BABA BANDA SINGH BAHADUR ENGINEERING COLLEGE

Department of Applied Sciences

Question Bank

Semester: Third **Subject:** Mathematics Paper-III **Code:** BTAM304-18

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1. If $f(x,y)=\frac{x-y}{2x+y}$, then show that $\lim_{x\to 0}\left[\lim_{y\to 0} f(x,y)\right]\neq \lim_{y\to 0}\left[\lim_{x\to 0} f(x,y)\right]$. Also show that the function is discontinuous at the origin.

2. Let f be a function defined by $f(x,y)=\frac{x^2-y^2}{x^2+y^2}, (x,y)\neq(0,0)$. Show that two iterated limits $\lim_{x\to 0}\left\{\lim_{y\to 0} f(x,y)\right\}$ and $\lim_{y\to 0}\left\{\lim_{x\to 0} f(x,y)\right\}$ exist, but the simultaneous limit $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.

3. Let $f(x,y)$ be a function defined as $f(x,y)=\begin{cases}\frac{xy}{x^2+y^2} & ;(x,y)\neq(0,0)\\ 0 & ;(x,y)=(0,0)\end{cases}$ show that $f_x(0,0)$ and $f_y(0,0)$ exists, although $f(x,y)$ is discontinuous at $(0,0)$.

4. Show that the function $f(x,y)=\begin{cases}\frac{x^3+2y^3}{x^2+y^2} & ;(x,y)\neq(0,0)\\ 0 & ;(x,y)=(0,0)\end{cases}$ possesses partial derivatives of first order.

5. Let $f:R^2\to R$ be a function defined as $f(x,y)=\begin{cases}\frac{x^2y}{x^4+y^2} & ;(x,y)\neq(0,0)\\ 0 & ;(x,y)=(0,0)\end{cases}$ show that $f_x(0,0)$ and $f_y(0,0)$ exists, although $f(x,y)$ is discontinuous at $(0,0)$.

6. If $z=\log\left(x^2+xy+y^2\right)$, prove that $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=2.$

7. If $u=e^{xyz}$, prove that $\frac{\partial^3 u}{\partial x\partial y\partial z}=\left(1+3xyz+x^2y^2z^2\right)e^{xyz}.$

8. If $f(x,y)=x^3y-xy^3$, find the value of $\left\{\frac{1}{\frac{\partial f}{\partial x}}+\frac{1}{\frac{\partial f}{\partial y}}\right\}_{(1,2)}$.

9. If $f(x,y)=\begin{vmatrix} x^2 & y^2 & z^2 \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix}$ then show that $f_x+f_y+f_z=0.$

10. If $u=\frac{y}{z}+\frac{z}{x}+\frac{x}{y}$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0.$

11. Find the value of n, so that the equation $V=r^n\left(3\cos^2\theta-1\right)$ satisfies the equation $\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right)=0.$

12. If $\theta=t^n e^{-\frac{r^2}{4t}}$ what value of n will make $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right)=\frac{\partial\theta}{\partial t}$?

13. If $u=\frac{1}{\sqrt{x^2+y^2+z^2}}$; $x^2+y^2+z^2\neq 0$, show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$.

14. If $V=f(r)$ and $r^2=x^2+y^2+z^2$, then prove that $V_{xx}+V_{yy}+V_{zz}=f''(r)+\frac{2}{r}f'(r)$.

15. If $V=r^m$ where $r^2=x^2+y^2+z^2$, show that $V_{xx}+V_{yy}+V_{zz}=m(m+1)r^{m-2}$.

16. If $z=f(x+ay)+\phi(x-ay)$ prove that $\frac{\partial^2 z}{\partial y^2}=a^2\frac{\partial^2 z}{\partial x^2}$.

17. If $\frac{x^2}{a^2+u}+\frac{y^2}{b^2+u}+\frac{z^2}{c^2+u}=1$, prove that $\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial x}\right)^2=2\left(x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}\right)$

18. If $u=\log(x^3+y^3+z^3-3xyz)$, show that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=-\frac{9}{(x+y+z)^2}$.

19. If $x^x y^y z^z=c$, show that at $x=y=z$, $\frac{\partial^2 z}{\partial x\partial y}=-\left[y\log ey\right]^{-1}$.

20. State and prove Euler's theorem for a function of three variables.

21. If $T=\dfrac{x^3y^3}{x^3+y^3}$, prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=3T$.

22. If $v=\tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$, prove that $x\dfrac{\partial v}{\partial x}+y\dfrac{\partial v}{\partial y}=\sin 2v.$

23. If $z=\sin^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$, prove that $x\dfrac{\partial z}{\partial x}+y\dfrac{\partial z}{\partial y}=\tan z$.

24. If $\sin u=\dfrac{x^2y^2}{x+y}$, prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=3\tan u$.

25. If $u=\sin^{-1}\left(\dfrac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}\right)$, show that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}+z\dfrac{\partial u}{\partial z}+3\tan u=0$.

26. If $u=\cos\left(\dfrac{xy+yz+zx}{x^2+y^2+z^2}\right)$, show that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}+z\dfrac{\partial u}{\partial z}=0$.

27. If $g=\log\left(\dfrac{x^5+y^5+z^5}{xy+yz+zx}\right)$, show that $x\dfrac{\partial g}{\partial x}+y\dfrac{\partial g}{\partial y}+z\dfrac{\partial g}{\partial z}=3$.

28. If $u=\tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$, prove that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\sin 4u-\sin 2u=2\cos 3u\sin u$$

29. If $u=\dfrac{x^2y^2}{x^2+y^2}$, prove that $x\dfrac{\partial^2 u}{\partial x^2}+y\dfrac{\partial^2 u}{\partial y\partial x}=\dfrac{\partial u}{\partial x}$.

30. If $u=\left(x^2+y^2\right)^{\frac{1}{3}}$, show that $x^2\dfrac{\partial^2 u}{\partial x^2}+2xy\dfrac{\partial^2 u}{\partial x\partial y}+y^2\dfrac{\partial^2 u}{\partial y^2}=-\dfrac{2u}{9}$.

31. If $u=\sin^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that

(i) $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\dfrac{1}{2}\tan z.$

(ii) $x^2\dfrac{\partial^2 u}{\partial x^2}+2xy\dfrac{\partial^2 u}{\partial x\partial y}+y^2\dfrac{\partial^2 u}{\partial y^2}=-\dfrac{\sin u\cos 2u}{4\cos^3 u}$.

32. If $u=\operatorname{cosec}^{-1}\left(\frac{x^{\frac{1}{2}}+y^{\frac{1}{2}}}{x^{\frac{1}{3}}+y^{\frac{1}{3}}}\right)^{\frac{1}{2}}$, show that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{\tan u}{144}\left(13+\tan^2 u\right)$.

33. If $u=x^2\tan^{-1}\frac{y}{x}-y^2\tan^{-1}\frac{x}{y}$, then evaluate $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$.

34. If $z=xyf\left(\frac{y}{x}\right)$ and z is a constant then show that $\frac{f'\left(\frac{y}{x}\right)}{f\left(\frac{y}{x}\right)}=\frac{x\left(y+x\frac{dy}{dx}\right)}{y\left(y-x\frac{dy}{dx}\right)}$

35. If $x=r\cos\theta, y=r\sin\theta$, show that $\frac{\partial^2 r}{\partial x^2}+\frac{\partial^2 r}{\partial y^2}=\frac{1}{r}\left[\left(\frac{\partial r}{\partial x}\right)^2+\left(\frac{\partial r}{\partial y}\right)^2\right]$.

36. If z is a function of x, y and $x=e^u+e^{-v}$ and $y=e^{-u}-e^v$, show that $\frac{\partial z}{\partial u}-\frac{\partial z}{\partial v}=x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}$.

37. If $u=\tan^{-1}\left(\frac{y}{x}\right)$, where $x=e^t-e^{-t}$ and $y=e^t+e^{-t}$, Show that $\frac{du}{dt}=-\frac{2}{e^{2t}+e^{-2t}}$.

38. If $u=f\left(x-y,y-z,z-x\right)$, show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$

39. If $u=f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$

40. If $u=x\log\left(xy\right)$, where $x^3+y^3+3xy=1$, find $\frac{du}{dx}$.

41. If $u=\sin^{-1}\left(x-y\right)$, $x=3t, y=4t^3$ show that $\frac{du}{dt}=3\left(1-t^2\right)^{-\frac{1}{2}}$.

42. If $x^y=y^x$, show that $\frac{dy}{dx}=\frac{y\left(y-x\log y\right)}{x\left(x-y\log x\right)}$ using partial derivatives.

43. If $u=f\left(\frac{y-x}{xy},\frac{z-x}{xz}\right)$ show that $x^2\frac{\partial u}{\partial x}+y^2\frac{\partial u}{\partial y}+z^2\frac{\partial u}{\partial z}=0$.

44. If $f\left(x,y\right)=0, \phi\left(y,z\right)=0$, show that $\frac{\partial f}{\partial y}.\frac{\partial \phi}{\partial x}.\frac{dz}{dx}=\frac{\partial f}{\partial x}.\frac{\partial \phi}{\partial y}$

45. Show that $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=\frac{\partial^2 z}{\partial s^2}+\frac{\partial^2 z}{\partial t^2}$, where $x=s\cos\alpha-t\sin\alpha$ and $y=s\sin\alpha+t\cos\alpha$

46. If z is a function of x and y and u and v be two other variables, such that $u = lx + my, v = ly - mx$, show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \left(l^2 + m^2\right)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$.

47. If $z = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$, show that $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0$.

48. Find the equation of the tangent plane and normal line to the surface $xyz = 6$ at the point (1, 2, 3).

49. Find the equation of the tangent plane and normal line to the surface $\frac{x^2}{2} - \frac{y^2}{3} = z$ at the point (2, 3, -1).

50. Show that the plane $ax + by + cz + d = 0$ touches the surface $px^2 + qy^2 + 2z = 0$, if $\frac{a^2}{p} + \frac{b^2}{q} + 2cd = 0$.

51. Find the condition that the plane $lx + my + nz + p = 0$ touches the conicoid $ax^2 + by^2 + cz^2 + d = 0$.

52. Examine for maximum and minimum value of the function $x^3 - 3axy + y^3$.

53. Examine for maximum and minimum value of the function $x^3 + y^3 - 63(x - y) + 12xy$.

54. Discuss the maxima and minima of $x^3 y^2 (1 - x - y)$.

55. If a real number k (> 0) is divided into three parts such that the sum of their products taken two at a time is maximum, find the numbers.

56. Show that of all triangles with given parameter, the one with maximum area is equilateral.

57. Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube.

58. Find the dimensions of the triangle whose perimeter is 9cms, such that its area is maximum.

59. Find the dimensions of the rectangular box, open at the top, of maximum capacity whose surface is 432 square cm.

60. Find the points on the surface $z^2 = xy + 1$ nearest to the origin.

61. If $u = a^3x^2 + b^3y^2 + c^3z^2$ where $x^{-1} + y^{-1} + z^{-1} = 1$, show that the stationary value of u is given by $x = \frac{\sum a}{a}, y = \frac{\sum a}{b}, z = \frac{\sum a}{c}$.

62. Find the dimensions of the rectangular parallelepiped of maximum volume that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

63. Prove that stationary value of $x^m y^n z^p$ under the condition $x+y+z=a$ is $m^m n^n p^p\left(\frac{a}{m+n+p}\right)^{m+n+p}$.

64. Sum of three positive numbers is one. Determine the maximum value of their product.

65. If $u=ax^2+by^2+cz^2$ with the conditions that $lx+my+nz=0$ and $x^2+y^2+z^2=1$, prove that the stationary values of u satisfy the equation $\frac{l^2}{a-u}+\frac{m^2}{b-u}+\frac{n^2}{c-u}=0$.

66. Show that the plane $3x+12y-6z-17=0$ touches the conicoid $3x^2-6y^2+9z^2+17=0$. Find also the point of contact.

67. Find the equation of the tangent plane and the normal line to the surface $z^2=4\left(1+x^2+y^2\right)$ at (2, 2, 6).

68. At a distance of 50 meters from the foot of the tower the elevation of its top is 30^0. If the possible errors in measuring the distance and elevation are 2cm and 0.05 degrees, find the approximate error in calculating the height.

69. A rectangular box, open at the top, is to have a volume of 32 cubic feet. Find the dimensions of the box requiring least material for its construction.

70. The temperature T at any point (x,y,z) in space is $T=400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2+y^2+z^2=1$.

71. Evaluate the following integrals

(a) $\int_0^3\int_0^1\left(x^2+3y^2\right)dydx$

(b) $\int_0^1\int_0^1(x+2)\,dydx$

(c) $\int_1^2\int_0^x\frac{dydx}{x^2+y^2}$

(d) $\int_0^2\int_0^{\sqrt{2x}} xy\ dydx$

(e) $\int_0^1 \int_x^{\sqrt{x}} \left(x^2+y^2\right) dydx$

72. Evaluate $\iint_R y\ dxdy$, where R is the region bounded by the parabolas $y^2=4ax$ and $x^2=4ay$

73. Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\ dydx$ by changing the order of integration.

74. Evaluate $\int_0^\infty \int_0^x xe^{-\frac{x^2}{y}}\ dydx$ by changing the order of integration.

75. Evaluate $\int_0^1 \int_x^1 \sin y^2\ dydx$ by changing the order of integration.

76. Change the order of integration in $\int_{-a}^{a} \int_0^{\sqrt{a^2-y^2}} f\left(x,y\right)\ dxdy$

77. Evaluate the double integral $\iint_R xy\ dxdy$, where R is the domain bounded by $x-$axis, ordinate $x=2a$ and the curve $x^2=4ay$.

78. Use the transformation $x+y=u$ and $y=uv$ to show that $\int_0^1 \int_0^{1-x} e^{\frac{y}{x+y}} dydx = \frac{e-1}{2}$.

79. Evaluate $\iint dxdy$ over the area bounded by $x=0, y=0, x^2+y^2=1$ and $5y=3$.

80. Evaluate $\iint r^3 drd\theta$ over the area included between the circles $r=2\sin\theta$ and $r=4\sin\theta$.

81. Evaluate $\iint r\sin\theta\ drd\theta$ over the area of the cardioid $r=a\left(1+\cos\theta\right)$ above the initial line.

82. Change the order of integration in $\int_0^a \int_y^a \frac{x}{x^2+y^2} dxdy$ and hence evaluate the same.

83. Change the order of integration in $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}} dxdy$ and hence evaluate the same.

84. Evaluate $\iint \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}} dxdy$ over the positive quadrant of the circle $x^2+y^2=1$.

85. Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} xyz \, dzdydx$.

86. Evaluate $\iiint \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$, the integral being extended to the positive octant of the sphere $x^2+y^2+z^2=1$.

87. Evaluate $\iint_R (x^2+y^2)dxdy$ where R is the region bounded by the four hyperbolas $x^2-y^2=2,9$ and $xy=2,4.$

88. Evaluate $\iint \sqrt{\frac{36-4x^2-9y^2}{36+4x^2+9y^2}}dxdy$ over the region bounded by the ellipse $4x^2+9y^2=36$ and the coordinate axes lying in the first coordinate.

89. Show that $\iiint_V x^2 dxdydz = \frac{4\pi}{15}$ where $V=\{(x,y,z): x^2+y^2+z^2 \le 1\}$.

90. Using the transformation $x+y=u, y=uv$ show that $\iint \sqrt{xy(1-x-y)}dxdy = \frac{2\pi}{105}$, where the integration being taken over the area of the triangle bounded by the lines $x=0, y=0, x+y=1$.

91. Find the area outside the circle $r=a$ and inside the cardioid $r=a(1+\cos\theta)$.

92. Find by double integration, the area of one loop of the leminscate $r^2=a^2\cos 2\theta$

93. Find the area enclosed by the cardioid $r=a(1+\cos\theta)$.

94. Find the area bounded by the curves $y^2=4ax$ and $x^2=4ay$.

95. Find the area bounded by the parabolas $y^2=4-x$ and $y^2=4-4x$.

96. Find the volume bounded by the cylinder $x^2+y^2=4$ and the planes $y+z=4, z=0$.

97. Find the volume of a right circular cone with base radius r and height h by triple integration.

98. Find the volume of a cylinder with base radius r and height h.

99. Prove that the volume enclosed by the cylinders $x^2+y^2=2ax, z^2=2ax$ is $\frac{128}{15}a^3$.

100. Find the volume of the unit sphere using triple integration.

101. Find the volume of the tetrahedron bounded by co-ordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

102. The axes of two right circular cylinders of the same radius a, intersect at right angles. Prove that the volume inside both the cylinders is $\frac{16a^3}{3}$.

103. Calculate the volume of the solid bounded by the surfaces $x=0, y=0, z=0$ and $x+y+z=1$.

104. Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

105. Find the area lying between the parabola $y^2=4x-x^2$ and the line $y=x$.

106. Evaluate $\iiint z\left(x^2+y^2\right)dxdydz$ over the volume of the cylinder $x^2+y^2=1$ intercepted by the planes $z=2$ and $z=3$.

Discuss the convergence/divergence of the following series 107-128:

107. $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+.........\infty$

108. $1+\frac{2^2}{\lfloor 2}+\frac{3^2}{\lfloor 3}+\frac{4^2}{\lfloor 4}+............\infty$

109. $\sum_{n=1}^{\infty}\left[\sqrt{n^2+1}-n\right]$

110. $\sum_{n=1}^{\infty}\left[\sqrt[3]{n^3+1}-n\right]$

111. $\sum_{n=0}^{\infty}\frac{2n^3+5}{4n^5+1}$

112. $\sum\left[\sqrt{(n^4+1)}-\sqrt{(n^4-1)}\right]$

113. $\frac{1}{2\sqrt{1}}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^6}{5\sqrt{4}}+.............\infty$

114. $1+\frac{1+\alpha}{1+\beta}+\frac{(1+\alpha)\,(1+2\alpha)}{(1+\beta)\,(1+2\beta)}+\frac{(1+\alpha)\,(1+2\alpha)\,(1+3\alpha)}{(1+\beta)\,(1+2\beta)\,(1+3\beta)}+........$

115. $x+\frac{2^2x^2}{\lfloor 2}+\frac{3^3x^3}{\lfloor 3}+\frac{4^4x^4}{\lfloor 4}+\frac{5^5x^5}{\lfloor 5}+............\infty$

116. $\frac{1}{1^2}+\frac{1+2}{1^2+2^2}+\frac{1+2+3}{1^2+2^2+3^2}+...........\infty$

117. $1+\frac{\alpha.\beta}{1.\gamma}x+\frac{\alpha(\alpha+1)\beta(\beta+1)}{1.2.\gamma(\gamma+1)}x^2+\frac{\alpha(\alpha+1)(\alpha+2)\beta(\beta+1)(\beta+2)}{1.2.3.\gamma(\gamma+1)(\gamma+2)}x^3+......\infty$

118. $\sum\frac{4.7......(3n+1)}{1.2......n}.x^n$.

119. $\frac{a+x}{\lfloor 1}+\frac{(a+2x)^2}{\lfloor 2}+\frac{(a+3x)^3}{\lfloor 3}+.......\infty$

120. $x^2(\log 2)^q+x^3(\log 3)^q+x^4(\log 4)^q+............\infty$

121. $\sum\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{\frac{3}{2}}}$

122. $\sum\frac{n^{n^2}}{(n+1)^{n^2}}$.

123. $\sum\frac{1}{\left(1+\frac{1}{n}\right)^{n^2}}$

124. $\frac{1}{2}+\frac{2}{3}x+\left(\frac{3}{4}\right)^2x^2+\left(\frac{4}{5}\right)^3x^3+.......\infty$

125. $\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-3}+.......\infty$

126. $\sum_{n=1}^{\infty}\frac{\cos n\pi}{n^2+1}$

127. $\frac{1}{6}-\frac{2}{11}+\frac{3}{16}-\frac{4}{21}+\frac{5}{26}-.........\infty$

128. $\sum_{n=2}^{\infty}\frac{(-1)^{n-1}x^n}{n(n-1)},0<x<1$

129. State Cauchy's Integral Test.

130. State D'Alembert's Ratio Test.

131. Prove that the series $\frac{\sin x}{1^3}-\frac{\sin 2x}{2^3}+\frac{\sin 3x}{3^3}-......$ converges absolutely.

132. Discuss the absolute convergence of

(i) $\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{n+1}$

(ii) $\frac{1}{\sqrt{(1^3+1)}}-\frac{1}{\sqrt{(2^3+1)}}x+\frac{1}{\sqrt{(3^3+1)}}x^2-......$

133. State and prove the NASC for the differential Mdx + Ndy = 0 to be an exact differential equation.

Solve the following differential equations 134-148:

134. $ye^{xy}dx+\left(xe^{xy}+2y\right)dy=0$

135. $\left(1+e^{\frac{x}{y}}\right)dx+\left(1-\frac{x}{y}\right)e^{\frac{x}{y}}dy=0$

136. $\left(y^2e^{xy^2}+4x^3\right)dx+\left(2xye^{xy^2}-3y^2\right)dy=0$

137. $\left(x^4e^x-2mxy^2\right)dx=2mx^2ydy$

138. $[\cos x\tan y+\cos(x+y)]dx+\left[\sin x\sec^2 y+\cos\left(x+y\right)\right]dy=0$

139. $xdy-ydx-3x^2y^2e^{y^3}dy=0$

140. $xdy-ydx=\left(x^2+y^2\right)dx$

141. $xdy-ydx=xy^2dx$

142. $\frac{dy}{dx}=\frac{x^2+y^2+1}{2xy}$

143. $\left(xy^2+2x^2y^3\right)dx+\left(x^2y-x^3y^2\right)dy=0$

144. $xe^{x^2+y^2}dx+y\left(e^{x^2+y^2}+1\right)dy=0$, y(0)=0

145. $\left(x\tan\frac{y}{x}-y\sec^2\frac{y}{x}\right)dx+\sec^2\frac{y}{x}dy=0$

146. $\left(2xy\cos x^2-2xy+1\right)dx+\left(\sin x^2-x^2\right)dy=0$

147. $xdy-ydx=x\sqrt{x^2-y^2}dx$

148. $\left(2x^2y-3y^4\right)dx+\left(3x^3+2xy^3\right)dy=0$

149. Find the integrating factor of the equation $\left(xy^2-e^{\frac{1}{x^3}}\right)dx-x^2ydy=0$ and solve it.

150. Show that x^ay^b is an integrating factor of the differential equation $\left(a+1\right)ydx+\left(b+1\right)xdy=0$ and hence solve it.

151. Verify that the differential equation $xdx+ydy=\frac{a^2\left(xdy-ydx\right)}{x^2+y^2}$ is exact and solve it.

152. Verify that the differential equation $\left(2x+e^{x}\sin y\right)dx+e^{x}\cos y\,dy=0$ is exact.

153. Show that the equation $\left(\cot y+x^{2}\right)dx=x\operatorname{cosec}^{2}y\,dy$ is exact and solve it.

Solve the following differential equations 154-214:

154. $x^{2}\left(\frac{dy}{dx}\right)^{2}+3xy\frac{dy}{dx}+2y^{2}=0$

155. $xy\left(\frac{dy}{dx}\right)^{2}-\left(x^{2}+y^{2}\right)\frac{dy}{dx}+xy=0$

156. $yp^{2}+\left(x-y\right)p-x=0$

157. $\frac{dy}{dx}-\frac{dx}{dy}=\frac{x}{y}-\frac{y}{x}$

158. $x^{2}\left(\frac{dy}{dx}\right)^{4}+2x\frac{dy}{dx}-y=0$

159. $y-2px=\tan^{-1}\left(xp^{2}\right)$

160. $y^{2}\log y=pxy+p^{2}$

161. $p^{3}-4pxy+8y^{2}=0$

162. $p\left(p+y\right)=x\left(x+y\right)$

163. $p^{2}+2py\cot x=y^{2}$

164. $y+px=x^{4}p^{2}$

165. $p=\tan\left(x-\frac{p}{1+p^{2}}\right)$

166. $\left(y-px\right)\left(p-1\right)=p$

167. $p=\log\left(px-y\right)$

168. $p^{2}\left(x^{2}-1\right)-2pxy+y^{2}-1=0$

169. $e^{3x}\left(p-1\right)+p^{3}e^{2y}=0$

170. $(y+px)^2 = x^2 p$

171. $\sin px \cos y = \cos px \sin y + p$

172. $(px-y)(py+x) = 2p$

173. $y = 3x + \log p$

174. $\frac{dy}{dx} + y\cot x = 5e^{\cos x}, y\left(\frac{\pi}{2}\right) = 4$

175. $\frac{dy}{dx} + y\cot x = 4x\cos ecx, y\left(\frac{\pi}{2}\right) = 0$

176. $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$

177. $(1+x)\frac{dy}{dx} - y = e^x(x+1)^2$

178. $\sqrt{1-y^2}\,dx = (\sin^{-1}y - x)\,dy$

179. $(1+y^2) + \left(x - e^{-\tan^{-1}y}\right)\frac{dy}{dx} = 0$

180. $\tan y\frac{dy}{dx} + \tan x = \cos y \cos^2 x$

181. $x\frac{dy}{dx} + y = e^{-x} - xy$

182. $(x+2y^3)\frac{dy}{dx} = y$

183. $(x+y)\,dy = a^2dx$

184. $x\log x\frac{dy}{dx} + y = (\log x)^2$

185. $(1-x^2)\frac{dy}{dx} + 2xy = x\sqrt{1-x^2}$

186. $\sin y\frac{dy}{dx} + \tan x = \cos x\left(2\cos y - \sin^2 x\right)$

187. $x\frac{dy}{dx}+y=x^3y^4$

188. $\frac{dy}{dx}-\frac{y}{2y\log y+y-x}=0$

189. $e^y\frac{dy}{dx}=e^x\left(e^x-e^y\right)$

190. $\left(2x\log x-xy\right)dy+2ydx=0$

191. $\frac{dy}{dx}-y\tan x=-y^2\sec^2 x$

192. $\left(xy^2-e^{\frac{1}{x^3}}\right)dx-x^2ydy=0$

193. $y-\cos x\frac{dy}{dx}=y^2\left(1-\sin x\right)\cos x, y\left(0\right)=2$

194. $y\left(2xy+e^x\right)dx=e^xdy$

195. $\left(x^4e^x-2mxy^2\right)dx+2mx^2ydy=0$

196. $\frac{dy}{dx}+\frac{y}{x}\log y=\frac{y}{x^2}\left(\log y\right)^2, \quad y\left(1\right)=2$

197. $\frac{dy}{dx}-x^2y=y^2e^{\frac{-x^3}{3}}$

198. $e^y\left(\frac{dy}{dx}+1\right)=e^x$

199. $\left(D^2+1\right)^3\left(D^2+D+1\right)^2 y=0$

200. $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=0, \quad y\left(0\right)=0, y'\left(0\right)=0$

201. $\frac{d^4y}{dx^4}-y=\cos x\cosh x$

202. $\frac{d^2y}{dx^2}+y=e^{2x}+\cosh 2x+x^3$

203. $\frac{d^2y}{dx^2}+a^2y=\sec ax$

204. $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=2e^x\cos\frac{x}{2}$

205. $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=e^{e^x}$

206. $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=\sin(e^x)$

207. $\frac{d^2y}{dx^2}+4y=x\sin x$

208. $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x\sin x$

209. $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+3y=\sin 3x.\cos 2x$

210. $\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=2^{x+3}+\cos(x+3)$

211. $\frac{d^2y}{dx^2}-y=3^x$

212. $\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}+\frac{dy}{dx}=x^2e^x+\sin^2 x$

213. $(D-1)^2(D+1)^2\,y=\sin^2\frac{x}{2}+e^x+x$

214. $\frac{d^2y}{dx^2}+\frac{dy}{dx}=x^2+2x+4$

Solve the following differential equation by the method of variation of parameters:

215. $\frac{d^2y}{dx^2}+4y=\sec 2x$

216. $\frac{d^2y}{dx^2}+16y=32\sec 2x$

217. $\frac{d^2y}{dx^2}+y=x\sin x$

218. $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=\frac{e^{3x}}{x^2}$

219. $\frac{d^2y}{dx^2}+y=\cos ecx$

Solve the following differential equations220-225-:

220. $\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}=\frac{12\log x}{x^2}$

221. $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=\log x\sin(\log x)$

222. $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+8y=65\cos(\log x)$

223. $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+2y=x\log x$

224. $(1+2x)^2\frac{d^2y}{dx^2}-6(1+2x)\frac{dy}{dx}+16y=8(1+2x)^2$

225. $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=\sin 2[\log(1+x)]$