QUESTION BANK

Digital Signal Processing

PART-A (2 Mark Questions) Unit-I

1. Differentiate between a static system and a dynamic system.
2. Draw the basic block diagram of a DSP system. Give two advantages of digital signal pro- cessing over analog signal processing.
3. What are energy and power signals? Give examples for each.
4. Define a causal system. Determine the given system is causal or non-casual y(n)=x(-n+2).
5. Give the general form of difference equation of an Nth order linear time invariant (LTI) discrete time system. Define natural response and forced response.
6. Evaluate the step response for the LTI system represented by the following impulse re- sponse. $h(n) = \delta(n) - \delta(n-1)$
7. Define a stable system. Determine the range of 'a' for which the system is stable when the given impulse response of a system is $h(k) = a^k u(k)$.
8. Define a linear system. Show that the discrete time system described by the input-output relationship y(n)=nx(n) is linear?
9. Determine if a discrete time signal described by the input output relation y(n)= x(n)coswn is time invariant.
10. Determine whether the system described by the input-output equation y(n)=y(n-1)+x(n) is a BIBO stable?

PART-A (2 Mark Questions) Unit-II

1. What is the basic difference between the Fourier series representations for continuous-time and discrete-time periodic signals?
2. What is the relationship between Fourier series coefficients of a periodic sequence and DFT.
3. An input sequence x(n)={2,1,0,1,2} is applied to a DSP system having an impulse sequence h(n)={5,3,2,1}. Determine the output sequence produced by linear convolution.
4. Given two sequences of length N=4 defined by x_1 (n) ={1, 2, 2, 1} and x_2 (n) ={2, 1, 1, 2}, determine the output sequence produced by circular convolution.
5. Give the number of multiplications and additions required for computation of N-point DFT by expressional method and FFT method respectively. Calculate the same for a sequence of length N=1024.
6. Distinguish between linear and circular convolution of two sequences.
7. State time reversal property and circular time shift property of DFT.
8. Calculate the 4-point DFT of a sequence x(n)={1,1,0,0}.
9. Differentiate between DIT and DIF algorithm used to compute DFT of a sequence.
10. What is zero padding? Mention its importance in signal processing.

PART-A (2 Mark Questions) Unit-III

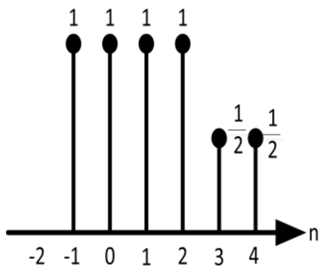
1. Given $X(Z) = \frac{Z}{(Z2 - 5Z + 6)}$, Sketch the region of convergence (ROC) if the sequence x(n) is a two sided sequence.
2. Determine the Z-transform and region of convergence (ROC) of the signal x(n)=[3(2n)-(3n)]u(n).
3. Define ROC with respect to Z-transform. Mention the properties of region of convergence.
4. Determine Z-transform and ROC of the finite-duration signal x(n)={1,2,5,7,0,1}
5. Calculate the unit sample response of the system described by the following difference equation y(n)=0.5y(n-1)+2x(n).
6. With reference to Z-transform, state the initial and final value theorem.
7. State the scaling and time reversal properties of Z-transform. Also give the details of corresponding ROC.
8. Calculate the Z-transform of even and odd components of a signal.
9. Define system function. Calculate the system function of a system described by the equation y(n)=x(n)+3x(n-1)+2y(n-1)-y(n-2)
10. What are the different types of realizations available? List out the basic building blocks of realization structures.

PART-A (2 Mark Questions) Unit-IV

1. What are the properties of butterworth filter? Give the equation for the order 'N' of butterworth filter.
2. Name the different types of window functions that are used to design FIR filters. Which window has the smallest ripple i.e. peak amplitude of sidelobes.
3. How phase distortions and delay distortions are introduced in filter characteristics?
4. Why impulse invariant method is not preferred in the design of IIR filters other than low pass filter?
5. What is meant by frequency warping? What is the cause of this effect?
6. Under what conditions a finite duration sequence h(n) will yield constant group delay in its frequency response characteristics and not the phase delay?
7. List out the applications for which the symmetrical impulse response can be used?
8. List out few advantages and disadvantages of Finite Impulse Response (FIR) filters.
9. State the condition for a digital filter to be causal and stable?
10. What is a Kaiser window? In what way is it superior to other window functions?

PART-B (5 Mark Questions) Unit-I

1. A discrete time domain signal x(n) is as shown in the Figure bellow. Based on this signal plot the following signals.



(a) x(n-2)   (c) x(n+2)   (e) $x(n-1)\delta(n-3)$

(b) x(4-n)   (d) x(n)u(2-n)

2. Find the even and odd parts of the following signal. Is this decomposition unique ?

$$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$$

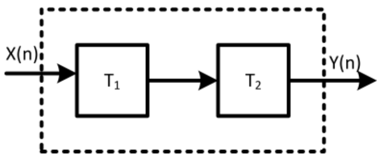
3. Examine the following systems with respect to the basic properties of the systems.

(a) y(n)=cos x(n)   (d) y(n) = x(n)u(n)

(b) y(n)=x(-n+2)   (e) y(n)=x(2n)

(c) y(n) =|x(n)|

4. Two discrete systems T_1 and T_2 are connected in cascade to form a new system. Prove the following statements.



(a) If T_1 and T_2 are linear then ‘ T ’ is also linear.

(b) If T_1 and T_2 are TIV then ‘ T ’ is TIV.

(c) If T_1 and T_2 are causal, then `T ' is casual.

(d) If T_1 and T_2 are LTI, then ‘ T ' is LTI.

(e) If T_1 and T_2 are LTI, by interchanging them ‘ T ’ holds the same.

5. Let ‘ T ‘ be an LTI , relaxed , and BIBO stable system with input x(n) and output y(n). Show that

(a) If x(n) is periodic with period N, the output y(n) tends to a periodic signal with the same period.

(b) If x(n) is an energy signal , the output y(n) will also be an energy signal.

6. The following input-output pairs have been observed during the operation of TIV systems

$$x_1(n) = \{\underset{\uparrow}{1}, 0, 2\} \xrightarrow{T} y_1(n) = \{0, 1, 2\}$$

$$x_2(n) = \{\underset{\uparrow}{0}, 0, 3\} \xrightarrow{T} y_2(n) = \{0, \underset{\uparrow}{1}, 0, 2\}$$

$$x_3(n) = \{\underset{\uparrow}{0}, 0, 0, 1\} \xrightarrow{T} y_3(n) = \{1, \underset{\uparrow}{2}, 1\}$$

What is the conclusion of the system . Find the IR of the system.

7. If $y(n) = x(n) * h(n)$, find the convolution of the following signals.

$$x(n) = \left(\frac{1}{2}\right)^n u(n) \quad h(n) = \left(\frac{1}{4}\right)^n u(n)$$

8. Find the zero-Input response of the system with impulse response $h(n) = a^n u(n)$ to the input signal x(n)=u(n)-u(n-10).

9. Find the step response of the LTI system if the impulse response is $h(n) = a^{-n}u(-n); \quad 0 < a < 1$

10. Consider the linear constant-coefficient difference equation

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = 2x(n-1)$$

Determine y(n) for $n \geq 0$, when $x(n) = \delta(n)$ and y(n)=0;n<0.

11. A casual linear LTI system is described by the difference equation

$$y(n) - 5y(n-1) + 6y(n-2) = 2x(n-1)$$

(a) Find the homogeneous response of the system

(b) Find the step response of the the system

12. Find the frequency response $H(e^{jw})$ of the LTI system whose input-output satisfy the difference equation

$$y(n) - \frac{1}{2}y(n-1) = x(n) + 2x(n-1) + x(n-2)$$

13. Consider the following difference equation

$$y(n) - \frac{5}{6}y(n-1) + \frac{1}{6}y(n-2) = \frac{1}{3}x(n-1)$$

What are the impulse response, frequency response and step response of the system.

14. Consider a system with input x(n) and output y(n) that satisfy the difference equation.

    $y(n)=ny(n-1)+x(n)$

    (a) If $x(n) = \delta(n)$, find the y(n)

    (b) Is this system LTI

15. Consider the following difference equation

$$y(n) - \frac{1}{4}y(n-1) - \frac{1}{8}y(n-2) = 3x(n)$$

    Find the general form of the homogeneous solution to this difference equation.

16. Consider the following difference equation

$$y(n) + \frac{1}{5}y(n-1) - \frac{2}{5}y(n-2) = x(n)$$

    Find the general form of homogeneous solution to this equation.

17. Find the zero-state response of the following systems.

$$y(n) - 0.5y(n-1) = \cos\frac{n\pi}{2}$$

18. Input-output relationship of a system may be defined as $y(n) - \frac{1}{2}y(n-1) = x(n)$ with $y(-1)=-1$. Find the response due to the following inputs.

$$x(n) = (0.5)^n\cos(0.5n\pi)u(n)$$

19. It is known that the response of the system $y(n) + \alpha y(n-1) = x(n)$ is given by $y(n) = [5 + 3(0.5)^n]u(n)$

    (a) Identify the natural response and forced response

    (b) Identify the values of α and y(-1)

    (c) Identify zero-state and zero input

    (d) Identify the input x(n)

20. It is know that the response of the system $y(n)+0.5y(n-1)=u(n)$ and given as

$$y(n) = [5(0.5)^n + 3(-0.5)^n]u(n)$$

    (a) Identify zero input and zero-state response

    (b) What is the zero input response of the system $y(n) + 0.5y(n-1) = x(n)$ if $y(-1)=10$

21. Classify the following systems

    (a) $y(n) = x\left(\frac{n}{3}\right)$

(b) $y(n) = \cos n\pi\, x(n)$

(c) $y(n) = (1 + \cos n\pi)x(n)$

(d) $y(n) = \cos[n\pi\, x(n)]$

(e) $y(n) = \cos[n\pi + x(n)]$

PART-B (5 Mark Questions) Unit-II

1. The first five points of the eight point DFT of a real-valued sequence are $\{0.25,\ 0.125 - j0.3018,\ 0,\ 0.125 - j0.0518,\ 0\}$. Determine the remaining three points.

2. Compute the eight-point circular convolution for the following sequence

$$x_1(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$$

$$x_2(n) = \sin\frac{3\pi}{8}n; \quad 0 \le n \le 7$$

3. Let X(k), $0 \le k \le N-1$, be the N-point DFT of the sequence x(n), $0 \le n \le N-1$ we define

$$\tilde{X}(k) = \begin{cases} X(k), & 0 \le k \le k_c,\ N-k_c \le k \le N-1 \\ 0, & k_c < k < N-k_c \end{cases}$$

and we compute the inverse N-point DFT of $\tilde{X}(k)$, $0 \le k \le N-1$. What is the effect of this process on the sequence. Explain.

4. Determine the circular convolution of the sequences $x_1(n) = \{1, 2, 3, 1\}$, $x_2(n) = \{4, 3, 2, 2\}$ using the time-domine formula.

5. Consider a finite duration sequence $x(n) = \{0, 1, 2, 3, 4\}$

   (a) Sketch the sequence s(n) with six-point DFT $S(k) = W_2^{*}\ X(k)$ k=0, 1,...6

   (b) Sketch the sequence y(n) with six-point DFT Y(k) = Re|X(k)|

   (c) Sketch the sequence v(n) with six-point DFT V(k) = Im|X(k)|

6. Consider the sequences $x_1(n) = \{0, 1, 2, 3, 4\}$, $x_2(n) = \{0, 1, 0, 0, 0\}$,$x_3(n) = \{1, 0, 0, 0, 0\}$ and their 5 point DFT.

   (a) Determine a sequence y(n) so that $Y(k) = X_1(k)\ X_2(k)$

   (b) Is there a sequence $x_3(n)$ such that $S(k) = X_1(k)\ X_3(k)$

7. Determine the eight-point DFT of the signal x(n)={1, 1, 1, 1, 1, 1, 0, 0 } and sketch its magnitude and phase.

8. Let x(n) be an N-point real sequence with N-point DFT X(k)[N even]. In addition , x(n) satisfies the following symmetry property.

$$x\left(n+\frac{N}{2}\right) = -x(n); \quad n = 0, 1, 2, \ldots \frac{N}{2} - 1$$

   (a) Show that X(k)=0; k= even .

   (b) Show that the values of this odd harmonic spectrum can be computed by equating the N/2 point DFT of a complex modulated version of the original sequence x(n).

9. (a) Determine the Fourier Transform $X(\omega)$ of the signal $x(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1, 0\}$

(b) Compute the 6-point DFT V(k) of the signal $v(n)=\{3, 2, 1, 0, 1, 2\}$

(c) Is there any relation between x(n) and v(n).

10. Suppose we have two-point sequence x(n) and h(n) as follows.

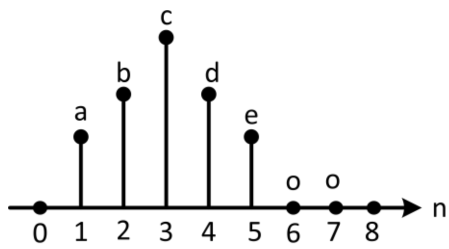
$$x(n) = \cos\frac{\pi n}{2} \; ; \quad n = 0, 1, 2, 3$$

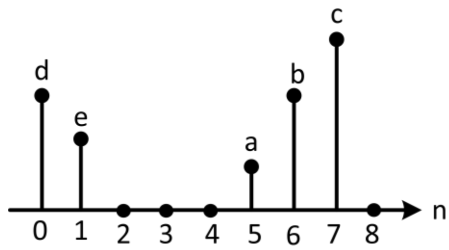
$$h(n) = 2^n; \quad n = 0, 1, 2, 3$$

(a) Calculate the 4-point DFT of x(n) and h(n)

(b) Also calculate the convolution directly and using Inverse DFT.

11. Two eight point sequence $x_1(n)$ and $x_2(n)$ shown in the Figure below. Their DFTs $X_1[k]$ and $X_2[k]$. Find the relationship between them.





PART-B (5 Mark Questions) Unit-III

1. Determine the z-transform of the following signals and sketch the corresponding pole-zero patterns

   (a) $x(n) = (1 + n)u(n)$

   (b) $x(n) = (na^n \sin\omega_0 n)u(n)$

   (c) $x(n) = \left(\frac{1}{2}\right)^n [u(n) - u(n -$

2. Determine the z-transform of the following signals

   (a) $x(n) = n(-1)^n u(n)$

   (b) $x(n) = n^2 u(n)$

   (c) $x(n) = (-1)^n u(n)$

3. Compute the convolution of the following signals by means of z-transform

$$x_1(n) = \begin{cases} \left(\frac{1}{3}\right)^n ; & n \geq 0 \\ \left(\frac{1}{2}\right)^{-n} ; & n < 0 \end{cases}$$

$$x_2(n) \quad \left(\frac{1}{2}\right)^n u(n)$$

4. The z-transform X(z) of a real signal x(n) includes a pair of complex-conjugate zeros & pair of complex conjugate poles. What happens to these pairs if we multiply x(n) by $e^{j\omega_0 n}$?

   (Hint: Use the scaling theorem in the z-domain)

5. Using long division , determine the inverse z-transform of

$$X(z) = \frac{1 + 2z^{-1}}{1 - 2z^{-1} + z^{-2}}$$

   if (a) x(n) is causal and

   (b) x(n) is anticausal

6. Let x(n) be a sequence with z-transform X(z). Determine in terms of X(z) the z-transform of the following signals

   (a) $x_1(n) = \begin{cases} x\left(\frac{n}{2}\right), & \text{if n is even} \\ 0, & \text{if n odd} \end{cases}$

   (b) $x_2(n) = x(2n)$

7. Determine the causal signal x(n) if its z-transform X(z) is given by

(a) $X(z) = \dfrac{1 + 3z^{-1}}{1 + 3z^{-1} + 2z^{-2}}$

(b) $X(z) = \dfrac{z^{-6} + z^{-7}}{1 - z^{-1}}$

8. Determine all possible signals x(n) associated with z-transform

$$X(z) = \frac{5z^{-1}}{(1 - 2z^{-1})(3 - z^{-1})}$$

9. Determine the convolution of the following pairs of signals by means of z-transform

$$x_1(n) = \left(\frac{1}{4}\right)^n u(n-1), \quad x_2(n) = \left[1 + \left(\frac{1}{2}\right)^n\right] u(n)$$

10. Prove that final value theorem for one-sided z-transform.

11. If X(z) is z-transform of x(n) show that

(a) $z[x^*(n)] = X^*(z^*)$

(b) $z[Re[x(n)]] = \frac{1}{2}[X(z) + X^*(z^*)]$

12. (a) Draw the pole-zero pattern for signal $x_1(n) = (r^n \sin\omega_0 n)u(n) \quad 0 < r < 1$

(b) Compute z-transform $X_2(z)$, which corresponds to pole-zero pattern in part(a)

(c) Compare $X_1(z)$ and $X_2(z)$ are they identical? If not,. indicate a method to derive $X_1(z)$ from pole-zero pattern.

13. Show that the roots of a polynomial with real coefficients are real or form complex conjugate pairs. The inverse is not true, in general.

14. Determine the signal x(n) with z-transform $X(z) = e^z + e^{1/z} \quad |z| \neq 0$

15. Determine, in closed form the causal signals x(n) whose z-transform are given by

(a) $X(z) = \dfrac{1}{1 + 1.5z^{-1} - 0.5z^{-2}}$

(b) $X(z) = \dfrac{1}{1 - 0.5z^{-1} + 0.6z^{-2}}$

Partially check your result by computing x(0), x(1), x(2) and x(∞) by an alternative method.

16. Determine all the possible signals that can have following Z-transforms

(a) $X(z) = \dfrac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$

(b) $X(z) = \dfrac{1}{1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}}$

17. Determine the signal x(n) with Z-transform

$$X(z) = \frac{1}{1 - \frac{10}{3}z^{-1} + z^{-2}} \quad \text{if } X(z) \text{ converges on unit circle}$$

18. Prove that the fibonacci series can be thought of as the impulse response of the system described by the difference equation

$$y(n) = y(n-1) + y(n-2) + x(n)$$

Then determine h(n) using Z-transform techniques.

19. Use the one-sided Z-transform to determine y(n), $n \geq 0$ in the following cases.

(a) $y(n) + y(n-1) - \frac{1}{4}y(n-2) = 0;\ y(-1) = y(-2) = 1$

(b) $y(n) - 1.5y(n-1) + 0.5y(n-2) = 0;\ y(-1) = 1;\ y(-2) = 0$

20. Compute zero-state response of the following pairs of system and input signals.

(a) $h(n) = \frac{1}{3}^{\,n} u(n),\ x(n) = \frac{1}{2}^{\,n} \cos\frac{\pi}{3}n\ \ u(n)$

(b) $h(n) = \frac{1}{2}^{\,n} u(n),\ x(n) = \frac{1}{3}^{\,n} u(n) + \frac{1}{2}^{\,-n} u(-n-1)$

PART-B (5 Mark Questions) Unit-IV

1. Design a FIR linear phase , digital filter approximating the ideal frequency response

$$H_d(\omega) = \begin{cases} 1, & \text{for} \quad |\omega| \le \frac{\pi}{6} \\ 0, & \text{for} \quad \frac{\pi}{6} < |\omega| \le \pi \end{cases}$$

   (a) Determine the coefficient of a 25-tap filter based on the window method with a rectangular window.

   (b) Determine and plot the magnitude and phase response of the filter.

   (c) Repeat part (a) and (b) using Hamming window.

   (d) Repeat part (a) and (b) using Barlett window.

2. Repeat Problem 1 for a band stop filter having the ideal response

$$H_d(\omega) = \begin{cases} 1, & \text{for} \quad |\omega| \le \frac{\pi}{6} \\ 0, & \text{for} \quad \frac{\pi}{6} < |\omega| \le \pi \\ 1, & \text{for} \quad \frac{\pi}{3} < |\omega| \le \pi \end{cases}$$

3. Re-design the filter of problem 1 using Hanning and Blackman window.

4. Re-design the filter of problem 2 using Hanning and Blackman window.

5. Determine the unit sample response $\{h(n)\}$ of a linear phase FIR filter of length M=4. For which the frequency response at $\omega = 0$ and $\omega = \frac{\pi}{2}$ is specified as

$$H_r(0) = 1, \quad H_r\left(\frac{\pi}{2}\right) = \frac{1}{2}$$

6. Determine the coefficients of $\{h(n)\}$ of a linear-phase FIR filter of length M=15 which has a symmetric unit sample response and a frequency response that satisfies the condition

$$H_r\left(\frac{2\pi k}{15}\right) = \begin{cases} 1, & k = 0, 1, 2, 3 \\ 0, & k = 4, 5, 6, \end{cases}$$

7. Repeat the filter design problem in 6 with frequency response specifications.

$$H_r\left(\frac{2\pi k}{15}\right) = \begin{cases} 1, & k = 0, 1, 2, 3 \\ 0.4 & k = 4 \\ 0, & k = 4, 5, 6, 7 \end{cases}$$

8. Convert the analog bandpass filter designed in to a digital filter by means of a bilinear transformation. There by derive the digital filter characteristics obtained by the alternative approach and verify that the bilinear transformation applied to the analog filter results in the same digital bandpass filter.

9. An ideal analog integrator is described by the system function $H_a(s) = \frac{1}{s}$. A digital integrator with system function H(z) can obtained by use of the bilinear transformation that is

$$H(z) = \frac{T}{2}\frac{1+z^{-1}}{1-z^{-1}} \equiv H_a(s)\Big|_{s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}}$$

   (a) Write the difference equation for the digital integrator relating the input x(n) to the output y(n).

   (b) Roughly sketch the magnitude $|H_a(j\Omega)|$ and phase θ() of the analog integrator.

   (c) It is easily verified that the frequency response of the digital integrator is

$$H(\omega) = -j\frac{T}{2}\frac{\cos(\omega/2)}{\sin(\omega/2)} = -j\frac{T}{2}\cot(\omega/2)$$

   Roughly sketch $|H(\omega)|$ and $\theta(\omega)$

   (d) Compare the magnitude and phase characteristics obtained in part(a) and part (b) how well does the digital integrator match the magnitude and phase characteristics of the analog integrator.

   (e) The digital integrator has a pole at z=1. If you implement this filter on a digital computer, what restrictions might you place on the input signal sequence x(n) to avoid computational difficulties.

10. Consider the pole-zero plot shown in below Figure

   (a) Does it represent an FIR filter

   (b) Is it a linear -phase system.

11. A digital low-pass filter is required to meet the following specifications:

Pass band ripple	:	≤ 1
pass band edge	:	4KHz
Stop band attenuation	:	≥ 40dB
Stop band edge	:	6KHz
Sample rate	:	24KHz

The filter is to be designed by performing a bilinear transformation on an analog system function. Determine what order butter worth, chebysher and elliptic analog designs must be used to meet the specifications in the digital implementations.

12. An IIR digital lowpass filter is required to meet the following specifications:

Pass band ridple	:	≤ 0.5dB
pass band edge	:	1.2KHz
Stop band attenuation	:	$\geq$ 40dB
Stop band edge	:	2.0KHz
Sample rate	:	8.0KHz

Use the design formulas in the book to determine the required filter order for

(a) A digital butterworth filter

(b) A digital chebyshev filter

(c) A digital elliptic filter

13. Determine the system function H(z) of hte lowest-order chebyshev digital filter that meets the following specifications:

(a) 1-dB ripple in the passband $0 \leq |\omega| \leq 0.3\pi$

(b) At least 60dB attenuation in the stopband $0.35\pi \leq |\omega| \leq \pi$. Use the bilinear transformation

14. Determine the system function H(z) of hte lowest-order chebyshev digital filter that meets the following specifications:

(a) 0.5dB ripple in the passband $0 \leq |\omega| \leq 0.3\pi$

(b) At least 50dB attenuation in the stopband $0.35\pi \leq |\omega| \leq \pi$. Use the bilinear transformation

15. Consider a causal continuous time system with impulse response $h_c(t)$ and system function.

$$H_c(s) = \frac{s+9}{(s+a)^2+b^2}$$

(a) Use impulse invariance to determine $H_1(z)$ for a discrete time systems such that $h_1[n] = h_c(nT)$.

(b) Use step invariance to determine $H_2(z)$ for a discrete time system such that $S_2[n] = S_c(nT)$, where

$$S_2[n] = \sum_{k=-\infty}^{n} h_2[k] \quad \text{and} \quad S_c(t) = \int_{-\infty}^{t} h_c(\tau)\,d\tau$$

(c) Determine the step response $S_1[n]$ of system 1 and the impulse response $h_2[n]$ of system 2. Is it true that $h_2[n] = h_1[n] = h_c(nT)$?. Is it is true that $S_1[n] = S_2[n] = S_c(nT)$?