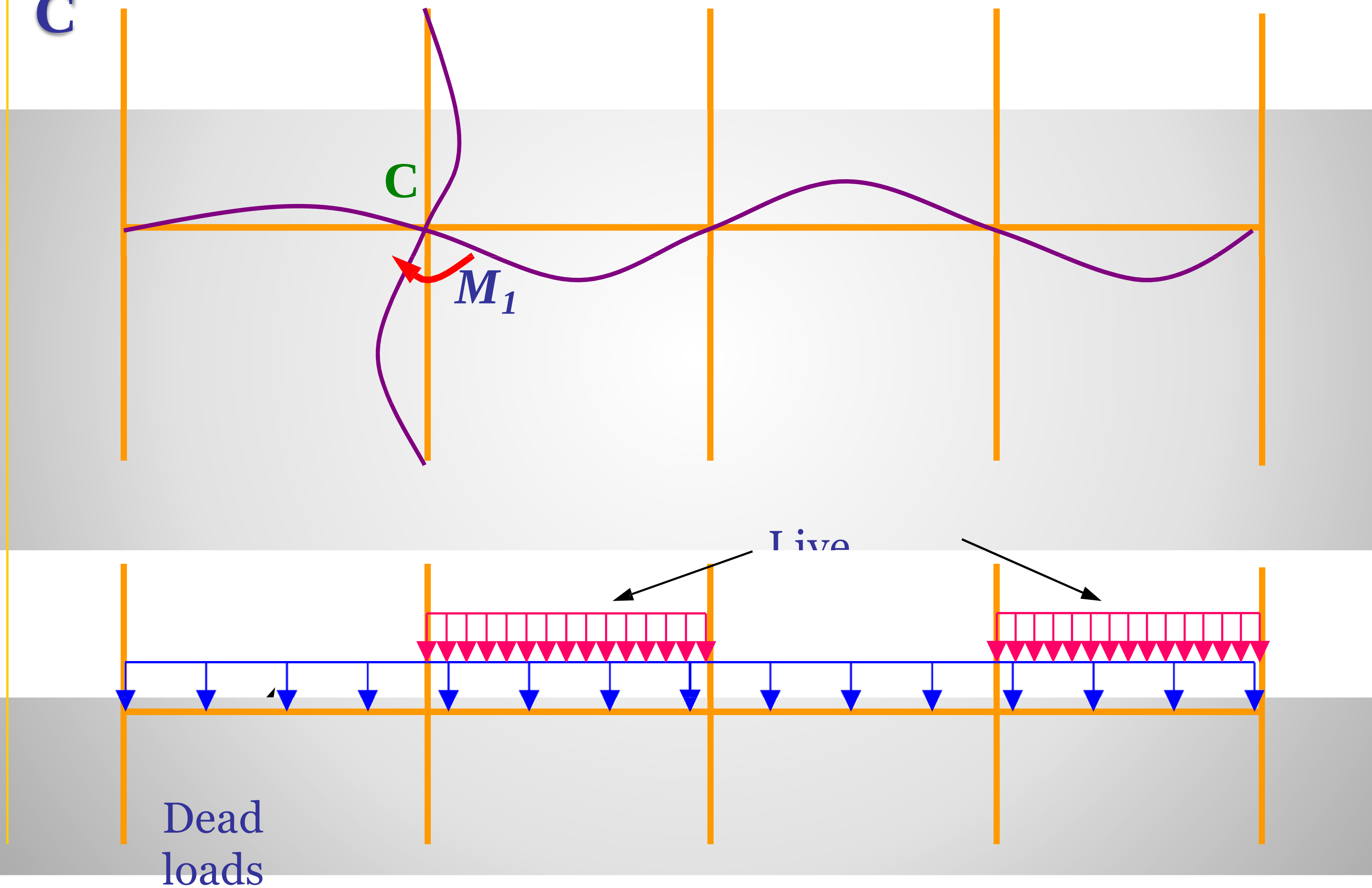
LL positions for maximum positive span moment at



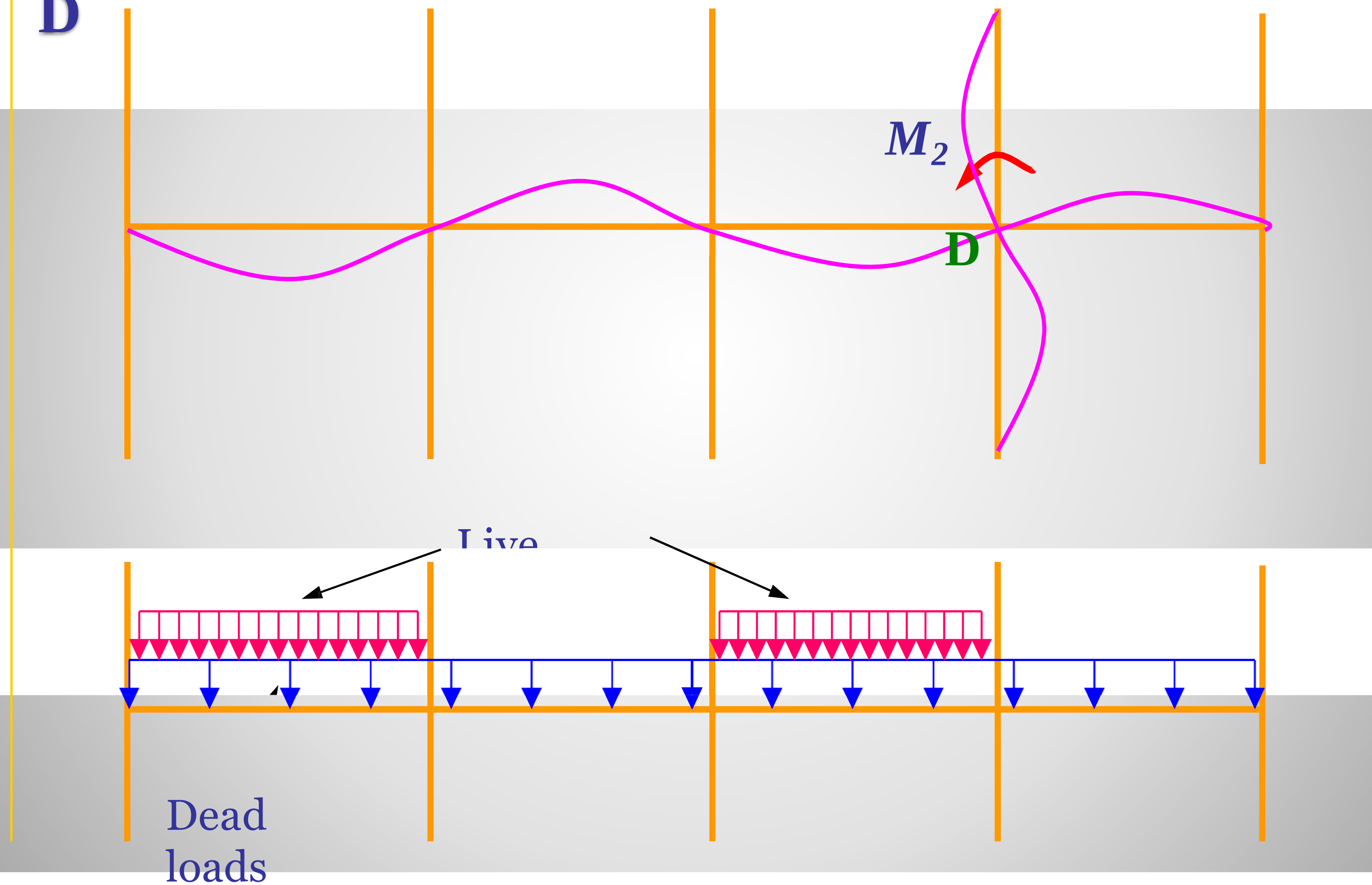
LL positions for maximum negative support moment at



LL positions for maximum column moment M_1 at C

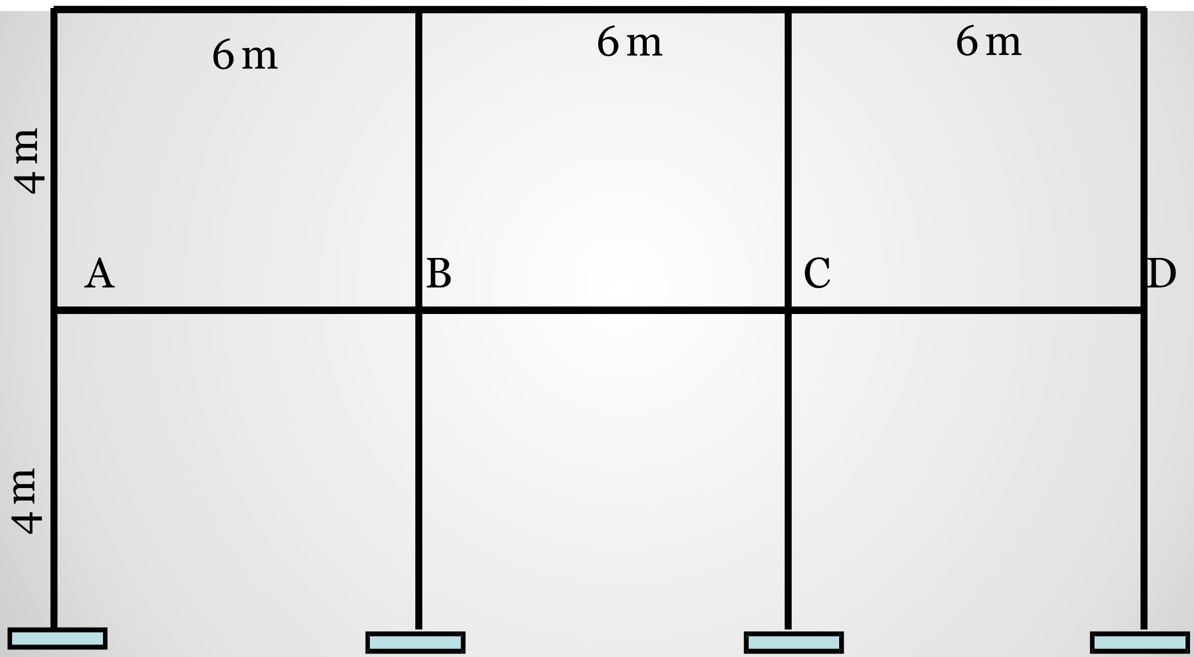


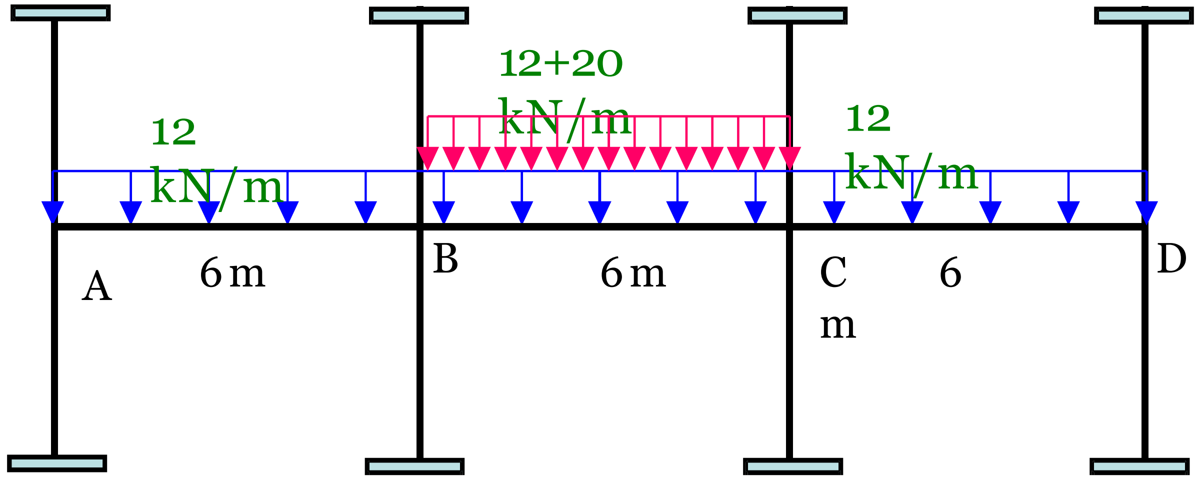
LL positions for maximum column moment M_2 at D



Problem 1: **Total dead load is 12 kN/m. Total live**

kN/m. Analyse the frame for midspan positive moment on BC.





Fixed end moments

$$FEM_{AB} = \frac{-wl^2}{12} \quad \frac{-12\times 6^2}{} = -36kNm$$

1

$$FEM_{B} = 36kNm$$

$$FEM_{BC} = \frac{-32\times 6^2}{12} = -96kNm$$

2

$$FEM_{CB} =$$

$$-FEM_{CD} = FEM_{DC} = 36kNm$$

Distribution factors

$$DF_{AB} = \frac{K_1}{K_1 + K_2 + K_3} = \frac{4EI/4}{4EI/6 + 4EI/4 + 4EI/4} = 0.25 = DF_{DC}$$

$$DF_{BA} = \frac{K_1}{K_1 + K_2 + K_3 + K_4} = \frac{4EI/6}{4EI/4 + 4EI/6 + 4EI/4 + 4EI/6} = 0.2$$

$$DF_{BC} = DF_{CD} = DF_{CB} = DF_{BA} = 0.2$$

A		B		C		D
0.25	0.2	0.2		0.2	0.2	0.25
*		*			*	
*		*			*	
*		*			*	
*		*			*	
	*	*			*	
		*				
		*				
		*				
		*				
		*				

DFs

FE

M

Dis

t

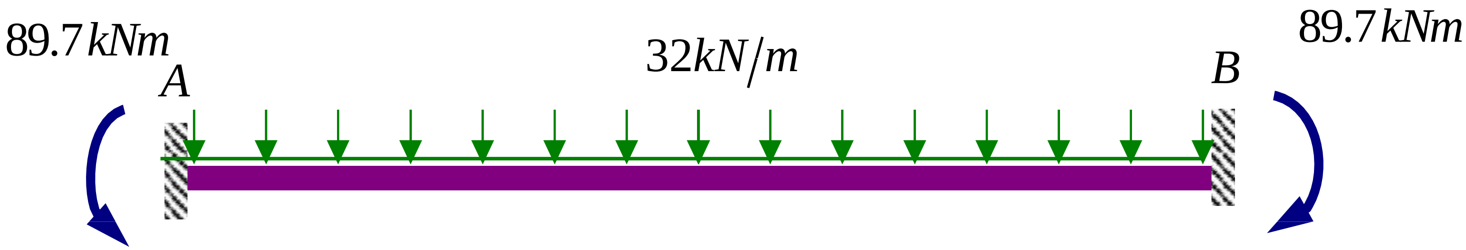
CO

Dist

Final
Moments

A	B		C		D
0.2	0.2		0.2		0.25

Moments



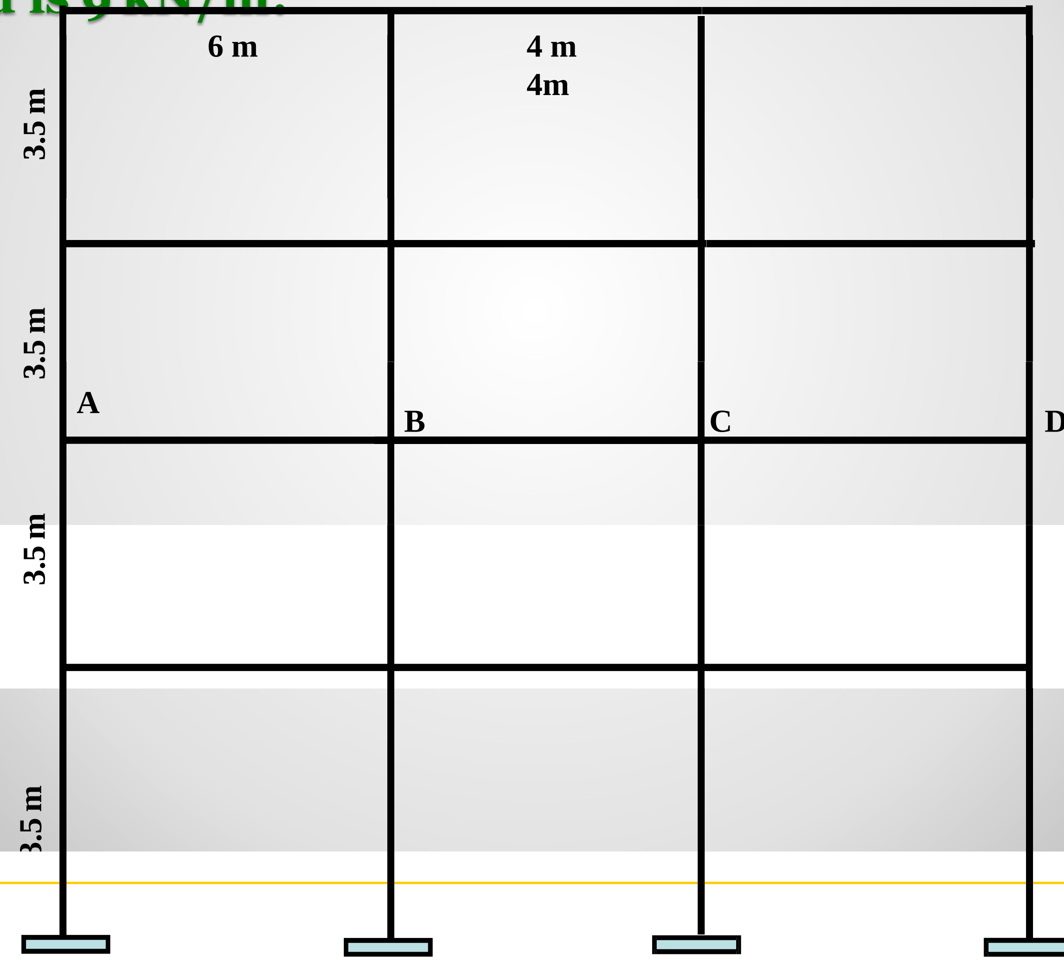
Midspan positive moment on
BC,

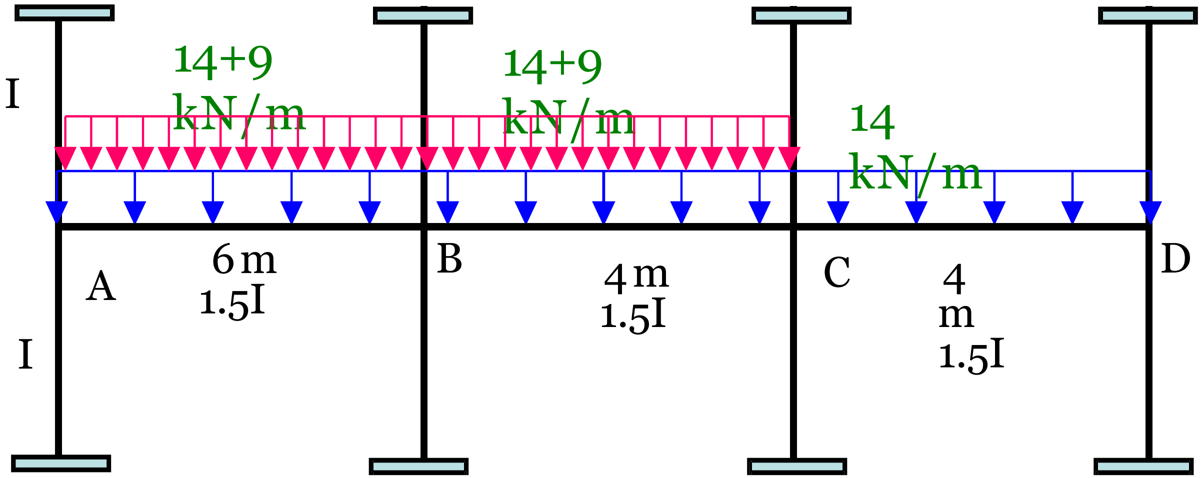
$$M_E = -89.7 - 32 \times \frac{3^2}{2} \quad \frac{32 \times 6}{2} \times 3 = 54.3$$

kNm

$+$

Problem 2: Analyse the frame for beam negative moment at B. Moment of inertia of beams is 1.5 times that of columns. Total dead load is 14 kN/m and total live load is 9 kN/m.





Fixed end moments

$$FEM_{AB} = \frac{-wl^2}{12} \quad \frac{-23\times 6^2}{12} = -69\,kNm \qquad FEM_{B} = 69\,kNm$$

$$-FEM_{BC} = FEM_{CB} = \frac{23\times 4^2}{12} =$$

$$-FEM_{CD} = FEM_{DC} = \frac{14\times 4^2}{12} =$$

Distribution factors

$$DF_{AB} = \frac{K_1}{K_1+K_2+K_3} = \frac{4E\ /}{4E(1.5I)/6+4EI/3.5+4EI/3.5} =$$

$$DF_{BA} = \frac{K_1}{K_1+K_2+K_3+K_4} = \frac{1.5I/6}{1.5I/6+I/3.5+I/3.5+1.5I/4} = 0.209$$

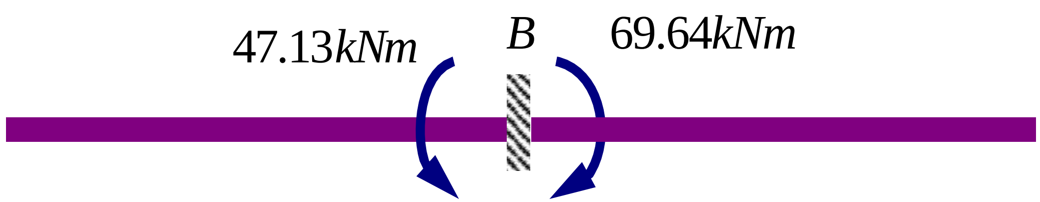
$$DF_{BC} = \frac{K_1}{K_1+K_2+K_3+K_4} = \frac{1.5I/4}{1.5I/6+I/3.5+I/3.5+1.5I/4} = 0.313$$

$DF_{CB} = 0.284$, $DF_{CD} = 0.284$, $DF_{DC} = 0.396$

	A	B	B	C	C	D
DFs	0.304	0.209	0.313	0.284	0.284	0.396
FE	*		*		*	
M	*		*			
Dis t	*		*			
CO	*		*			
Dist		*	*			
		*	*			
Final Moments		*	*			

A	B		C		D
0.30	0.209		0.284	0.284	0.396

Moments



Max. beam negative moment at B $= 69.64$ *kNm*

Approximate analysis for Horizontal

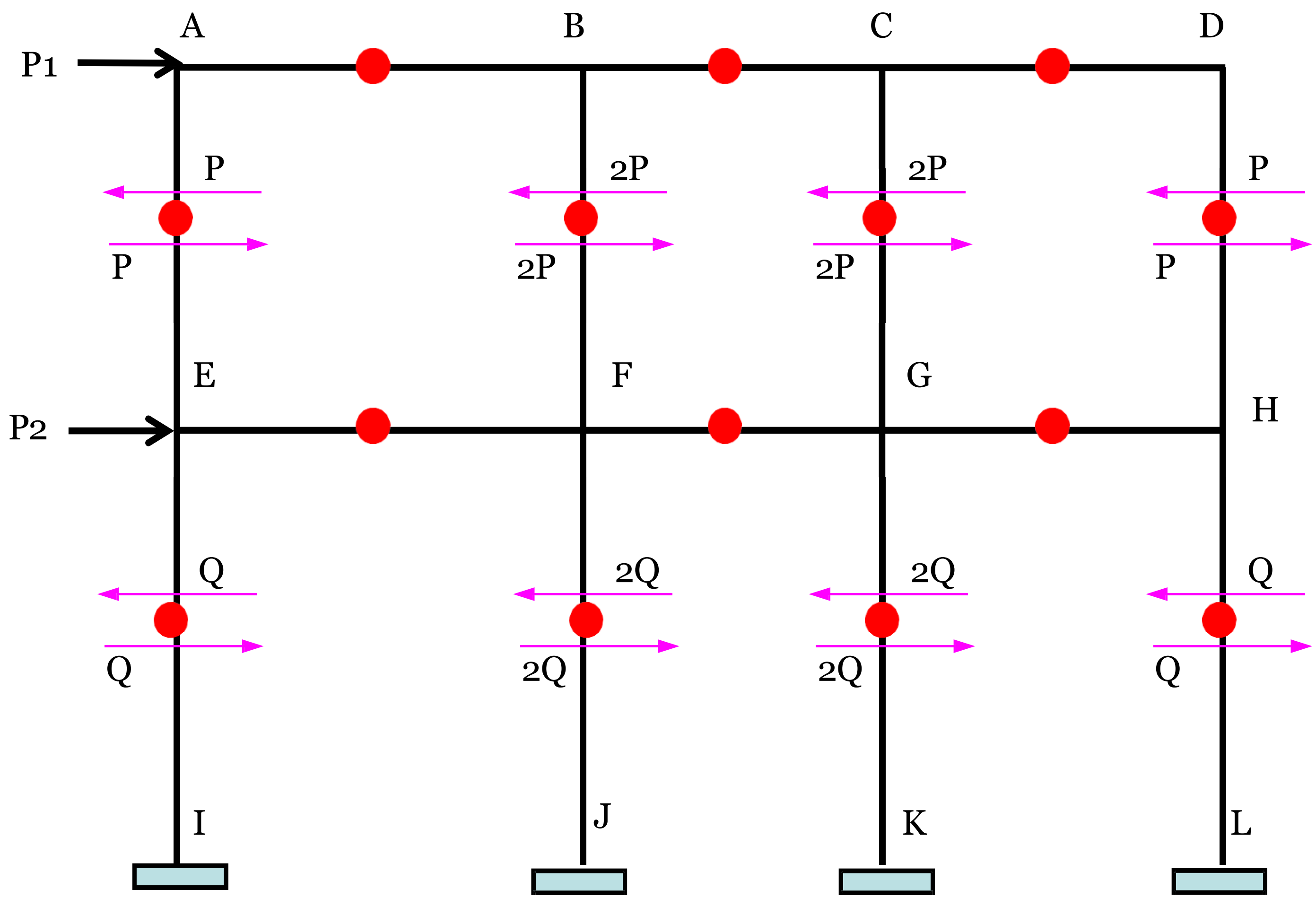
1. **Portal method**
2. **Cantilever method**
3. **Factor method**

PORTAL METHOD

Assumptions

1. The points of contraflexure in all the members lie at their midpoints.
2. Horizontal shear taken by each interior column is double that taken by each exterior column.

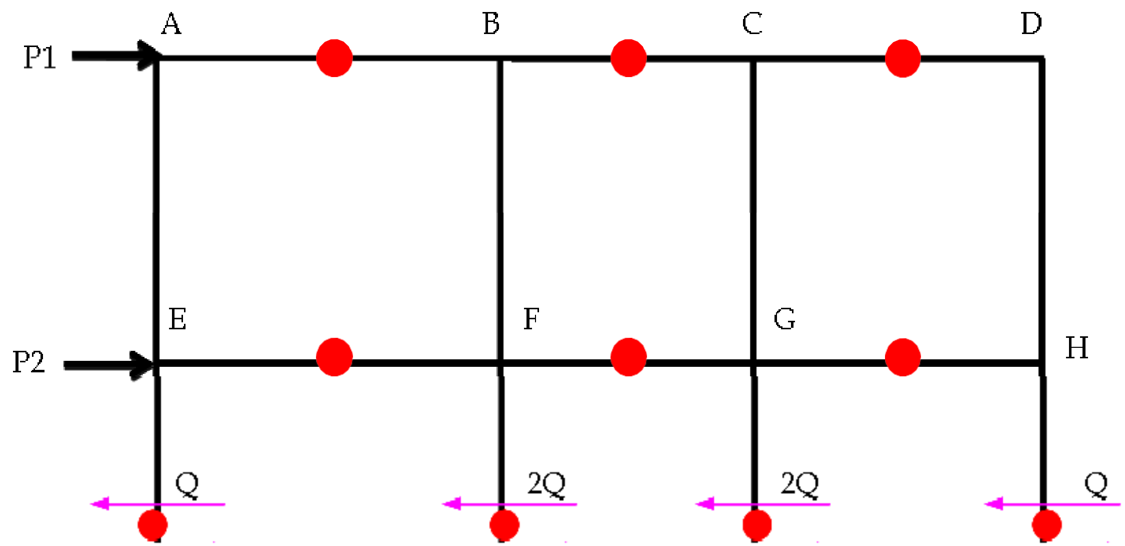
Horizontal forces are assumed to act only at the joints.



A
B
C
D
P1
P
2P
2P
P
P
2P
2P
P
E
F
G
H
P2
Q
2Q
2Q
Q
Q
2Q
2Q
Q
I
J
K
L

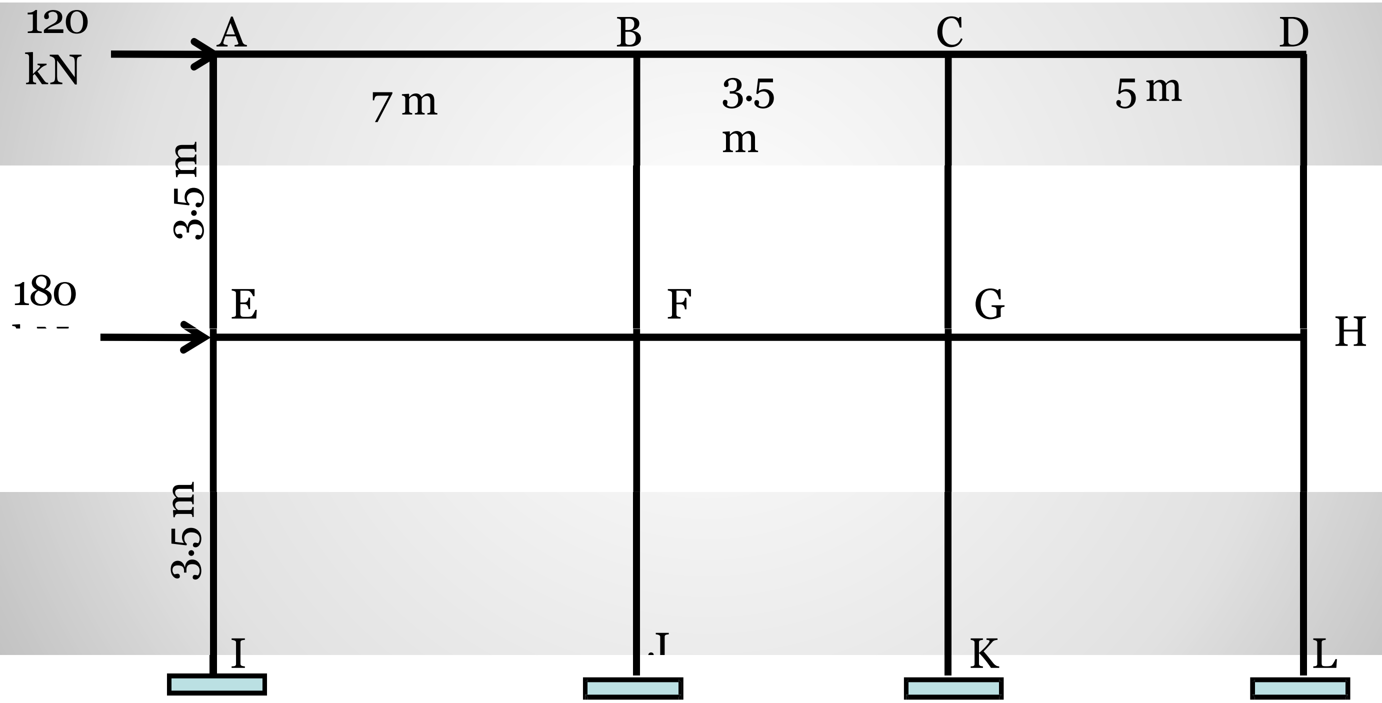
$P_1 = P + 2P + 2P + P$

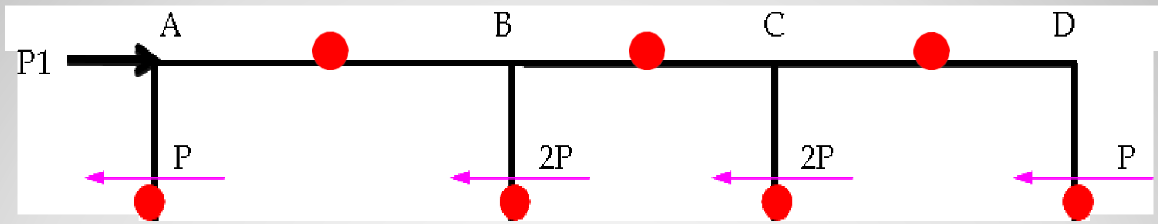
$\Rightarrow P = \frac{P_1}{6}$



$$P_1 + P_2 = Q + 2Q + 2Q + Q \qquad \Rightarrow Q = \frac{P_1 + P_2}{6}$$

Problem 3: Analyse the frame using portal



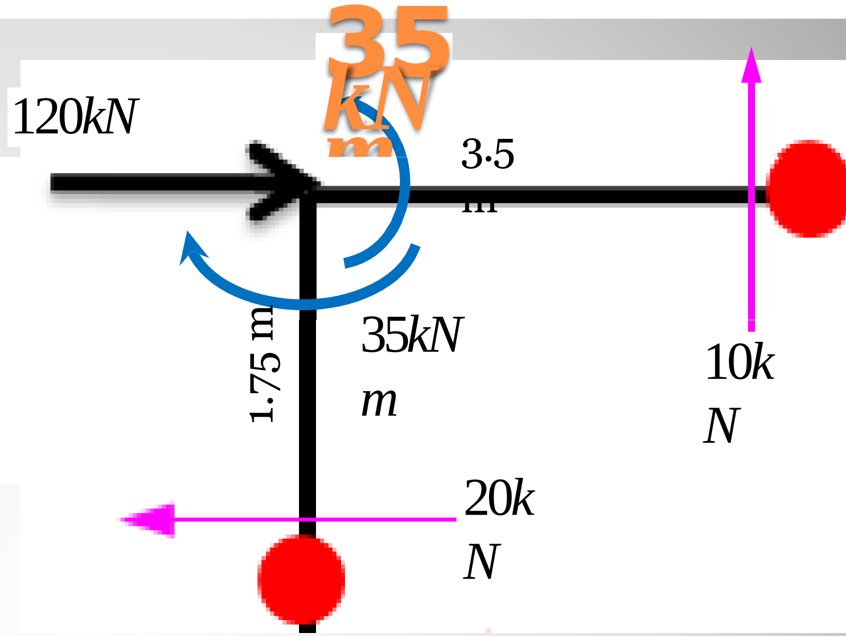


Horizontal shears:

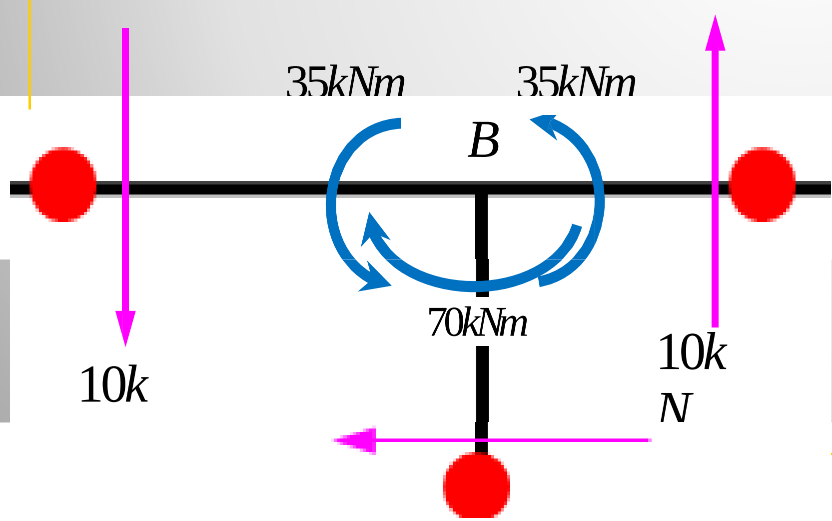
For the top storey, $P_1 = P + 2P + 2P + P \Rightarrow P = \frac{P_1}{6} = \frac{120}{6} = 20kN$

For the bottom storey, $Q = \frac{P_1 +}{6} = \frac{120+180}{6} =$

Moments:

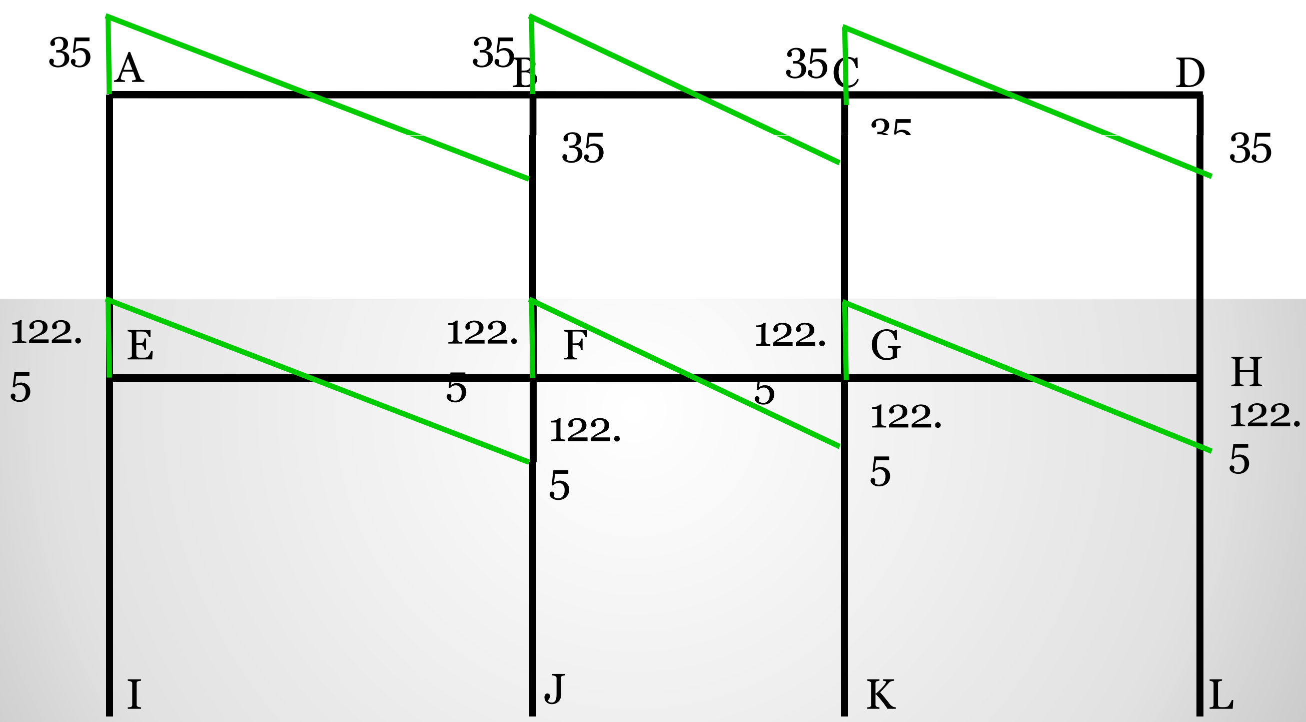


35
kN
m
120kN
3.5
m
1.75 m
35kN
m
10k
N
20k
N

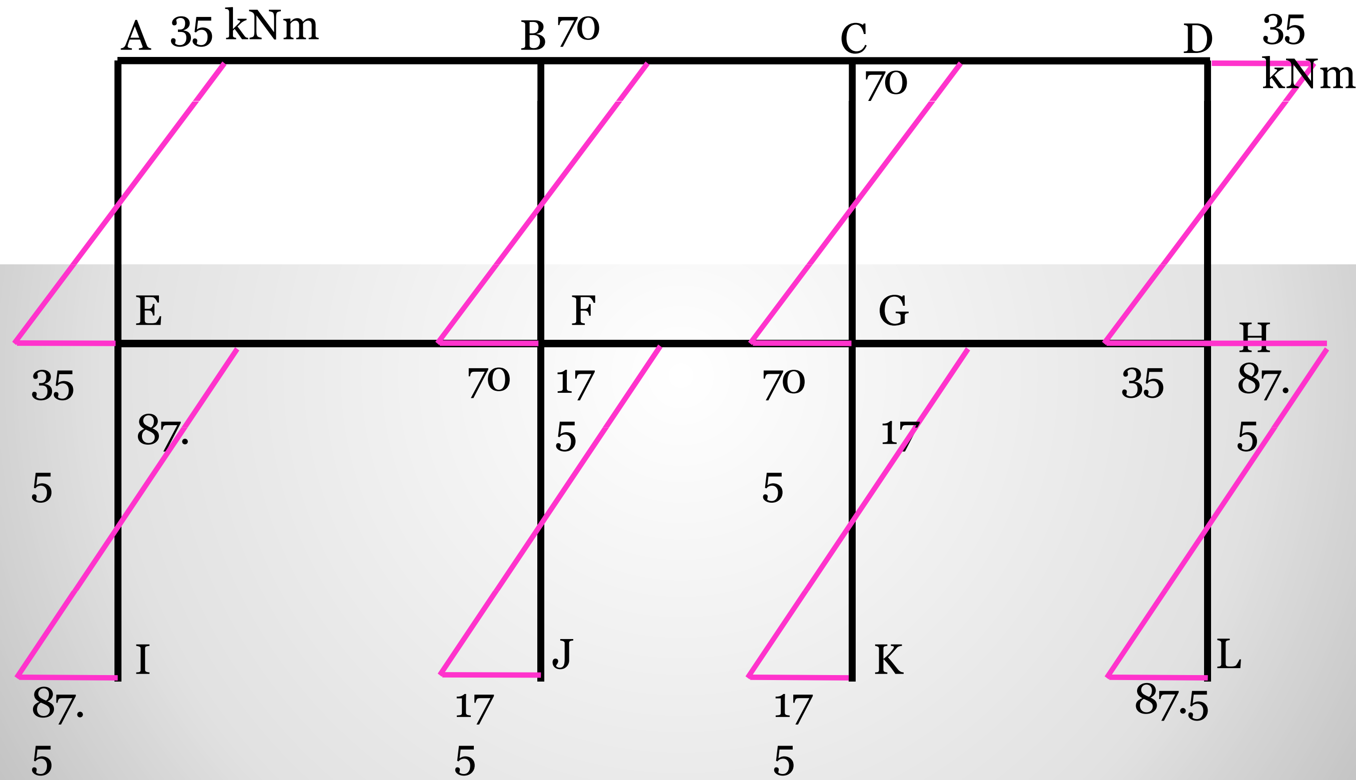


35kNm
35kNm
B
70kNm
10k
10k
N

Beam

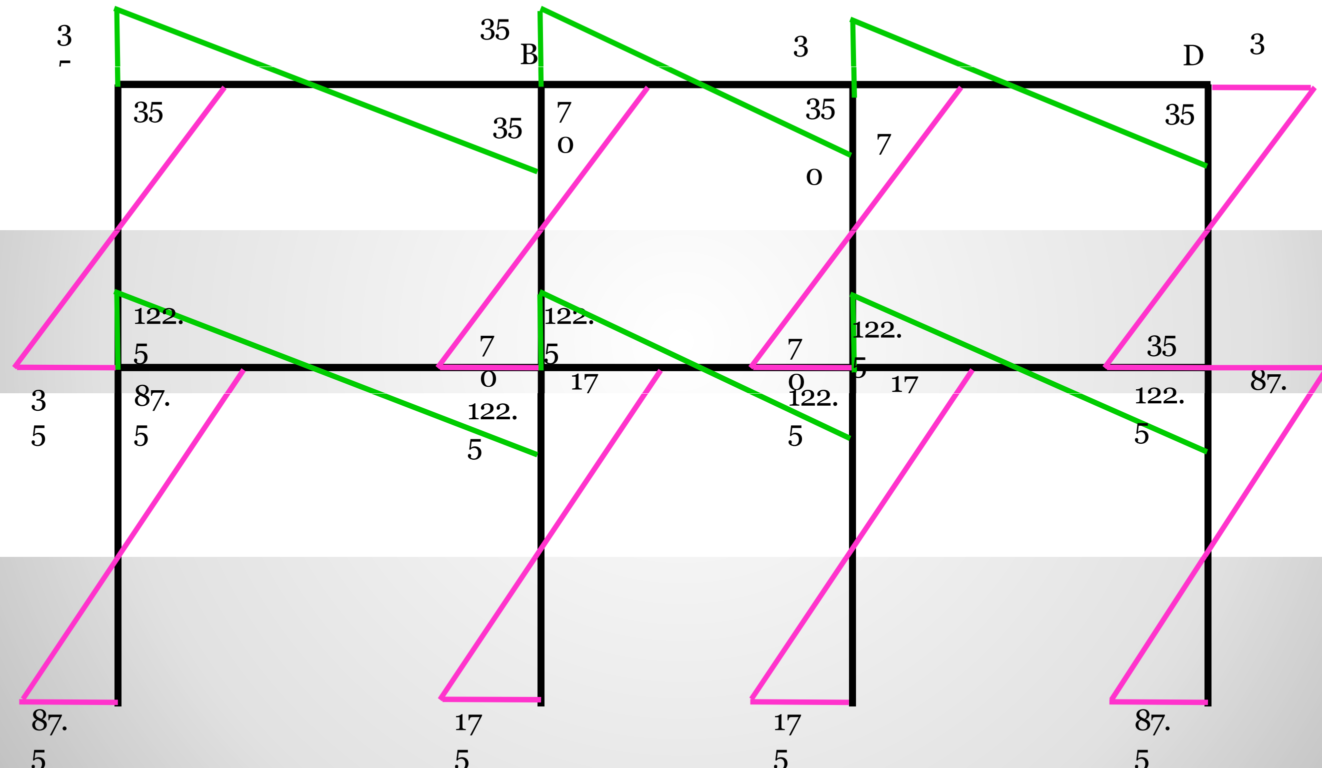


Column

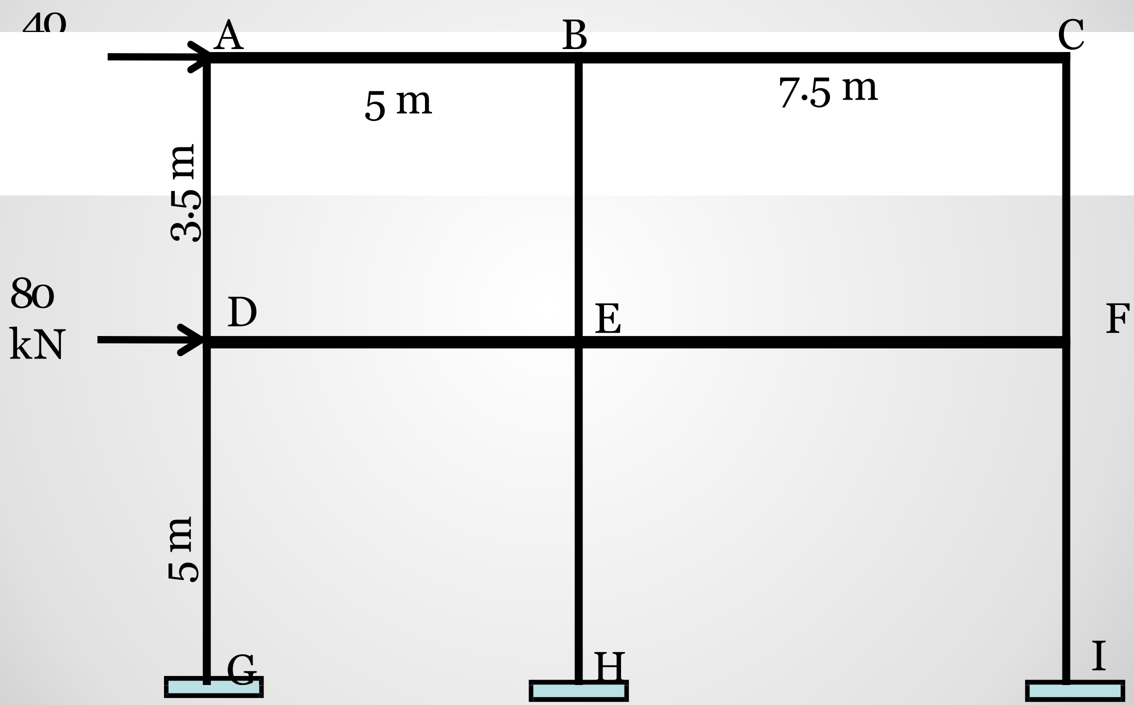


A
35 kNm
B
70
C
70
D
35
kNm
E
F
G
H
35
87.
5
70
17
5
70
17
5
35
87.
5
I
J
K
L
87.
5
17
5
17
5
87.5

Beam and Column



Home work

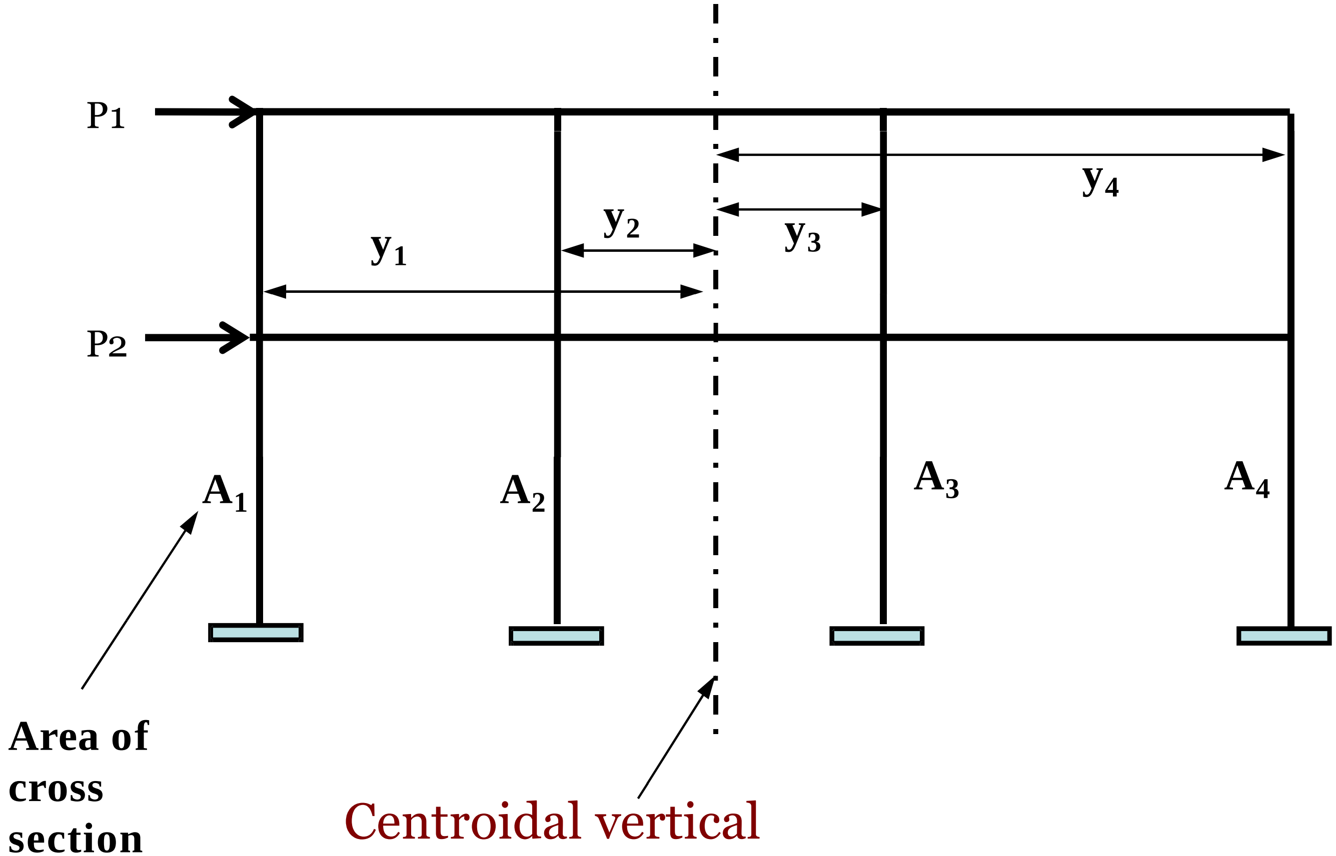


CANTILEVER

- Frame considered as a vertical cantilever

Assumptions

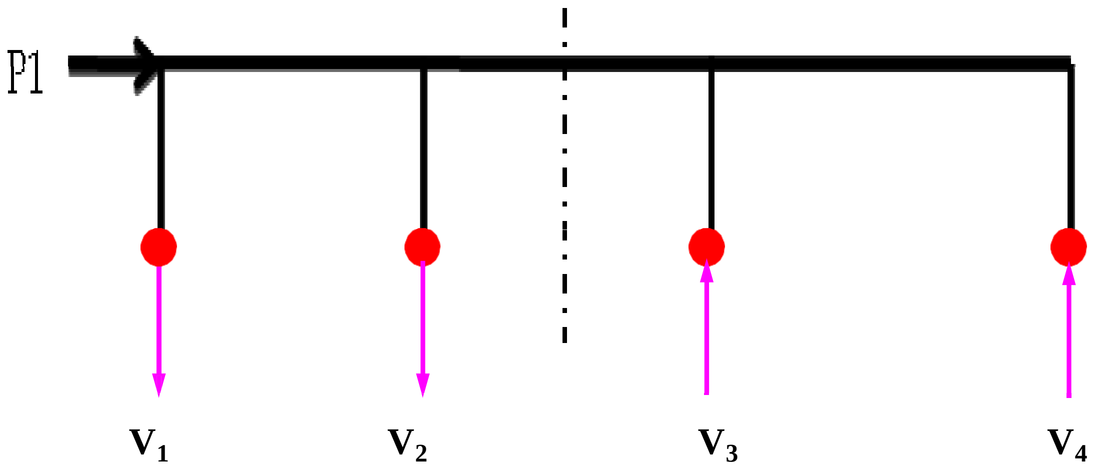
1. The points of contraflexure in all the members lie at their midpoints.
2. The direct stresses (axial stresses) in the columns are directly proportional to their distance from the centroidal vertical axis of the frame.



Centroidal vertical axis of the frame

To locate centroidal vertical axis of the frame,

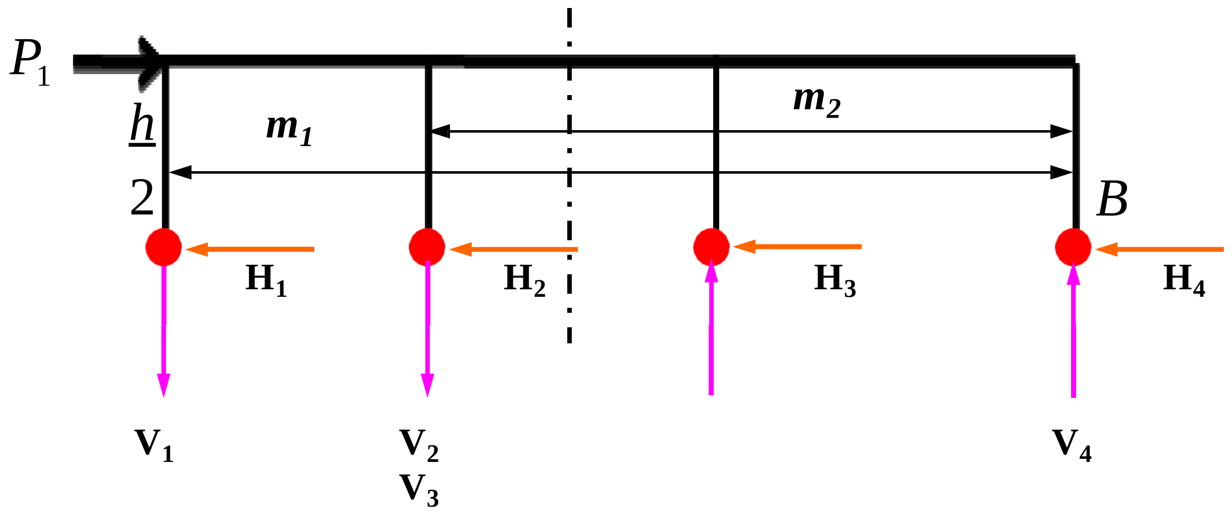
$$\bar{y} = \frac{A_1 d_1 + A_2 d_2 + A_3 d_3 + A_4 d_4}{A_1 + A_2 + A_3 + A_4}$$



$$o_x = \frac{My}{I}$$

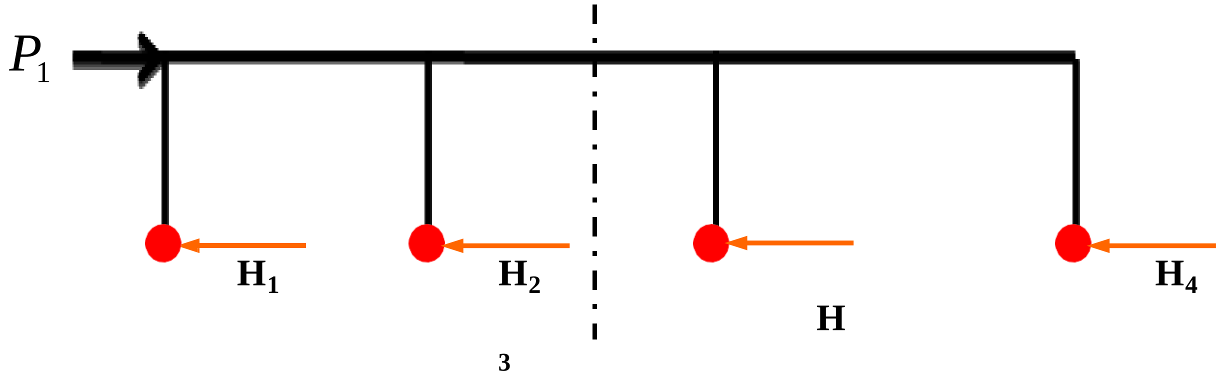
$\frac{M}{I}$ is constant at a given height (of the 'vertical cantilever').

$$\frac{\sigma_1}{\sigma_4} = \frac{\sigma_2}{\quad} = \frac{\sigma_3}{\quad} = \frac{\quad}{\quad} \Rightarrow \frac{V_1/A_1}{y_1} = \frac{V_2/A_2}{y_2} = \frac{V_3/A_3}{y_3} = \frac{V_4/A_4}{y_4} \qquad \text{———} \quad (1)$$



$$\sum M_B \Rightarrow P_1 \frac{h}{2} = V_1 m_1 + V_2 m_2 - V_3 m_3 - V_4 m_4 \qquad \text{———— (2)}$$

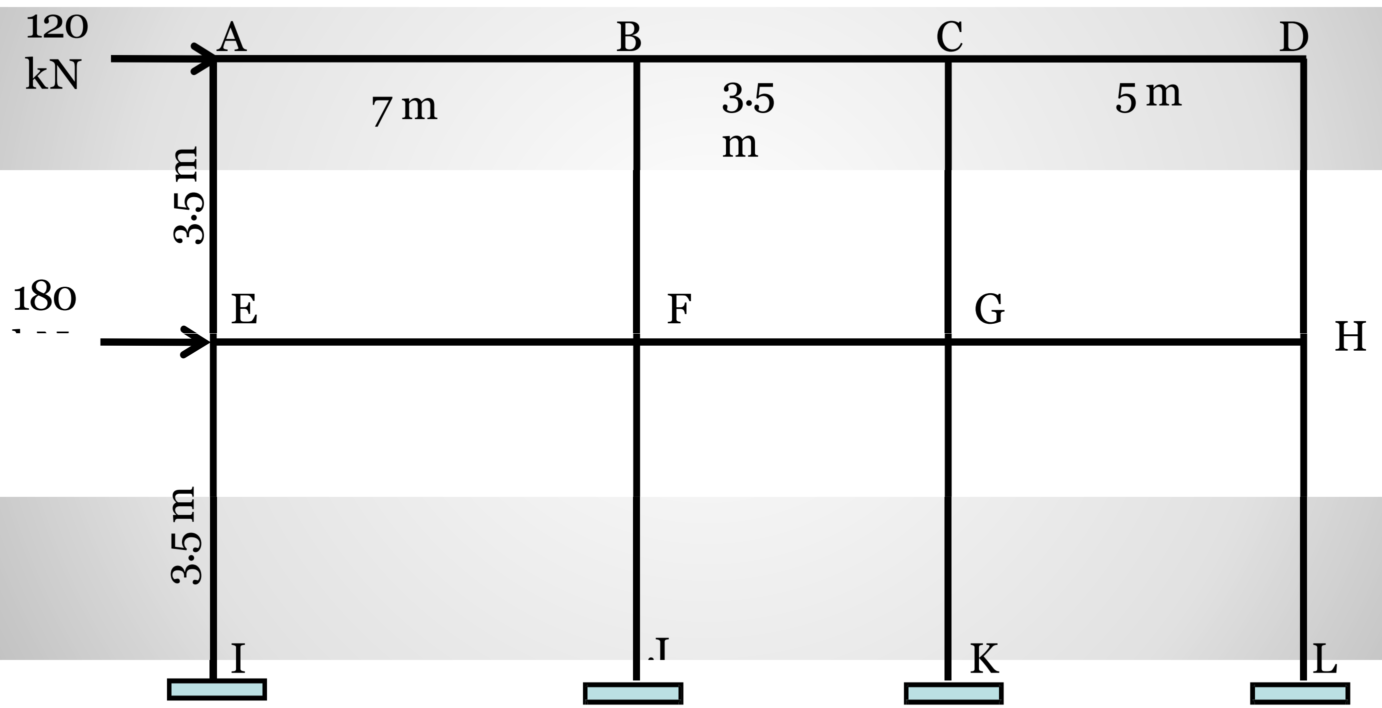
From (1) and (2), V_1, V_2, V_3, V_4 can be found.



$$P_1 = H_1 + H_2 + H_3 + H_4$$

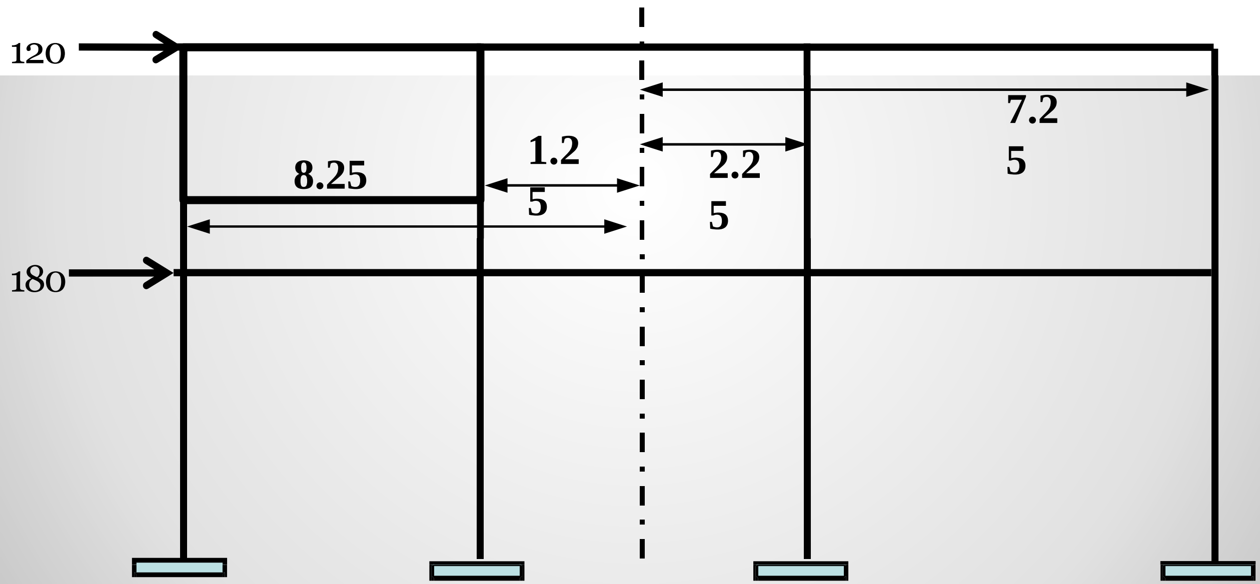
Problem 4: **Analyse the frame using cantilever**

the columns have the same area of cross section.



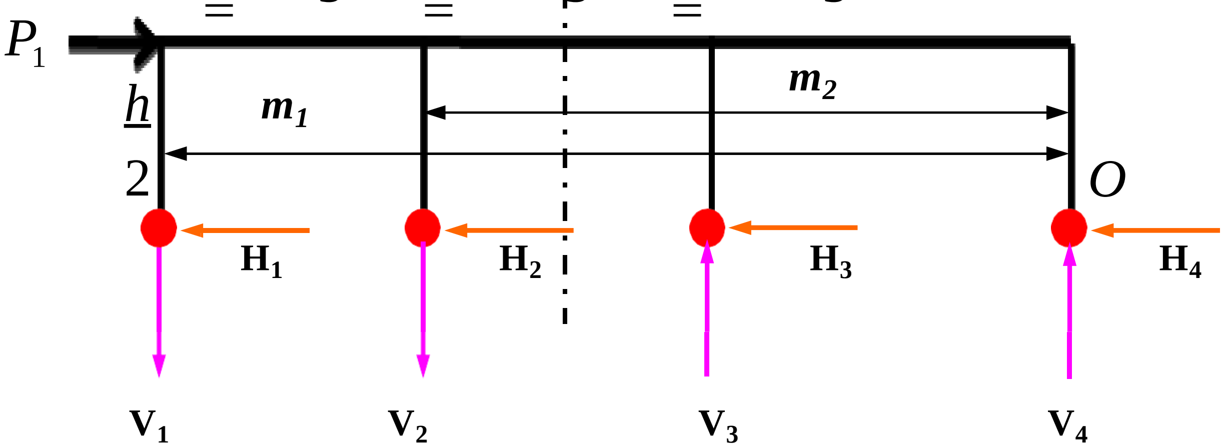
To locate centroidal vertical axis of the

$$\bar{y} = \frac{A_1\times 0 + A_1\times 7 + A_1\times 10.5 + A_1\times 15.5}{A_1 + A_1 + A_1 + A_1} = \frac{33}{4} = 8.25$$



Also, $\frac{V_1/A_1}{8.25} = \frac{V_2/A_2}{1.25} = \frac{V_3/A_3}{2.25} = \frac{V_4/A_4}{7.25} \Rightarrow \frac{V_1}{8.25} = \frac{V_2}{1.25} = \frac{V_3}{2.25} = \frac{V_4}{7.25}$

$$V_2 = \frac{1.25V_1}{8.25}, \quad V_3 = \frac{2.25V_1}{8.25}, \quad V_4 = \frac{7.25V_1}{8.25}$$



For the top storey,

$$\sum M_O \Rightarrow P_1 \frac{h}{2} = V_1 m_1 + V_2 m_2 - V_3 m_3 - V_4 m_4$$

$$\Rightarrow 120 \times \frac{3.5}{2} = V_1 \times 15.5 + V_2 \times 8.5 - V_3 \times 5 - V_4 \times 0$$

$$\Rightarrow 120\times 3.5 = V_1\times 15.5+\left(\frac{1.25V_1}{8.25}\right)\times 8.5-\left(\frac{2.25V_1}{8.25}\right)\times 5$$

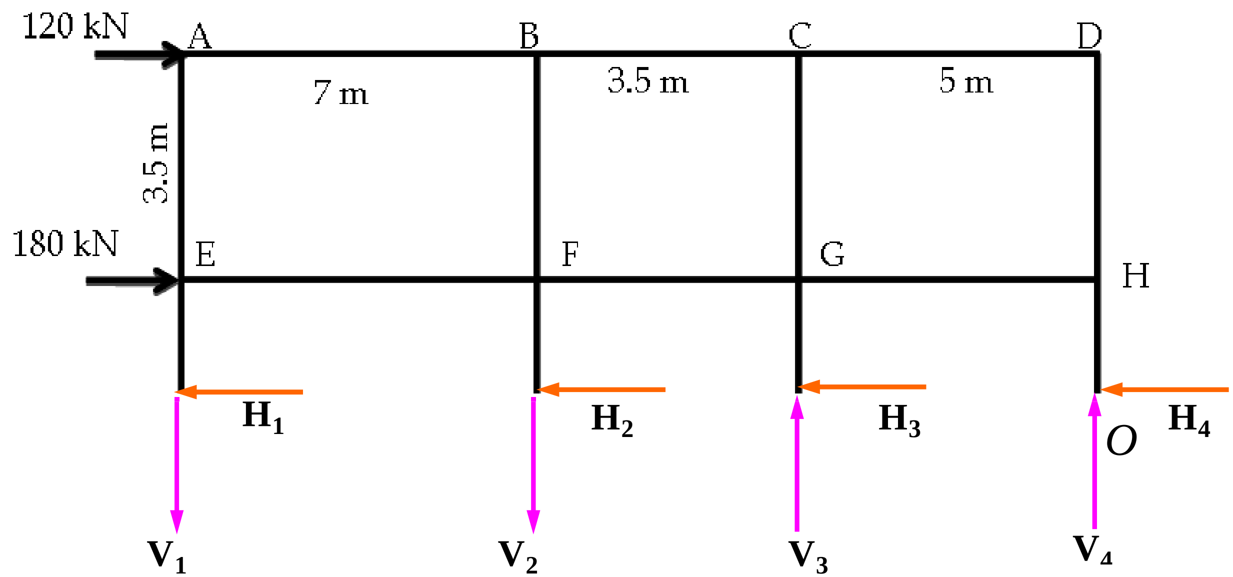
$$\Rightarrow V_1 = 13.615\,kN$$

$$V_2 = \frac{1.25\times 13.615}{8.25} = 2.063\,kN,$$

$$V_3 = \frac{2.25\times 13.615}{8.25} = 3.713\,kN,$$

$$V_4 = \frac{7.25\times 13.615}{8.25} = 11.965\,kN$$

$$Check:\ 13.615+2.063-3.713-11.965 = 0$$



For the bottom storey,

$$\sum M_O = 0 \Rightarrow 120 \times \left(3.5 + \frac{3.5}{2}\right) + 180 \times \frac{3.5}{2} = V_1 \times 15.5 + V_2 \times 8.5 - V_3 \times 5 - V_4$$

$$\Rightarrow 120 \times \left(3.5 + \frac{3.5}{2}\right) + 180 \times \frac{3.5}{2} = V_1 \times 15.5 + \left(\frac{V_1}{8.25}\right) \times 8.5 - \left(\frac{2.25V_1}{8.25}\right) \times 5$$

$$\Rightarrow V_1 = 61.267kN$$

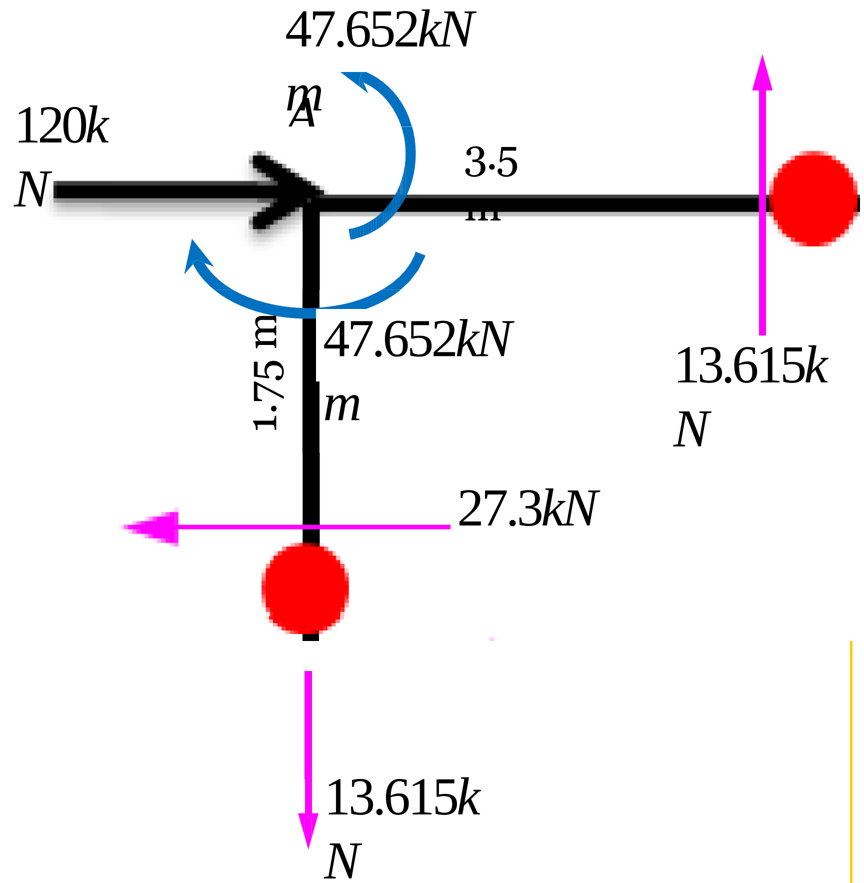
$$V_2 = \frac{1.25 \times 61.267}{8.25} = 9.283kN;$$

$$V_3 = \frac{2.25 \times 61.267}{8.25} = 16.709kN;$$

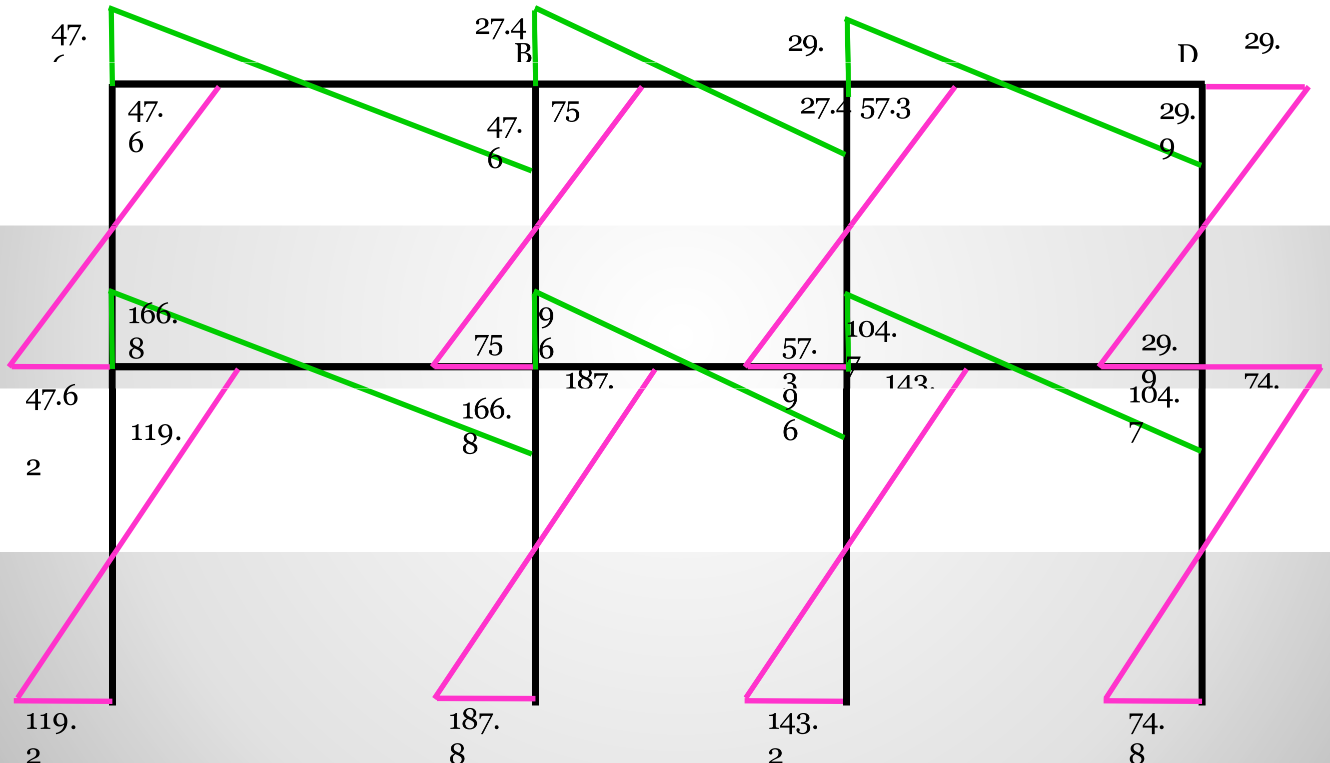
$$V_4 = \frac{7.25 \times 61.267}{8.25} = 53.841kN$$

$$Check:\ 61.267 + 9.283 - 16.709 - 53.841 = 0$$

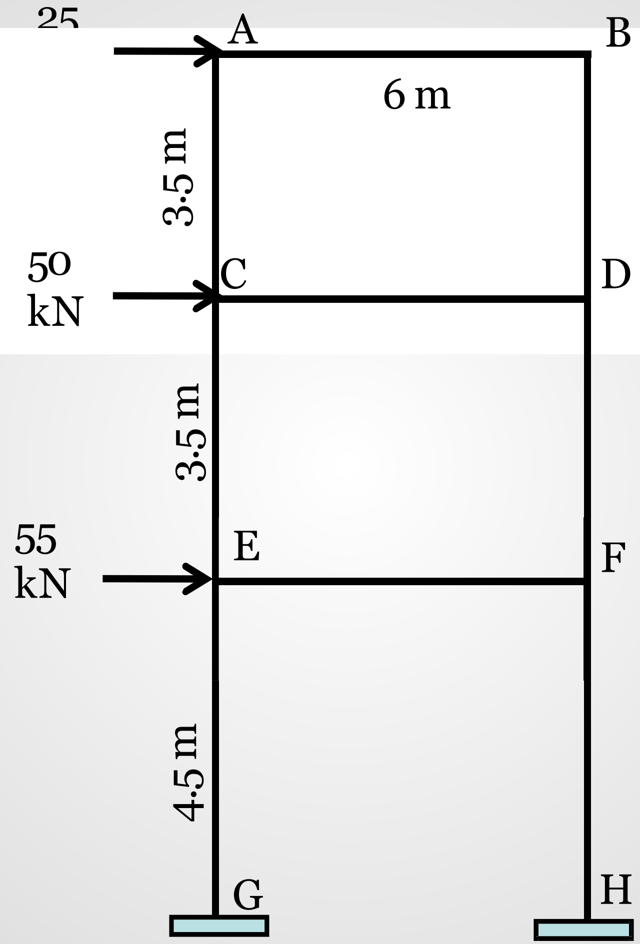
Moments:



Beam and Column



Home



FACTOR

- More accurate than Portal and Cantilever methods
- Specially useful when moments of inertia of various members are different.

Basis:

- At any joint the total moment is shared by all the members in proportion to their stiffnesses
- Half the moment gets carried over to the far end

Girder and column factors:

- Relative stiffness of a $k = \dfrac{I}{L}$

Girder factor at a joint

$$g = \frac{\sum k,\ of\ all\ columns\ meeting\ at\ the\ joint}{\sum k,\ of\ all\ members\ meeting\ at\ the\ joint}$$

Column factor at a joint

$$c = \frac{\sum k,\ of\ all\ beams\ meeting\ at\ the\ joint}{\sum k,\ of\ all\ members\ meeting\ at\ the} = 1 - g$$

Moment factor for a member

$$C = c_m k, \text{ for a column}$$

$$G = g_m k, \text{ for a beam}$$

$where\ c_m = c + half\ of\ column\ facor\ of\ far$

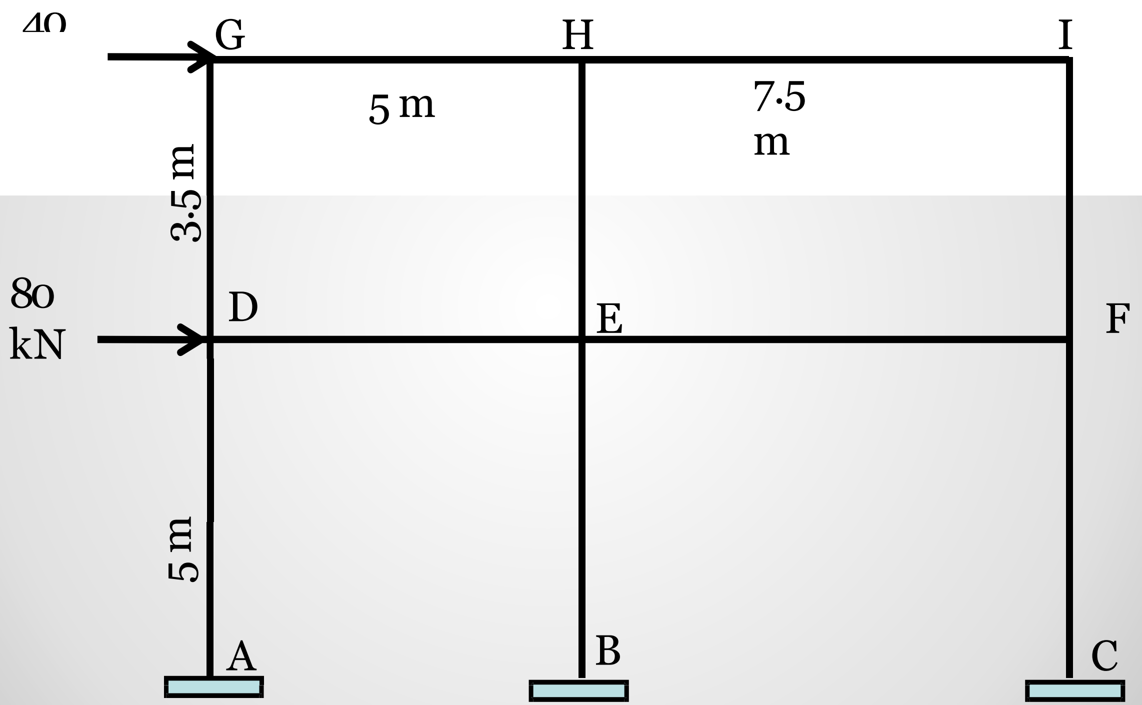
$end\ and\ \ g_m = g + half\ of$

$\sum C \rightarrow sum\ of\ column\ moment\ factors\ for\ a$

storey

$\sum G \rightarrow sum\ of\ beam\ moment\ factors\ for\ a$ *joint*

Problem 5: Analyse the frame using factor



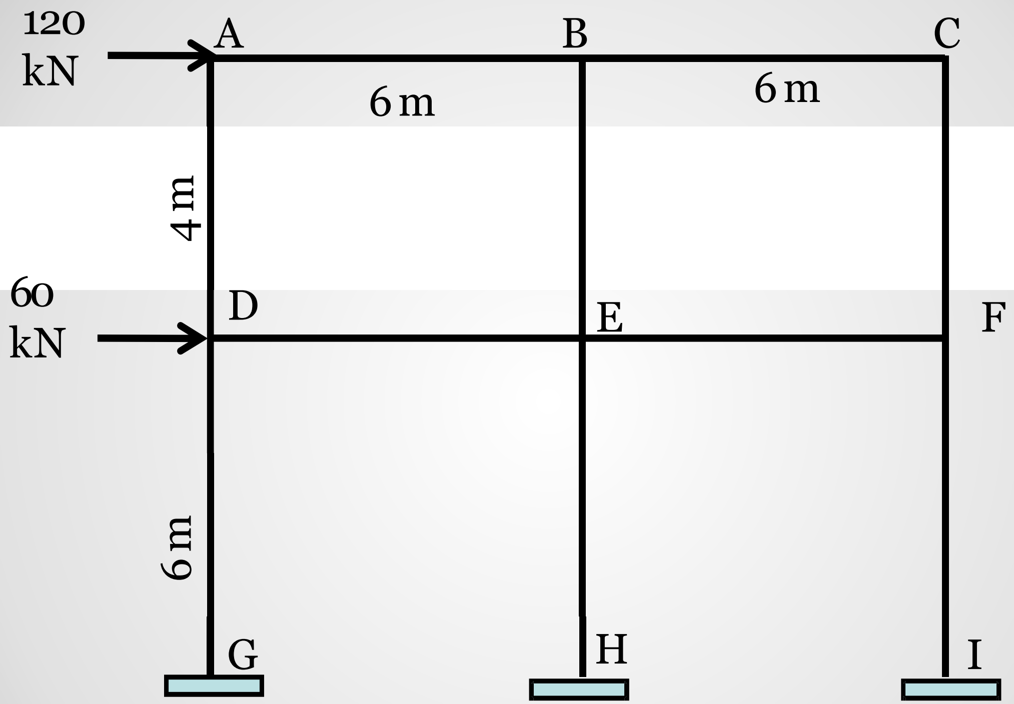
Total column moment

$$Total\ \ column\ moment\ above\ ABC\ = 40 \times 8.5 + 80 \times 5 = 740 kNm$$

1	2		3		4	5		6	7		8		9	10	11	12	
JOINT	MEMBER		k=I/L		Σk	FACTOR		c/2, g/2 from far end	5+6		MOMENT FACTOR		Total Col. Mom, M_T	Col. Mom, $M_C = M_T \times C/\Sigma C$	DF_B =G/ΣG	Beam	
	Col	Beam	Col	Beam		c =Σk(beams)/Σ k	g =Σk(columns)/Σk		c_m	g_m	C= c_m × k	G = g_m × k				Mo = MDF	$m_c \times B$
D	DA		0.2		0.686	0.29		0.5	0.79		0.158		740	99.4			
		DE		0.2			0.71	0.3		1.01		0.202			1	122	.6
	DG		0.286			0.29		0.3	0.59		0.169		140	23.2			
E		ED		0.2	0.819		0.59	0.36		0.95		0.19			0.59	83.2	5
	EH		0.286			0.41		0.27	0.68		0.194		140	26.6			
		EF		0.133			0.59	0.4		0.99		0.132			0.41	57.8	5
	EB		0.2			0.41		0.5	0.91		0.182		740	114.5			
F		FE		0.133	0.772		0.79	0.3		1.09		0.145			1	133	.03
	FI		0.286			0.21		0.16	0.37		0.106		140	14.6			
	FC		0.2			0.21		0.5	0.71		0.142		740	89.3			
G	GD		0.286		0.486	0.59		0.15	0.74		0.212		140	29.1			
		GH		0.2			0.41	0.23		0.64		0.128			1	29.	1
H		HG		0.2	0.772		0.46	0.21		0.67		0.134			0.559	16.	5
	HE		0.286			0.54		0.21	0.75		0.215		140	29.5			

$$storey, \sum C = 0.158 + 0.182 + 0.142 + 0.23 + 0.242 + 026$$

Home work



Summar

Approximate Methods of Analysis of Multi-storey Frames

- **Analysis for vertical loads** - Substitute frames-Loading conditions for maximum positive and negative bending moments in beams and maximum bending moment in columns
- **Analysis for lateral loads** - Portal method–Cantilever method– Factor method.

- Influence Line for Reaction , Moment & Shear for Indeterminate Structure

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CED
BBSBEC, FGS.

Influence Lines

Variation of

Reaction, Shear, Moment or Deflection

at a SPECIFIC POINT

due to a **concentrated** force **moving** on member

Influence Lines

SIGNIFICANCE

- Influence lines are important in the design of structures that resist large **live loads**.

- If a structure is subjected to a live or moving load, **the variation in shear and moment** is best described using influence lines.

- Once the influence line is drawn, **the location of the live load which will cause the greatest influence** on the structure can be found very quickly

Influenc e Lines

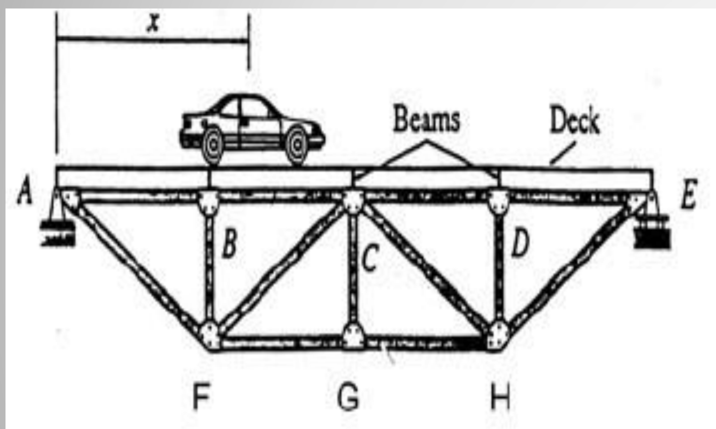


Figure 1. Bridge Truss Structure Subjected to a Variable Position Load

- As the car moves across the bridge, the forces in the truss members change with the position of the car and the maximum force in each member will be at a different car location.

- The design of each member must be based on the maximum probable load each member will experience

- If a structure is to be safely designed, members must be proportioned such that the maximum force produced by dead and live loads is less than the available section capacity.

Influence Lines

Structural analysis for variable loads consists of two steps:

1.Determining the positions of the loads at which the response function is maximum;

AND

2.Computing the maximum value of the response function.

Response Function = support reaction, axial force, shear force, or bending moment.

Influence Lines

INFLUENCE LINE VS SFD/BMD

- shear and moment diagrams represent the effect of fixed loads at all points along the member.

- Influence lines represent the effect of a moving load only at a specified point on a member

Influence Lines

TYPES OF INFLUENCE LINES

- **Reaction I.L.**
- **Shear I.L.**
- **Moment I.L.**
- **Floor Girder I.L.**
- **Truss Bar force I.L.**

Influence Lines

Structure type

- Determinate
- Indeterminate

Influence lines

- **For Determinate Structure**
- **For Indeterminate Structure**

Influence Lines

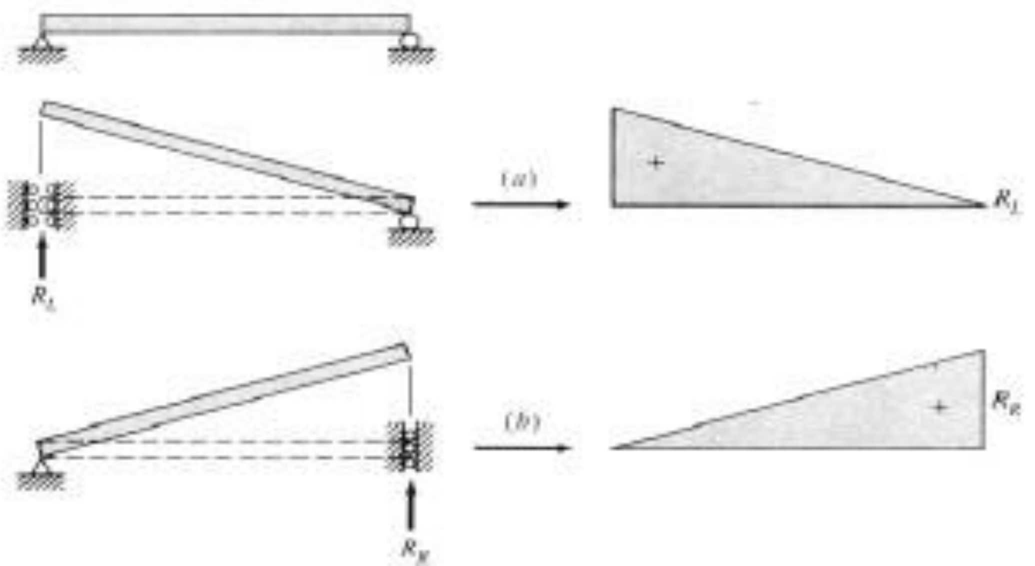
Methods of constructing the shape of Influence Lines

- **Tabulation Method**.
- **Muller –Breslau principles.**

Influence Lines

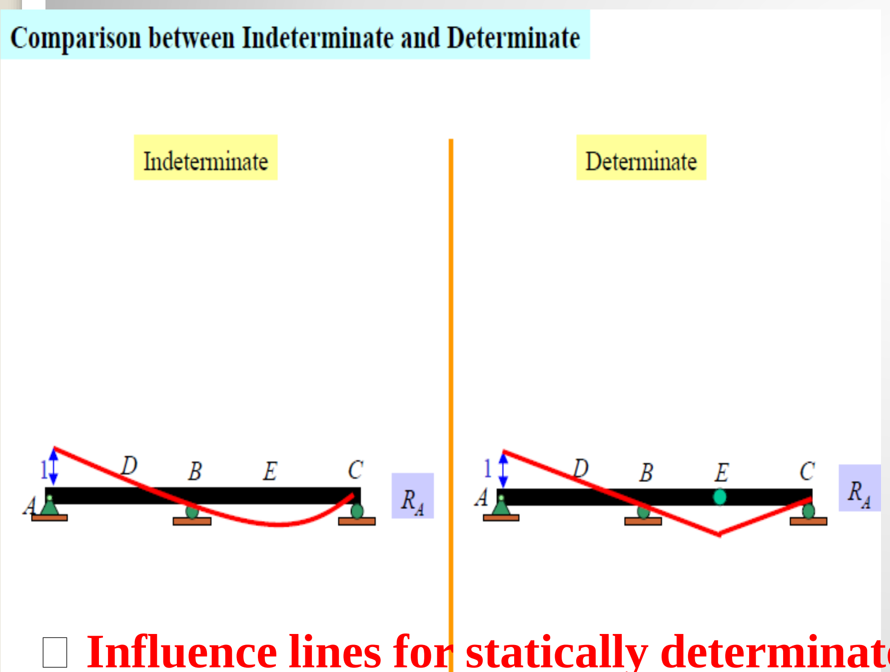
Muller-Breslau Principle

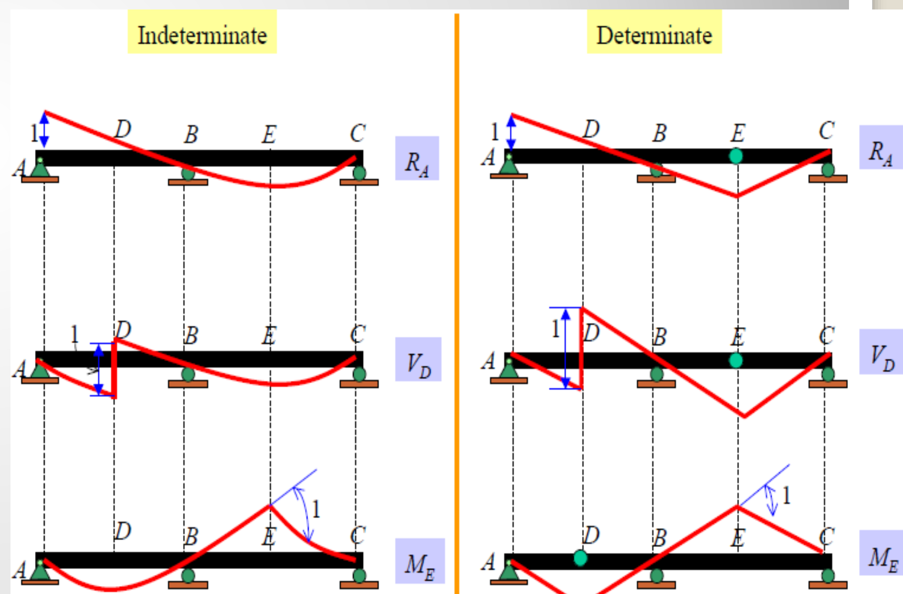
- Müller-Breslau Principle: "If a function at a point on a beam, such as reaction, or shear, or moment, is allowed to act without restraint, the deflected shape of the beam, to some scale, represent the influence line of the function".



Influenc e Lines

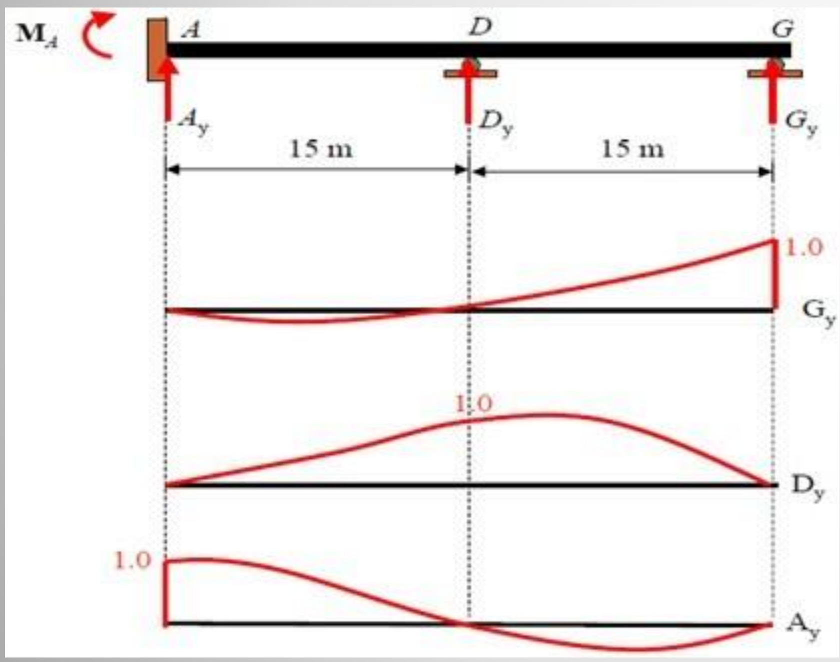
Indeterminate VS Determinate

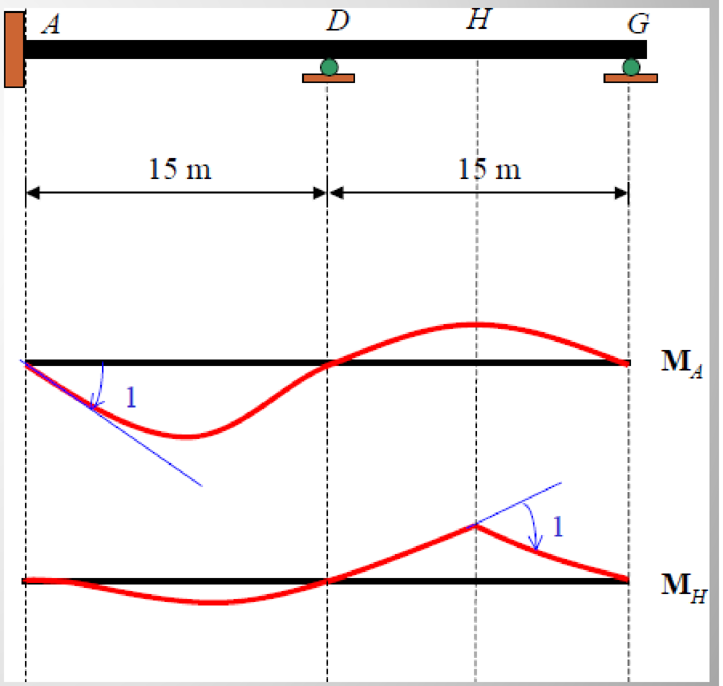




- Influence lines for statically determinate structures are always piecewise linear.
- For indeterminate structures, the influence line is not straight lines!

Influenc e Lines





Influenc
e Lines